Ergodic Throughput in Coordinated Multi-point Downlink under Distributed Power Constraints

Mahesh Nagulapalli†, Mahendra Pratap Singh†, Anil Kumar‡, Sibi Raj B Pillai†
†Department of Electrical Engineering, Indian Institute of Technology Bombay, Mumbai, India.
maheshnagulapalli718@gmail.com, 24m1226@iitb.ac.in, bsraj@ee.iitb.ac.in
‡INSA, INRIA, Lyon, France. anil.kumar@inria.fr

***Abstract*—Coordinated multi-point systems comprise distributed access points (APs) serving a common downlink user. Although the transmissions here are centrally coordinated, the non-colocated APs may have to respect different power constraints, thus necessitating novel power control and beamforming solutions. Numerical solutions that maximize the throughput of a fixed fading matrix under diverse distributed power constraints have appeared in the recent past. However, the time varying fading model where each AP adapts its transmit power and beamforming vector based on the channel matrix, in order to maximize the ergodic throughput, has received much less attention. Note that here each AP should respect its own specified long-term average power constraint. This paper proposes novel power control and beamforming solutions that not only respect the distributed power constraints but also achieve near-optimal ergodic throughput in popular fading models.**

I. Introduction

In small cell wireless scenarios, it is increasingly common for several non-colocated access points (APs) to serve a common downlink user. The availability of good backhaul connectivity facilitates coordinated transmissions; however, the distributed APs may have to respect different energy consumption requirements. This calls for per-AP power constraints. Note that an AP may host multiple transmit antennas, and the same applies to the receiving user equipment (UE) as well. The distributed constraints here make the communication solutions differ from that of standard point to point MIMO. For the latter, appropriate waterfilling of the singular values of the fading matrix achieves capacity, for a fixed fading channel, as well as in the ergodic time-varying setup [1].

Due to the importance of distributed constraints in modern wireless systems, several works in the past decade focused on characterizing the capacity of such systems, see [2], [3], [4], [5], [6] and the references therein. All these works primarily considered the capacity of a fixed fading channel under distributed constraints. For example, [3] analyzes the capacity of a fixed fading matrix under per-antenna power constraints. The optimal solution is shown to satisfy an appropriate matrix equality whenever the transmit covariance is of full rank. Notice that the full rank condition is not on the channel matrix, but on the transmit covariance. The rank of the transmit covariance is related to the number of independent parallel data streams that can be supported [1]. It should be noted that even when the channel is full rank, the optimal transmit covariance may be rank-deficient to facilitate opportunistic schemes that exploit the unequal singular values of the channel, particularly so at low average power availability. This problem is exacerbated in an ergodic setup in which the opportunistic communication should also target the higher singular values across the time variations of the channel.

It is evident that the solutions for fixed fading channel under per-antenna power constraints may not easily generalize to ergodic setups. Furthermore, the channel itself is rank deficient when the number of transmit antennas N is higher than the number of receive antennas M. Recently, [6] has proposed a computational approach to evaluate the optimal covariance matrix under per-antenna and group power constraints, again for a fixed fading matrix. Adapting such a solution to widely employed time-varying fading models not only involves solving the power control problem for a continuum of matrix realizations, but also the power allocated in each realization is not given apriori, as only long-term average power constraints are considered.

In the specific case of a symmetric model where the fading distributions and average power per transmit antenna are identical, the ergodic capacity can be evaluated by replacing all the per-antenna constraints by a sum-power constraint [7]. However, with different power sources and energy availability, asymmetric APs are more the norm than the exception. Optimal power control in the ergodic setup under different per-AP power constraints was recently established for the MISO case, where the receiver has a single antenna [8]. The MISO case for the fixed fading setup was previously considered in [2], which also notes that in the absence of CSIT, per antenna power constraints can be easily accommodated in the known ergodic capacity evaluation, by employing independent inputs at each of the antennas. With modern wireless systems often hosting multiple receive antennas, it is imperative to identify near-optimal power adaptation schemes in MIMO systems, under different per-AP power constraints. The main contribution of the current paper is in proposing power control laws and beamforming solutions that not only respect the distributed power constraints at the APs, but also achieve near optimal average throughput in several relevant time-varying fading environments.

II. System Model and Distributed Power Control

We consider a downlink coordinated multi-point (CoMP) system where a collection of N_{ap} distributed APs jointly serve a downlink UE having M receive antennas. The APs are geographically separated but are interconnected through high-capacity backhaul links to a central coordination unit. This central entity enables coherent joint transmissions, facilitating the distributed APs to operate

as a virtual antenna array. However, equipped with an independent power supply and radio front end, each AP is subject to its own average power budget. This results in a collection of per-AP power constraints, in contrast to the classical sum-power assumption where all APs are assumed to share a common energy pool. The presence of distributed constraints significantly alters the optimal beamforming and power allocation strategies, necessitating new analytical and algorithmic approaches.

A. MIMO Channel Model

Let N denote the total number of transmit antennas, aggregated over the $N_{\rm ap}$ APs. The downlink model at the UE can be expressed as

$$\bar{y} = H\bar{x} + \bar{z}, \tag{1}$$

where $H \in \mathbb{C}^{M\times N}$ denotes the channel gain matrix between the N antenna virtual transmitter and the M antenna UE, $\bar{y} \in \mathbb{C}^{M\times 1}$ represents the UE received vector, and $\bar{x} \in \mathbb{C}^{N\times 1}$ is the transmitted signal vector aggregated across all APs. W.l.o.g we will index the transmit antennas as $1, 2, \ldots, N$. The term $\bar{z} \in \mathbb{C}^{M\times 1}$ models the additive white Gaussian noise (AWGN), assumed to be independent and identically distributed (IID) according to $\mathcal{CN}(0, \mathbf{I}_M)$, it is also independent of $(\bar{x}, H)$. While it is customary to identify random variables using bold face letters or other typesets, the underlying randomness in this paper is mostly clear from the context. Therefore, we simply use an overbar to identify vectors (as in $\bar{y}$), the randomness and appropriate distribution will be implicit. For a matrix B, the $(l,k)-$th entry will be denoted as B_{lk}. Furthermore, $\mathbf{I}_M$ will denote the $M\times M$ identity matrix, and $\mathbb{I}_{\{A\}}$ stands for the indicator function which takes unit value when A is true and zero otherwise.

The fading matrix is time-varying, we assume that the dyamic variations obey an underlying ergodic principle. Furthermore, the fading matrix is known to the transmitters as well as the receiver, i.e. system has both CSIT and CSIR. These assumptions are well justified in several practical systems that serve mobile terminals [1]. Although our results are true for more general fading models, in this paper we will focus mainly on Rayleigh fading, where the entries of the matrix H are assumed to be IID complex circularly symmetric Gaussian. Furthermore, the averages will be evaluated for IID realizations across time (or transmission blocks) as well, as the solutions will not change for any ergodic variation that maintains the same singular value distribution of the fading matrix.

B. Per-AP Power Constraints and Ergodic Capacity

Given a channel matrix H, the transmit vector $\bar{x}$ can be chosen as a function of H. In order to capture the power constraints succinctly, let us define the conditional input covariance matrix given H as $Q_H = \mathbb{E}[\bar{x}\bar{x}^\dagger|H]$. Under the ergodic channel variations, we can take $Q = \mathbb{E}[Q_H]$. A long-term average sum-power constraint can now be imposed as $\mathrm{tr}(Q) \le P_{\rm tot}$. As discussed earlier, since the APs are equipped with their own power sources, it is natural to constrain the average transmit power of each AP. Let S_k be the set of transmit antenna indexes that belong to AP k. In order to respect an average sum-power constraint of $P_{\rm Ak}$ at AP k, we then have

$$\sum_{i\in S_k} Q_{ii} \le P_{\rm Ak}, \quad k = 1, 2, \ldots, N_{\rm ap}. \tag{2}$$

Let $\mathcal{Q}$ be the class of covariance matrices such that (2) is satisfied. Given the above system description, the ergodic downlink capacity of the CoMP system with distributed power constraints is given by

$$C = \max_{Q_H:\mathbb{E}[Q_H]\in\mathcal{Q}} \mathbb{E}_H\left[\log\det\left(\mathbf{I}_M + HQ_HH^\dagger\right)\right], \tag{3}$$

where the maximization is over all feasible transmit covariance matrices Q_H satisfying the specified per-antenna power constraints. Notice that the objective function is nothing but the average mutual information $I(\bar{x};\bar{y}|H)$ achieved by using a Gaussian input distribution of covariance matrix Q_H, on observing the channel realization H.

In many systems, it is reasonable to assume that the fading distribution encountered by each transmit antenna of an AP is identical, i.e. link statistics are similar from each AP antenna. In this case, the long-term average per-antenna transmit power at AP k is identical across the set S_k. To substantiate this, note that swapping the labels among S_k will not lead to a change in the objective function for any feasible transmission scheme. The concavity of mutual information will now imply that the per-antenna average transmit powers are identical in S_k. This permits an immediate simplification of the per-AP power constraint, i.e. we can replace (2) by

$$Q_{ii} \le \frac{P_{A_k}}{|S_k|}, \ \text{ for } i \in S_k,\ k = 1, \ldots, N_{\rm ap}. \tag{4}$$

Taking $P_{ai} = \sum_{k=1}^{N_{\rm ap}} \frac{P_{A_k}}{|S_k|}\mathbb{I}_{\{i\in S_k\}}$, it is thus sufficient to only consider the per-antenna constraints $Q_{ii} \le P_{ai}, \forall i$.

C. Distributed Constraints and Coordinated Beamforming

In the case of identical per-antenna constraints, i.e. $P_{ai} = P_a, \forall i$, and identical fading links from each of the transmit antennas, the above optimization problem has a simple answer. In this case, the per-antenna power constraints can be replaced by a single constraint $\mathrm{tr}(Q) \le NP_a$, and the well-known optimal waterfilling solution will indeed allocate identical long-term average power for each transmit antenna [7]. However, the symmetry among the transmitters is lost in the presence of distributed constraints, and the optimization problem becomes more challenging.

Although (3) is a convex problem, the main challenges in solving it are two-fold. First, we need to specify an appropriate power constraint at each AP, conditioned on a given channel realization H. Once the AP power constraints are given, the optimal distributed beamforming directions for each fixed channel matrix and power constraint have to be identified. The latter itself is an important problem, and algorithmic solutions are usually required for the solutions [6]. However, solving such a problem for every channel realization is only possible for a small number of transmit and receive antennas. In order to illustrate, we plot the optimal throughput for a 2×2 MIMO system, encountering IID normalized Rayleigh fading, in Fig. 1. The optimization is performed using CVXPy, where the

long-term average power constraints at each of the two transmit antennas are in the ratio $\alpha P_{\text{tot}} : (1-\alpha)P_{\text{tot}}$, the long-term average sum-power being P_{tot}. For comparison, we also plot the ergodic capacity under the sum-power constraint, as an upper bound.

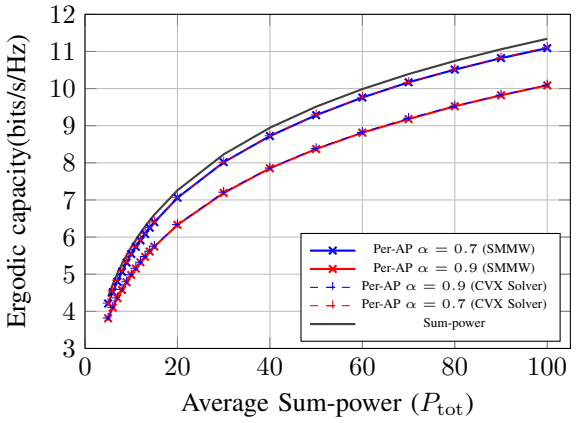

Fig. 1. Throughput for Asymmetric APs

Fig. 1 was obtained by first discretizing the fading distribution and then solving the discrete optimization problem with thousands of variables. Clearly, such computations are not tractable when there are many APs on the transmitter side, and alternative solutions are required. It is natural to ask whether such alternate schemes can achieve rates close to the ergodic capacity while respecting the power constraints. To answer this question, we have also plotted the rates obtained by a different scheme, which we call Singular Mode Multiview Waterfilling (SMMW), in Fig. 1. Remarkably, SMMW in this case, achieves almost the same rates as the optimal.

III. Singular Mode Multiview Waterfilling

Notice that the power control law should suggest suitable transmit symbols from each of the antennas for every given channel matrix, i.e., we have to specify $\bar{x}(H)$ such that the per AP power constraints are met in the long term. Using singular value decomposition, we can express $H = U\Lambda V^\dagger$, where U is a $N_r \times N_r$ unitary matrix and the $N_r \times N_t$ diagonal matrix $\Lambda = \text{diag}(\nu_1, \ldots \nu_r)$, with $r = \min\{M, N\}$. Since

$$U^\dagger \bar{y} = \Lambda V^\dagger \bar{x} + U^\dagger \bar{z}, \tag{5}$$

with $U^\dagger \bar{z}$ identically distributed as $\bar{z}$, we can equivalently propose the transmit vector $\bar{x}(\Lambda, V^\dagger)$ for each realization of $\Lambda V^\dagger = U^\dagger H$. Denoting the phase of a complex value c by $\Theta(c)$, let us define $\phi_{lk} = \exp(-j\Theta(V_{lk})), 1 \le l, k \le N$. To save notation, we suppressed the dependency of ϕ_{lk} on the matrix V.

Our strategy is to send r independent parallel data streams and compute the effective mutual information over the channel. We remind the reader that different APs coordinate and send each stream of data, i.e., the transmitted symbols across the different antennas may be correlated. Let us first consider the case $N \ge M$, i.e., the number of transmit antennas is greater. While this regime is more interesting for the distributed AP model under consideration here, our results have a natural extension to higher values of M, this will be discussed later. With $r = M$, let $\bar{d} = [d_1, \ldots, d_r]^T$ have IID Gaussian distributed entries according to $\mathcal{CN}(0, \mathbf{I}_r)$. The entries of the vector $\bar{d}$ represent data symbols that are generated from r Gaussian codebooks, to support r independent streams of messages using typical random coding arguments in information theory [1], [9]. On encountering $\Lambda V^\dagger$ as $U^\dagger H$, the transmitter will send the vector

$$\bar{x}(\Lambda, V^\dagger) = W\bar{d}, \tag{6}$$

where column i of the $N \times r$ matrix W is given by

$$\bar{w}_i = \begin{bmatrix} \sqrt{P_{1i}} & 0 & \cdots & 0 \\ 0 & \sqrt{P_{2i}} & \cdots & 0 \\ 0 & \cdots & 0 & \sqrt{P_{Ni}} \end{bmatrix} \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \\ \phi_{Ni} \end{bmatrix}. \tag{7}$$

Essentially, ϕ_{lk} in (7) nullifies the phase of V_{lk} encountered in the channel, whereas the diagonal power scaling matrix should not only facilitate opportunistic exploitation of the singular values, but also ensure that the per-antenna power constraints are met. We now propose a power scaling law that meets both objectives. For positive $\lambda_1, \ldots, \lambda_N$ (Lagrange parameters), define

$$P_{ki} = \frac{|V_{ki}|^2}{|V_{ki}|^2 + \sum\limits_{\substack{l=1 \\ l \ne k}}^{N} \frac{\lambda_l}{\lambda_k} |V_{li}|^2} \left[\frac{1}{\lambda_k} - \frac{1}{\nu_i^2 \sum\limits_{l=1}^{N} \frac{\lambda_l}{\lambda_k} |V_{li}|^2} \right]^+. \tag{8}$$

The parameters $\lambda_i, 1 \le i \le N$ are chosen to meet the per-antenna average power constraint specified in (4). We term this allocation the SMMW allocation to obtain a deeper insight into this otherwise clumsy-looking formula; an elegant interpretation is now in order. This will also justify the name SMMW.

Let us first consider the MISO model with $M = 1$ and $H = [h_1, h_2, \ldots, h_N]$. In this case $V = \frac{H^\dagger}{\|H\|}$, the lone singular value is $\nu_1 = \|H\|$, and the formula (8) becomes

$$P_k = \frac{|h_k|^2}{\sum_{l=1}^{N} \frac{\lambda_l}{\lambda_k} |h_l|^2} \left[\frac{1}{\lambda_k} - \frac{1}{\sum\limits_{l=1}^{N} \frac{\lambda_l}{\lambda_k} |h_l|^2} \right]^+. \tag{9}$$

Let us imagine that the transmit antenna k considers $\tilde{h}[k] = (\sqrt{\lambda_k})^{-1}(\sqrt{\lambda_1}h_1, \ldots, \sqrt{\lambda_N}h_N)$ as the effective MISO channel to accommodate the per-antenna power constraints. A water-filling power allocation for this modified channel is computed at antenna k, using $\|\tilde{h}[k]\|^2$. Then, a power proportional to the individual link gain is allocated to that antenna, based on the modified channel, as in (9). Thus, each transmit antenna creates its own view of the fading vector; this justifies the terminology of *Multiview Waterfilling*. The proposed power allocation scheme is indeed optimal for MISO [8] as stated below.

Lemma 1. *Choosing* $x_i = \exp(-j\Theta(h_i))\sqrt{P_i}d$, *with* $d \sim \mathcal{CN}(0,1)$ *maximizes* $I(\bar{x}; \bar{y}|H)$ *s.t.* $\mathbb{E}|x_i(H)|^2 \le P_i, \forall i$.

Proof: The proof can be found in Appendix A. ∎

Also, a MISO channel realization has only a single eigenmode, whereas the multiple singular values for MIMO channels can be exploited in SMMW. We can now interpret

(8) as a modified water-filling on each eigenmode of the observed MIMO channel H. Notice that the N Lagrange parameters correspond to the N per-antenna power constraints. If the constraint is on the per-AP power, the number of Lagrange parameters can be reduced. Recall that the per-AP constraint is specified as the maximum sum-power for a set S_k of the transmit antennas.

Lemma 2. *Under the scheme in* (8)*, it is sufficient to choose $\lambda_i = \lambda_j$ if both i and j belong to the set S_k, in order to maximize the ergodic throughput.*

The proof of Lemma 2 follows from the link symmetry with respect to the antennas in S_k and the convexity of the objective function; the details are omitted, see Section II-B. Therefore, the number of parameters to be determined in the power allocation scheme is the same as the number of APs. In the extreme case where there is only a sum-power constraint, the power allocation in (8) will become the well-known optimal water-filling with a single Lagrange parameter, i.e. $\lambda_i = \lambda, \forall i$. Let us now consider the case with more receive antennas.

For $M > N$, notice that at least $M - N$ singular values are zero. We can remove those entries in $U^\dagger \bar{y}$ that correspond to any set of $(M - N)$ zero singular values. The resulting system effectively becomes an $N \times N$ MIMO, with the input constraints remaining the same. However, the fading distribution has changed. Nevertheless, the results already proposed for $M = N$ will now yield the ergodic throughput for $M > N$ as well. We will now present numerical validation and further discuss the implications of the proposed SMMW allocation based on the numerical results.

IV. Numerical Analysis

In this section, we analyze the performance of the proposed SMMW allocation under different MIMO system configurations. The results will be presented in two flavors. In the first, we will consider the ergodic MIMO throughput under IID Rayleigh fading and distributed power constraints, which was indeed mentioned as the main focus of the current paper. Subsequently, we will present the achievable rates, using the SMMW allocation in (8), for some fixed channel MIMO configurations employed in previous works [6], and compare the performance against existing results.

A. Ergodic Rayleigh Fading

Let us consider the $M \times N$ channel matrix H to have independent normalized Gaussian entries, which corresponds to Rayleigh fading MIMO channels with N transmit antennas. For $M = N = 2$, we have already presented ergodic capacity results in Fig. 1 and discussed in Section II-B. Let us now consider $N = 4, M = 2$. The ergodic throughput results under a sum-power constraint and 4 different per-antenna constraints are demonstrated in Fig. 2. In the sum-power constraint case, the answer is already known, and the throughput of SMMW allocation indeed matches the optimal one, providing a quick verification of the calculations. Fig. 2 also shows the ergodic throughput when the per-antenna constraints are in the ratio $1.5 : 1.7 : 0.8 : 6$ and $0.0001 : 0.1599 : 0.24 : 6$.

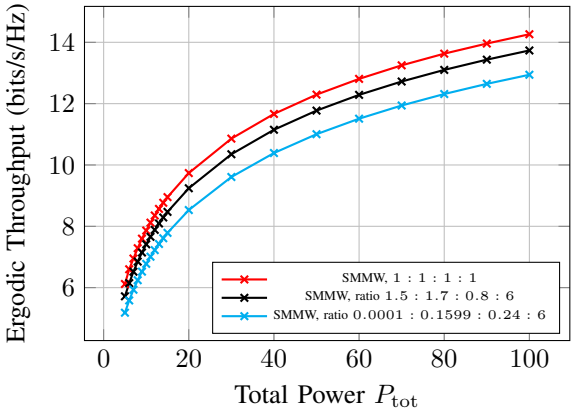

Fig. 2. 4×2 MIMO under per-antenna constraints.

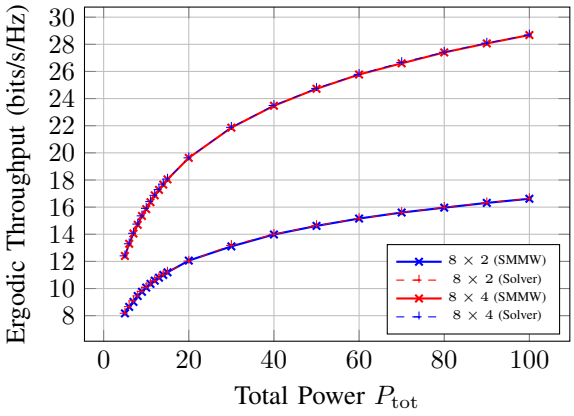

Fig. 3. 8×2 and 8×4 MIMO under per-antenna constraints.

In order to demonstrate the scalability, let us consider $N = 8$ and 2 antennas at the receiver. The results for this case are demonstrated in Fig. 3. Here also the throughput under a sum-power constraint results in optimal water-filling. Additionally, when the per-antenna constraints at antennas m and n are in the ratio $1 + n\epsilon : 1 + m\epsilon$ for all $1 \leq m, n \leq N$, the ergodic throughput is plotted against P_{tot} for $\epsilon = 0.1$.

Notice that the fading distribution does not explicitly appear in the allocation of (8). Rather, the Lagrange parameters are affected by the fading distribution, while calculating the per-antenna power consumption. Let us now consider an extreme case of the ergodic variation, i.e. a fixed channel with no variations.

B. Fixed Fading Matrix

While our allocation was motivated by water-filling for the ergodic setups, let us consider the fixed fading matrix as an extreme case with no time variations. Several results are already known for fixed H, primarily in terms of efficient algorithms to evaluate the capacity under distributed constraints, see [5], [6]. This provides an excellent opportunity to compare the performance of SMMW against established algorithms, though the existing algorithms cannot as such solve general time-varying ergodic problems. Quite surprisingly, SMMW performance is almost as good as the optimal for these fixed fading setups. In order to compare, let us employ the same fixed fading 4×4 MIMO matrices

employed in [6], corresponding to the full-rank and low-rank cases, respectively.

Fig. 4 shows the ergodic capacity computed using the optimal numerical solution, along with the ergodic throughput of SMMW allocation, when an identical per-antenna power is enforced for the fixed fading matrices. The performance difference is negligible across the SNR range, for both the full-rank and low-rank models. We also plot the results when the average power constraints are in the ratio $1:3:2:6$ at the antennas.

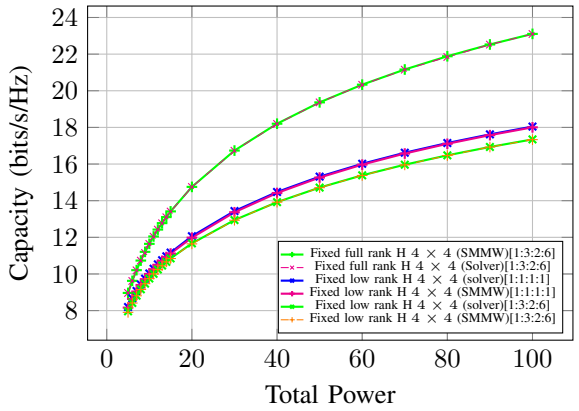

Fig. 4. capacity for 4×4 fixed channel matrix

C. Discussion

We have demonstrated that SMMW allocation provides near-optimal ergodic capacity solutions in time-varying Rayleigh fading models and in the fixed-fading MIMO examples. This remarkable quality provides a novel framework to solve power allocation problems under distributed per-antenna constraints. We have employed a numerical search algorithm to choose the Lagrange parameters in the examples presented in the paper. Nevertheless, we believe that more efficient computation with a gradient-descent-like approach should be possible. This is currently under development and not presented in the paper.

V. Conclusion and Future Scope

In conclusion, we have proposed a scheme tailored for the evaluation of ergodic capacity under distributed power constraints in wireless MIMO systems with coordinated multipoint transmissions. The proposed SMMW scheme not only accommodates a closed form power allocation formula in terms of water-filling, but also achieves close to optimal throughputs in diverse environments. Furthermore, the allocation was found to be near-optimal in several fixed-fading setups as well, for appropriately chosen parameters.

We considered the ergodic setup with all antennas from a given AP encountering identical links, and in this case, the per-AP constraint is equivalent to appropriate per-antenna constraints. More generally, one can consider different link distributions or diverse group power constraints [6]. SMMW should accommodate natural extensions to such cases; these are part of our future work.

Notice that choosing transmit covariance matrices to maximize ergodic throughput, which obeys distributed power constraints, is not an easy task. In fact, algorithmic solutions are needed, even for fixed channel settings [6]. Against this backdrop, we provide transmit covariance matrices that yield good achievable throughput performance, even in an ergodic setting. The explicit form allows covariance adaptation based on perceived channel conditions, and its solutions differ significantly from conventional water-filling. However, on the downside, our explicit solution does not satisfy the optimal KKT condition in general; this may require extensive numerical evaluation, making it impractical.

VI. Acknowledgment

The authors acknowledge several discussions with Kumar Appaiah during the early stages of this work. Research funding from Qualcomm 6G UR India is also acknowledged.

Appendix A
Proof of MISO Optimality

Notice that [8] already presented the optimal ergodic MISO power allocation under per-AP constraints, using the appropriate KKT conditions. More specifically, AP k sends $(\|\bar{h}_k\|)^{-1}(\bar{h}_k)^* \sqrt{\tilde{P}_i(\bar{h})}d$, where $\bar{h}_k$ is the fading vector of k-th AP, d denotes the normalized common data symbol for that transmission instant, and

$$\tilde{P}_k(\bar{h}) = \frac{\|\bar{h}_k\|^2}{\lambda_k \left(\sum_{l=1}^{N_{\text{ap}}} \frac{\|\bar{h}_l\|^2}{\lambda_l} \right)} \left(\frac{1}{\lambda_k} - \frac{1}{\sum_{l=1}^{N_{\text{ap}}} \lambda_k \frac{\|\bar{h}_l\|^2}{\lambda_l}} \right)^+ . \quad (10)$$

Notice that both (9) and (10) are nearly identical, except for a single term shown in the numerator. Denoting $l(k,i)$ as the index of i-th antenna at AP k, the transmitted symbol at antenna $l(k,i)$ can be expressed as

$$\frac{h^*_{l(k,i)}}{\|\bar{h}_k\|} \sqrt{\tilde{P}_k(\bar{h})}\, d = \frac{h^*_{l(k,i)}}{|h_{l(k,i)}|} \sqrt{P_{l(k,i)}(\bar{h})}\, d, \quad (11)$$

where $P_{l(k,i)}(\bar{h})$ is given in (9). Therefore, (9) indeed matches the optimal MISO power allocation under per-AP constraints, completing the proof.

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