On minimization of average AoI for static age-threshold irregular repetition slotted ALOHA

Minha Mubarak and Vineeth Bala Sukumaran

Abstract—**We consider the problem of minimizing average Age of Information (AAoI) in a grant free random access scenario where a large number of users transmit to a single base station using irregular repetition slotted ALOHA (IRSA). Each user implements a simple policy that transmits with a certain attempt probability only when the user's age exceeds a static AoI-threshold. An important design problem in this context is the optimization of the AoI-threshold and attempt probability to minimize AAoI. The characterization of AAoI (which is the optimization objective) as a function of the AoI threshold and the attempt probability is challenging since the age evolution processes of different users are coupled. The paper proposes a novel numerical approximation of the AAoI for static AoI-threshold IRSA. The approximation is shown to match the simulated AAoI for multiple cases. Then, the approximation is utilized to obtain optimal AoI thresholds as well as attempt probabilities for IRSA systems. Through simulations, we observe that the AAoI of optimized static AoI-threshold policies is lower than that of conventional IRSA, and IRSA with fixed AoI-thresholds and attempt probabilities for different number of users. The AAoI performance of optimized static AoI-threshold policies is also compared with an adaptive AoI-threshold policy from prior work. Although, the adaptive AoI-threshold policy has a smaller AAoI (but with slighly more computation at the base station), the performance of the optimized static AoI-threshold policy is observed to be comparable for some ranges of system parameters.**

Index Terms—**Age of Information, Irregular repetition slotted ALOHA, Age-Threshold policies, Attempt probability, Numerical approximation, Threshold optimization, Attempt probability optimization.**

I. Introduction

Timely delivery of status updates is a fundamental requirement in many emerging networked systems, including Internet of Things (IoT) [1] deployments, cyber–physical systems, autonomous vehicles, and remote monitoring applications. The performance of such systems needs to be studied using information freshness metrics such as Age of Information (AoI) [2]. In large-scale IoT networks consisting of a massive number of devices [3], centralized scheduling of transmissions is often impractical due to the significant signaling overhead and the sporadic nature of status updates. Random access protocols provide a scalable alternative that allows users to access the channel in a distributed manner without requiring coordination among devices. Irregular Repetition Slotted ALOHA (IRSA) [4] has attracted significant attention due to its ability to achieve high throughput by exploiting successive interference cancellation (SIC) at the receiver. In IRSA, users transmit multiple replicas of their packets in randomly selected slots within a frame according to a pre-defined degree distribution. The receiver iteratively resolves collisions by decoding singleton slots and performing SIC, which enables the recovery of packets even in highly loaded systems. In conventional IRSA, the transmission of packets by users are done independently of their AoI. AoI-aware transmission policies are expected to lead to a lower average AoI (AAoI). Therefore, we consider the AAoI performance of a static AoI-threshold policy in this paper. Under this policy, users attempt to transmit with an attempt probability only if their AoI exceeds a threshold value. We consider the optimization of the attempt probability as well as the threshold for minimizing AAoI.

Prior work: AoI has been widely studied in both *point-to-point* and *multiple access systems*. In point-to-point systems, Sun et al. [5] showed that zero-wait and constant-wait transmission policies are generally sub-optimal for minimizing AoI in links with feedback. In multi-user settings, AoI has also been investigated in conventional random access systems such as slotted ALOHA [6] and CSMA [7]. Kadota et al. [8] developed centralized scheduling policies to minimize AoI in wireless networks. AoI has been investigated in modern random access protocols with SIC, such as *IRSA* [4] and *frameless ALOHA* [9]. For IRSA systems, Munari et al. [10] derived a closed-form expression for the average AoI and demonstrated its advantages in massive machine-type communications (mMTC) scenarios compared to slotted ALOHA. Additional analyses of average AoI in IRSA systems appear in [11], [12]. These works primarily analyze AoI under fixed protocol parameters, such as fixed repetition distributions or access probabilities. In contrast, *age-aware transmission policies*, where transmission attempts depend on the current information age, can further improve freshness by prioritizing stale updates. Related ideas have also been explored in enhanced random access schemes such as *Mini-Slotted Threshold ALOHA (MiSTA)* [13]. An AoI-threshold based transmission policy was proposed and analyzed in [14] for slotted ALOHA systems. However, adapting such an analysis to IRSA is challenging because IRSA operates over frames in which multiple users can be decoded using successive interference cancellation. Asgari et al. [15] proposed an adaptive age-threshold–based modification of IRSA and demonstrated through analysis and simulations that the scheme can substantially reduce the average network AoI compared to conventional IRSA. This adaptive AoI-threshold based IRSA scheme in [15] requires the base station to main-

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tain the current AoI state for each user and computation of a threshold, which needs to be fed back to the users every frame. In contrast, we consider static AoI-threshold policies, which require feedback of packet reception acknowledgements, but do not require AoI state to be maintained or a threshold to be computed at the base station. Although static AoI-threshold policies have been considered in prior work [15], their AAoI has not been characterized.

The main contribution of this paper is *a novel numerical approximation* for the AAoI in an IRSA-based system where users employ a static AoI-threshold based transmission policy. The approximation (which has a complexity polynomial in the number of users) characterizes AAoI as a function of the AoI-threshold and the attempt probability. The numerical approximation is significant since it can be used as an objective function in numerically optimizing the AoI-threshold and attempt probability. Using simulations, we compare the AAoI for the optimized static AoI-Threshold policy with conventional IRSA and show improved AAoI performance. The AAoI for an adaptive AoI-threshold policy is observed to be smaller compared with the optimized static AoI-Threshold. For a fixed number of users, the adaptive policy requires the base station to maintain AoI state and compute the adaptive threshold, which is then fed back to the users. In contrast, the static policy just requires packet reception acknowledgement from the base station.

II. System Model and Problem Statement

We consider a discrete time system with slots indexed by $t \in \mathbb{Z}_+$ (here $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$). There are $M > 0$ users in the system. Let $[M] = \{1, 2, 3, \ldots, M\}$. These M users communicate to the base station (BS) using the Irregular Repetition Slotted ALOHA (IRSA) [4] protocol. The transmissions in IRSA are organized into frames. Each frame consists of N contiguous slots. Users are assumed to be synchronized at both the slot and frame levels. Frames are indexed by $m \in \mathbb{Z}_+$. The system evolves according to a transmission policy Π, which may be centrally implemented at the BS (which uses a feedback channel to communicate its decisions to the users) or distributed across users. In each frame, the policy Π determines whether a user transmits a packet.

If a user is scheduled to transmit in frame m, it follows the IRSA protocol. The user samples a value R, which is the number of packet replicas. In IRSA, for each transmission attempt, R is independently drawn from a repetition distribution $p_R[r]$, where $r \in \{2, 3, \ldots, N\}$. An equivalent representation of this distribution is given by the generating function $\Lambda(x) = \sum_r p_R[r] x^r$. The user then transmits r copies of the packet in r distinct slots, chosen uniformly at random without replacement from the N slots within the frame. If multiple users transmit in the same slot, a collision occurs, and the packets cannot be decoded directly, whereas a slot containing a single transmission can be decoded successfully. The BS collects all packet transmissions from the users transmitting in frame m across all N slots. Upon decoding a singleton transmission, IRSA employs successive interference cancellation (SIC) to iteratively resolve packet collisions across the frame. Each packet copy is assumed to include a header specifying the set of slots in which all replicas of that packet were transmitted.

For an IRSA system with $M_{active} \leq M$ transmitting users and N slots per frame, the load is defined as $g = M_{active}/N$. Let $p_s(g)$ denote the probability of successful decoding, i.e., the fraction of users whose packets are successfully decoded in a frame with load g. For a chosen degree distribution $\Lambda(x)$, IRSA performance is defined by a threshold g^*. Under the asymptotic conditions $M_{active}/N = g$ and $M_{active}, N \to \infty$, reliable data recovery is guaranteed ($p_s \approx 1$) provided the g remains less than g^* [4].

Age of information (AoI): We first consider a transmission scheme, where every user transmits a packet in every frame using the IRSA protocol. Let $m \in \mathbb{Z}_+$ index the frames. At the end of a frame, the BS uses SIC to decode packets. For a user u, let $W^u(t)$ denote the generation time slot of the most recently *successfully received* packet at the BS by time t. Then the Age of Information (AoI) of user u at time t is defined as:

$$\Delta^u(t) = t - W^u(t)$$

which measures how stale the freshest information available at the BS is. Let $D_m^u \in \{0, 1\}$ indicate whether the user transmits and user's packet has been decoded in the m^{th} frame. Equivalently, the AoI (units of slots) evolves as:

$$\Delta^u(t) = \begin{cases} N, & \text{if} \quad D_m^u = 1 \text{ and } t = (m+1)N, \\ \Delta^u(t-1) + 1, & \text{otherwise}, \end{cases}$$

for $t > 0$. We assume that $\Delta^u(0) = N$. The AoI resets to N if there is a successful transmission since the successfully received packet was generated at the start of that frame.

Static AoI-Threshold IRSA (S-AT-IRSA): We consider the setting where the BS broadcasts, at the end of each frame, a feedback signal indicating which packets were successfully decoded. Using this feedback, each user u can track its AoI process $\Delta^u(t)$. We propose an AoI-threshold based random access scheme in which a user is allowed to transmit in frame m only if its AoI at the beginning of the frame satisfies $\Delta_m^u = \Delta^u(mN) \geq \Gamma$ where Γ is a system-wide AoI threshold that may be included in the feedback broadcast.[1] Users whose AoI exceeds this threshold transmit with an access probability p_t. We denote this AoI-based variant of IRSA as *Static AoI-Threshold IRSA (S-AT-IRSA)*. The BS performs SIC-based decoding as in standard IRSA. We note that the policy is parameterized by Γ and p_t, let a S-AT-IRSA policy be denoted as $\Pi(\Gamma, p_t)$.

Problem Statement: For a given transmission policy Π, we define the long-term time-average AoI of user u as

$$\bar{\Delta}^u(\Pi) \triangleq \limsup_{T \to \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T} \Delta^u(t) \,|\, \Delta^u(0) = N \right].$$

[1]In scenarios where the number of users change slowly, the optimized parameters for different number of users can be broadcast.

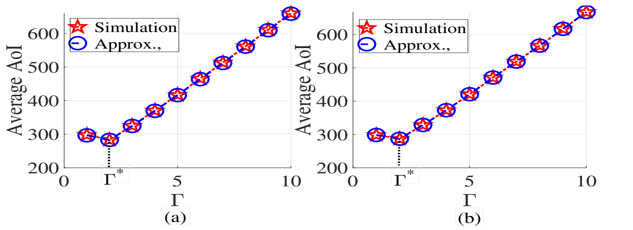

Fig. 1: Average AoI $\bar{\Delta}$ as a function of Γ for $N = 100$: (a) $M = 150, \Lambda(x) = x^3$ and (b) $M = 160, \Lambda(x) = 0.5x^2+0.28x^3+0.22x^8$. This is done for $p_t = 0.5$.

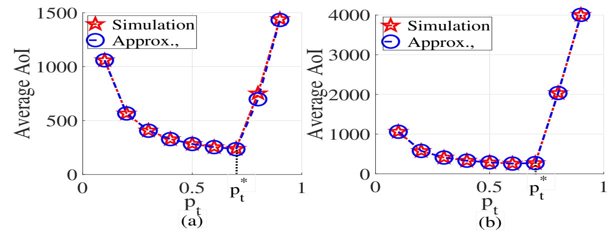

Fig. 2: Average AoI $\bar{\Delta}$ as a function of p_t for $N = 100$: (a) $M = 150, \Lambda(x) = x^3$ and (b) $M = 160, \Lambda(x) = 0.5x^2+0.28x^3+0.22x^8$. This is done for $\Gamma = 2$.

The average AoI across the network is then defined as $\bar{\Delta}(\Pi) = \frac{1}{M}\sum_{u=1}^{M}\bar{\Delta}^u(\Pi)$. The objective of this work is to minimize the network-wide AAoI by appropriately selecting the threshold parameter Γ and the probability p_t within the class of S-AT-IRSA policies, that is, $\min_{\Gamma,p_t}\ \bar{\Delta}(\Pi(\Gamma, p_t))$.

III. Optimal Threshold for S-AT-IRSA

For optimizing the objective function $\bar{\Delta}(\Pi(\Gamma, p_t))$, we first obtain an approximate characterization of the same as a function of Γ and p_t that is utilized for numerical optimization. Since users are homogeneous, we have that $\bar{\Delta}(\Pi(\Gamma, p_t)) = \bar{\Delta}^u(\Pi(\Gamma, p_t))$ and we consider $\bar{\Delta}^u(\Pi(\Gamma, p_t))$ for a single tagged user u. In the following, for brevity we use the notation $\bar{\Delta}$ for this average.

Approximate characterization of AAoI: We have the following approximation for $\bar{\Delta}$ by considering the AoI evolution $\Delta^u(t)$ of one tagged user.

Theorem 1. *The AAoI $\bar{\Delta}$ for a tagged user under S-AT-IRSA is approximated as*

$$\bar{\Delta} \approx \frac{q + \mathbb{E}[R]}{N\,(p_I + \mathbb{E}[P_{II}])}, \tag{1}$$

where $q = N((\Gamma-1)N) + (((\Gamma-1)N)((\Gamma-1)N-1))/2$ and $p_I = \Gamma - 1$. The quantities $\mathbb{E}R$ and $\mathbb{E}P_{II}$ are obtained numerically and explained in Appendix A. For fixed N, the approximation has a computational complexity $\mathcal{O}(M^3)$ in the number of users [2].

The proof is discussed in Appendix A. To motivate why the approximation of the objective function is useful, we compare the simulated values of $\bar{\Delta}$ and the approximations as a function of Γ (keeping p_t fixed) and p_t (keeping Γ fixed) for two different M and IRSA repetition distributions in Figs. 1 and 2 respectively. The simulated results presented here are obtained using Monte Carlo simulations. From these figures, it is evident that the approximations match with the simulated values of AAoI. From Fig. 1, we observe that there is an optimal value of threshold Γ^* at which the AAoI is minimum, which is the same for both simulated and approximate AAoI. Similar observations can be made for AAoI as a function of access probability p_t in Fig. 2.

The approximation in Theorem 1 therefore enables the numerical optimization of Γ and p_t for different system parameter

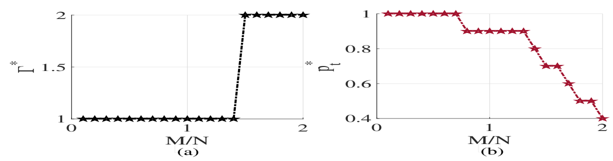

Fig. 3: (a) Γ^* as a function of offered load M/N (b) p_t^* as a function of offered load M/N. (for $N = 100$ and a repetition distribution: $\Lambda(x) = x^3$).

values. For example, the optimized Γ and p_t are shown as a function of M/N for $N = 100$ and $\Lambda(x) = x^3$ in Fig. 3.

IV. AAoI Performance of Optimized S-AT-IRSA

In this section, we compare the AAoI of optimized S-AT-IRSA, conventional IRSA, AT-IRSA [15], and variants of S-AT-IRSA (e.g., with fixed Γ and p_t) using Monte-Carlo simulations. AAoI as a function of M/N is illustrated in Fig. 4 for $N = 100$ and two different repetition distributions. These parameters are commonly used for IRSA performance analysis [4]. We also compare with a variant of S-AT-IRSA where $p_t = 0.5$, but the threshold is optimized. In both cases, we note that IRSA where all M users are active (i.e. $M_{active} = M$) has a sharply increasing AAoI once $M_{active}/N > g^*$ (where g^* depends on the repetition distribution). Threshold based policies maintain a $M_{active} < M$ and their AAoI does not show such a sharp increase. In all cases, the adaptive age-threshold policy has the minimum AAoI, but note that this policy requires AoI of every user to be maintained at the BS and the age-threshold needs to be adaptively computed at the end of every frame and fed-back to the users. The optimized S-AT-IRSA policy gives comparable performance to the adaptive policy and does not require such a computation at the BS. The policies with fixed threshold and fixed access probability and those with fixed access probability, yield larger AAoI, highlighting the importance of selecting the optimal threshold and optimal access probability as a function of g to optimize system performance.

V. Conclusions and Future Work

In this paper, we considered the problem of optimizing the threshold Γ and access probability p_t for static AoI-Threshold S-AT-IRSA policy for minimizing AAoI. The main challenge for this problem is the characterization of the objective function. We proposed a novel numerical approximation method utilizing a Markov chain to model the number of concurrent transmissions to address this challenge. The AAoI obtained

[2] A function is said to be $\mathcal{O}(M^3)$ if there exist constants $c > 0$ and M_0 such that it is bounded above by cM^3 for all $M \geq M_0$.

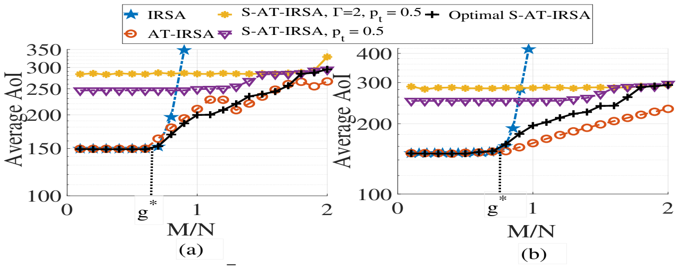

Fig. 4: Average AoI $\bar{\Delta}$ as a function of offered load M/N for $N = 100$ for different IRSA protocols with two different degree distributions (a) $\Lambda(x) = x^3$ ($g^* = 0.62$) and (b) $\Lambda(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$ ($g^* = 0.76$).

from numerical approximation has been shown to match simulated values. The approximate AAoI is then used to obtain the optimal Γ and p_t for S-AT-IRSA. The AAoI performance of optimized S-AT-IRSA is better than conventional IRSA as well as IRSA with fixed threshold (independent of the number of users M) showing that IRSA transmission policies with simple threshold structure can yield minimal AAoI provided that they are optimized correctly. In future, we plan to obtain a more rigorous theoretical basis (e.g. possibly using mean-field approach) for the current heuristic approximation. Furthermore, extending the approximation to consider non-homogeneous users, i.e., with different thresholds as well as attempt probabilities is also planned as part of future work.

Appendix A
Proof of Theorem 1: Approx. for S-AT-IRSA's AAoI

Consider a tagged user $u = 1$ (we suppress dependence on u in the notations, e.g., $\Delta_m = \Delta_m^1$). A typical sample path

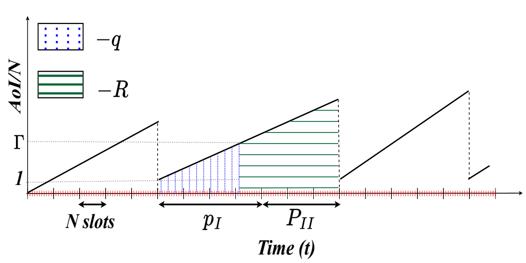

Fig. 5: A typical evolution of AoI Δ_m (and $\Delta(t)$) for the tagged user under S-AT-IRSA with threshold Γ. Black tick marks on the time axis indicate frame boundaries, while red marks show slot boundaries. The evolution consists of cycles with two phases with durations p_I, P_{II} and areas q, R as shown.

of the AoI evolution for the tagged user is shown in Fig. 5 under S-AT-IRSA. We consider the AoI embedded at IRSA frame boundaries, i.e. Δ_m. Since Δ_m takes values which are multiples kN of N, we consider Δ_m to be in units of N slots in this discussion (however, the AAoI is still calculated in terms of slots). From Fig. 5 we note that each time a packet is successfully delivered, the AoI is reset to 1. The evolution of Δ_m consists of two phases. In Phase-I, which begins immediately after a successful update with $\Delta_m = 1$, lasts until a m' such that $\Delta_{m'} = \Gamma$. During this phase, the tagged user remains silent and does not participate in IRSA transmissions. Since the AoI increases deterministically by one per frame, the duration of this phase is deterministic and given by $p_I = \Gamma - 1$.

Phase II starts once the AoI reaches Γ. In this phase, the user becomes active and participates in the IRSA protocol in every frame until a packet is successfully delivered at the end of that frame. The AoI continues to grow linearly during this phase, but its duration is random due to the random attempt process (controlled by p_t) and the probabilistic nature of SIC decoding in IRSA. Let P_{II} denote the random duration of Phase II in frames. We note that the evolution of AoI for the tagged user during Phase-II depends on the number of other users which are transmitting concurrently. To handle this dependence, we consider the number of users which have $\Delta_m^u \geq \Gamma$, i.e. $\sum_{u=1}^{M} \mathbb{I}\{\Delta_m^u \geq \Gamma\}$. We discuss an approximation for the steady-state distribution of this sum in the following, which is useful for our AAoI approximation.

An approximate steady-state distribution of $\sum_{u=1}^{M} \mathbb{I}\{\Delta_m^u \geq \Gamma\}$: Let $H_\Gamma(m) = \sum_{u=1}^{M} \mathbb{I}\{\Delta_m^u \geq \Gamma\}$ be a count variable denoting the number of users[3] whose AoI is greater than or equal to the threshold in frame m, and let $g(m) = H_\Gamma(m)/N$. The process $(H_\Gamma(m), m \in \mathbb{Z}_+)$ is not Markovian in general.[4] *We approximate its evolution by a Markov chain* $(\widetilde{H}_\Gamma(m), m \in \mathbb{Z}_+)$ *taking values in* $\{0, 1, \ldots, M\}$. The evolution of $\widetilde{H}_\Gamma(m)$ *heuristically approximates* the evolution of $H_\Gamma(m)$. Suppose $\widetilde{H}_\Gamma(m) = h$. We say that

$$\widetilde{H}_\Gamma(m+1) = \widetilde{H}_\Gamma(m) - J(m) + I(m), \tag{2}$$

where $J(m)$ is the number of users which leave the count variable $\widetilde{H}_\Gamma(m)$ since they are successfully decoded and $I(m)$ is the number of users which enter the count variable at $m+1$ since their AoI has increased to Γ. Suppose $\text{Bin}(n,p)$ denotes a Binomial distribution with parameters n and probability p. We note that $J(m) \sim \text{Bin}(T(m), p_s(T(m)/N))$, where $T(m) \sim \text{Bin}(\widetilde{H}_\Gamma(m), p_t)$ and $p_s(./N)$ is the probability of successful decoding for IRSA. Also $I(m)$ is the number of users which have AoI $= \Gamma - 1$ which have not transmitted and whose AoI increases to Γ at $m+1$. We note that there are $M - h$ users with AoI $< \Gamma$. Suppose there exists the following steady state probabilities: (a) $p_\Gamma = \mathbb{P}\{\Delta_m \geq \Gamma\}$ and (b) $p_{\Gamma'} = \mathbb{P}\{\Delta_m = \Gamma - 1\}$. We make the approximation that each user with $\Delta_m < \Gamma$ has an *AoI* which is uniformly distributed in $\{1, 2, \ldots, \Gamma - 1\}$. So for the $M - h$ users (for which AoI $< \Gamma$) the number of users with AoI being $\Gamma - 1$ is approximated as $\text{Bin}(M - h, p_{\Gamma'}/(1 - p_\Gamma))$. It can be shown that $p_{\Gamma'}/(1 - p_\Gamma) = 1/(\Gamma - 1)$. Therefore, $I(m)$ is approximated as $\text{Bin}(M - h, 1/(\Gamma - 1))$. This defines a finite-state, irreducible, and aperiodic Markov chain with transition

[3] In this definition $H_\Gamma(m)$ also considers the tagged user. For large number of users, we assume that the effect of a single user is negligible for our approximation.

[4] The process $(H_\Gamma(m))$ is Markovian if $\Gamma = 1$ or 2. For $\Gamma > 2$, a vector Markov process $(H_1(m), H_2(m), \ldots, H_\Gamma(m))$, where $H_\tau(m) = \sum_{u=1}^{M} \mathbb{I}\{\Delta_m = \tau\}$, for $1 \leq \tau < \Gamma$ and $\sum_{\tau=1}^{\Gamma} H_\tau(m) = M$ can be defined. In our approach, we consider one component of this process instead of analyzing the Γ (note: $\Gamma - 1$ is sufficient) dimensional process.

matrix $\mathcal{P}_h$ (of size $(M+1)\times(M+1)$) and stationary distribution Π_h. The stationary distribution Π_h is computed numerically for a given M by solving a linear system of equations.

Duration of Phase-II: Approximation for the tagged user using Π_h: We assume that when $\Delta_m = \Gamma$ for the tagged user (i.e., at the first slot of Phase II), the number of other users is given by $\widetilde{H}_\Gamma(m)$. Since $\widetilde{H}_\Gamma(m)$ is assumed to evolve independently, the number of other users is an independent sample from Π_h. Now, conditioned on $\widetilde{H}_\Gamma(m)=\eta$, let Y_η denote the number of frames required until the tagged user's AoI is reset due to a successful update. Suppose $T \sim \mathrm{Bin}(\eta, p_t)$ denotes the number of users which attempt with probability p_t. Let $b(\tau)$ denote the Binomial pmf $\binom{\eta}{\tau}p_t^\tau(1-p_t)^{\eta-\tau}$. Then, Y_η satisfies

$$Y_\eta = \begin{cases} 1, & \text{w.p.} \quad p_t p_s(\tau/N) b(\tau), \\ 1+X_{\eta,\tau}, & \text{w.p.} \quad ((1-p_t)+p_t(1-p_s(\tau/N)))b(\tau), \end{cases} \tag{3}$$

$\forall \tau \in \{0,\ldots,\eta\}$. Here $X_{\eta,\tau}$ is the next state $\widetilde{H}_\Gamma(1)$ given that $\widetilde{H}_\Gamma(0)=\eta$ and $T(0)=t$. That is

$$X_{\eta,\tau} \quad \sim \quad \eta - \mathrm{Bin}(\tau, p_s(\tau/N)) + \mathrm{Bin}(M-\eta, 1/(\Gamma-1)).$$

Let $\rho_{\eta,\tau}(\eta') = \mathbb{P}\{X_{\eta,\tau}=\eta'\}$ which can be obtained from the above binomial probabilities. We approximate Phase-II's duration as an independent sample from Y_η, where η is first sampled from Π_h.

Renewal process approximation for evolution of Δ_m^u and approximation of AAoI: With the above approximation for Phase-II, we have that the duration of time from when $\Delta_m^u = 1$ to the next time at which $\Delta_m^u=1$ is an independent sample from $\Gamma-1+Y_\eta$. Thus, we can consider the m-epochs at which $\Delta_m^u=1$ as renewal instants. By considering cumulative AoI over each renewal cycle, we apply the renewal reward theorem (RRT) [16, Theorem D.9] to obtain the average $\bar{\Delta}$.

We first note that a system of linear equations in $\mathbb{E}Y_\eta, \forall\eta$ can be obtained by taking expectations on either side of (3). We have $\mathbb{E}Y_\eta =$

$$1+\sum_{\tau=0}^{\eta}\left[(1-p_t)+p_t(1-p_s(\tau/N))\right]b(\tau)\sum_{\eta'=0}^{M}\rho_{\eta,\tau}(\eta')\mathbb{E}Y_{\eta'}.$$

This can be solved numerically to obtain $\mathbb{E}[Y_\eta]$ for all η. Then, we have $\mathbb{E}P_{II} = \sum_\eta \Pi_h(\eta)\,\mathbb{E}[Y_\eta]$. The expected duration of the renewal cycle is then $\Gamma-1+\mathbb{E}P_{II}$. For obtaining the AAoI, the cumulative AoI (in slots^2) for Phase I corresponds to the shaded area q and for Phase-II corresponds to R in Fig. 5. The area q is $N((\Gamma-1)N)+(((\Gamma-1)N)((\Gamma-1)N-1))/2$. Conditioned on the AoI value at the beginning of Phase II and the duration P_{II}, the area R can be expressed as $R = \Gamma\, P_{II}N^2 + \frac{N^2P_{II}(P_{II}-1)}{2}$. To compute $\mathbb{E}[R]$, we also require $\mathbb{E}[P_{II}^2] = \sum_\eta \Pi_h(\eta)\mathbb{E}Y_\eta^2$. From (3) we obtain $\mathbb{E}Y_\eta^2 =$

$$1+2\sum_{\tau=0}^{\eta}\left[(1-p_t)+p_t(1-p_s(\tau/N))\right]b(\tau)\sum_{\eta'=0}^{M}\rho_{\eta,\tau}(\eta')\mathbb{E}Y_{\eta'}$$
$$+\sum_{\tau=0}^{\eta}\left[(1-p_t)+p_t(1-p_s(\tau/N))\right]b(\tau)\sum_{\eta'=0}^{M}\rho_{\eta,\tau}(\eta')\mathbb{E}Y_{\eta'}^2.$$

This again yields a system of linear equations that can be solved numerically to obtain $\mathbb{E}Y_\eta^2$ and then $\mathbb{E}R$. Finally, the total average AoI accumulated over a renewal cycle is the sum of q and $\mathbb{E}R$. Using the Renewal Reward Theorem (RRT), the average AoI of the tagged user is given by $\bar{\Delta} = \frac{q+\mathbb{E}[R]}{N(\Gamma-1+\mathbb{E}[P_{II}])}$. We note that the method requires solution of three linear systems of equations ($M+1$ variables and equations) for a fixed N. Assuming that $p_s(g)$ has been computed using density evolution (which is $\mathcal{O}(M)$ assuming a fixed number of iterations), the computational complexity of the approximation is $\mathcal{O}(M^3)$.

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