E1 244: Detection and Estimation

Lecture 1: Introduction

Course information

- Instructor:
  - Sundeep Prabhakar Chepuri.
    Email: `spchepuri AT iisc.ac.in`
- Class schedule:
  - Tuesdays and Thursdays 2.00-3.30pm (Online via MS Teams).
- Course webpage:
  `https://ece.iisc.ac.in/~spchepuri/classes/e1244.html`

Course information

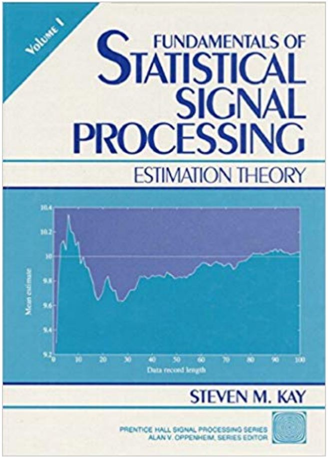
- Teaching Assistants

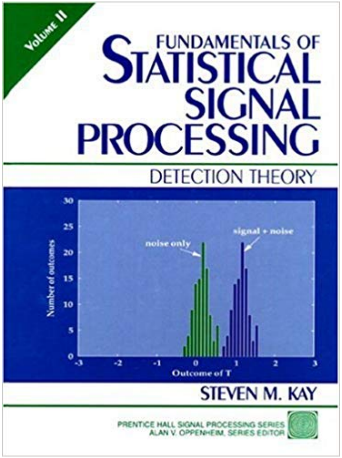
Sravanthi Gurugubelli
`sravanthig`

Prasobh Sankar
`prasobhr`

- Exercise/tutorial sessions:
  - By TAs, on 1st and 3rd Saturdays 11.00-12.30 pm

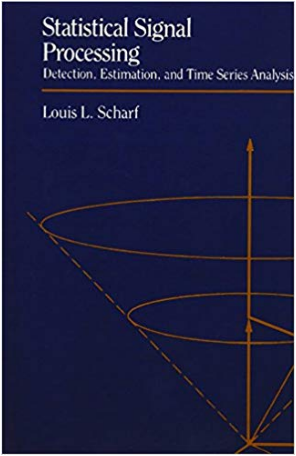
Textbooks

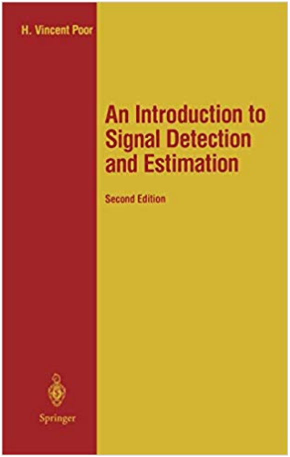




- Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory, S.M. Kay, Prentice Hall 1993, ISBN-13: 978-0133457117.
- Fundamentals of Statistical Signal Processing, Volume II: Detection Theory, S.M. Kay, Prentice 1993, ISBN-13: 978-0135041352.

Other resources



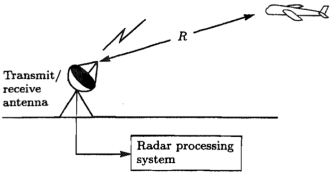


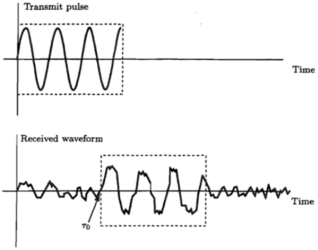
- Statistical Signal Processing, L.L. Scharf, Pearson India, 2010, ISBN-13: 978-8131733615.
- An Introduction to Signal Detection and Estimation, H.V. Poor, Springer, 2nd edition, 1998, ISBN-13: 978-0387941738.

Grading and course requirements

- **Prerequisite:** Matrix theory (or equivalent) and random processes (or equivalent).
- **Three assigments** (problem and programming set): 10% each, i.e., 30% in total
- **Midterm exam**: 20%
  - Open book exam.
- **Project**: 30%
- **Final exam**: 20%
  - Open book exam.

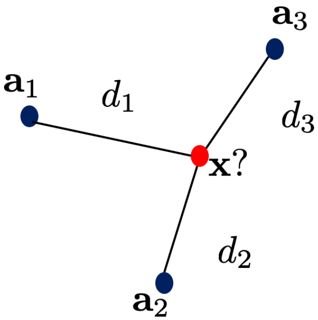
Estimation theory





Time delay estimation or ranging

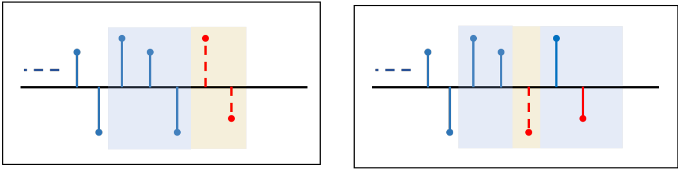
Estimation theory



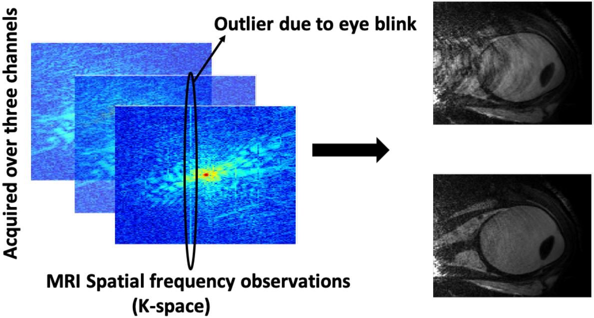
$$d_i = \|\mathbf{x} - \mathbf{a}_i\|_2 + \text{noise}, i = 1, 2, \ldots, 3$$

Sensor localization and positioning

Estimation theory

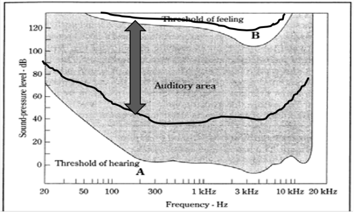


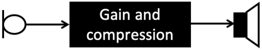
Time-series prediction and interpolation



Medical imaging

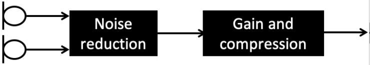
Estimation theory



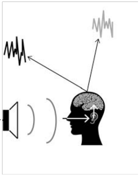


$y_1 = s + n_1$

s

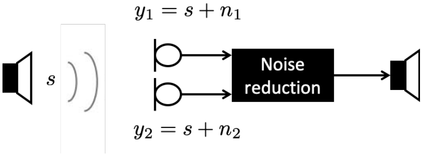


$y_2 = s + n_2$



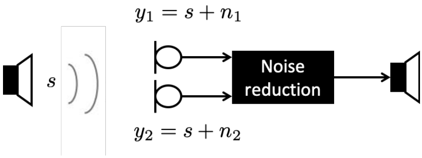
Denoising in hearing aids

A simplified model



How to determine $\hat{s}$ - an estimate of s?

A simplified model

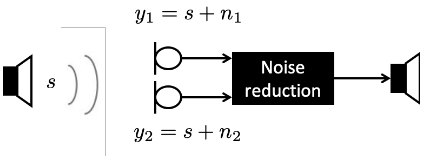


How to determine $\hat{s}$ - an estimate of s?

- $\hat{s}_1 = y_1$?
- $\hat{s}_2 = \frac{1}{N}\sum_{i=1}^{N} y_i$?

How good are these estimators? Are there better estimators?

A simplified model



How to determine $\hat{s}$ - an estimate of s?

- $\hat{s}_1 = y_1$?
- $\hat{s}_2 = \frac{1}{N}\sum_{i=1}^{N} y_i$?

Suppose noise at each microphone, n_i, has variance σ_i^2. Then, how about:

- $\hat{s}_3 = \frac{\sum_{i=1}^{N} \frac{y_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}$?

What is the recipe to determine optimal estimators?

Estimation theory

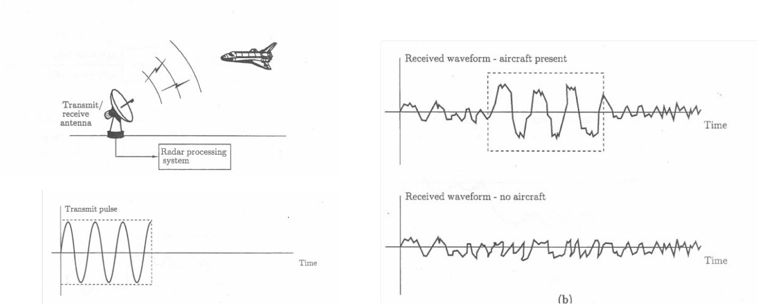
Typical formulation in estimation theory

$$y[n] = f_n(\boldsymbol{\theta}) + w[n].$$

The noise is usually assumed to stochastic, the parameter vector of interest $\boldsymbol{\theta}$ may be

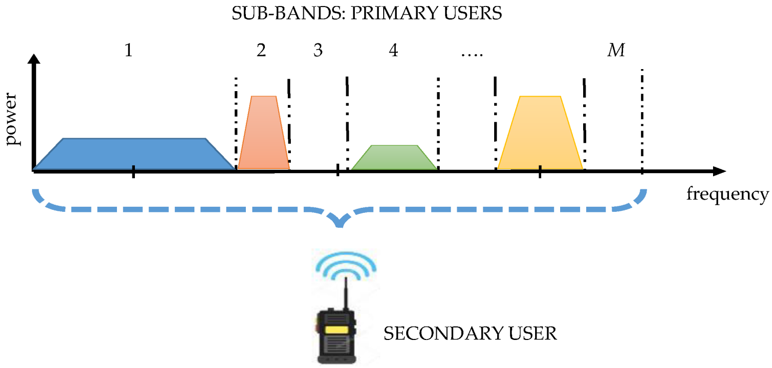
- an unknown deterministic quantity: classical estimation theory.
- an unknown random quantity: Bayesian estimation theory.

Detection theory



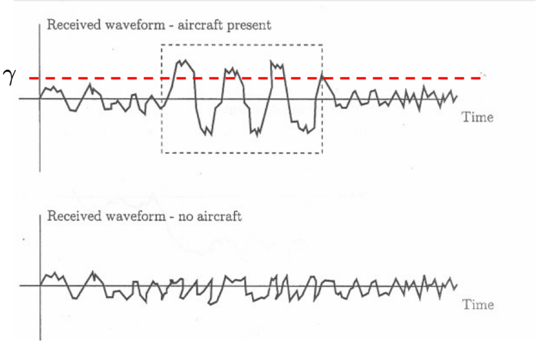
Radar - target detection

Delay/ranging might not be always required



Cognitive radio - spectrum sensing

Detection theory



How to choose γ?

Detection theory

Typical detection problem formulation, also referred to as binary hypothesis testing problem:

$$\text{(no target)} \quad \mathcal{H}_0 : x[n] = w[n]$$

$$\text{(target)} \quad \mathcal{H}_1 : x[n] = s[n] + w[n]$$

We wish to infer the state of nature, i.e., to decide on $\mathcal{H}_0$ or $\mathcal{H}_1$ using a detector of the form:

$$T(x[n]) > \gamma$$

- How to make an optimal decision: how to choose $T(\cdot)$ and γ?
- detection of deterministic signals: Neyman-Pearson detectors.
- detection with priors: Bayes detectors.
- detection with unknown parameters.

Content

- Review of linear algebra and random processes.
- Minimum variance unbiased estimator
- Cramér-Rao bound
- Maximum likelihood estimator
- Best linear unbiased estimator
- Least squares and recursive least squares.
- Structured covariance estimation
- Bayesian estimators (MMSE and MAP estimators)
- Kalman filtering
- Neyman-Pearson detector
- Bayes detector
- Multiple hypothesis testing
- Composite hypothesis testing
- Sequential probability ratio test (SPRT)

Course objectives

Generally, a solution to estimation and detection problems depend on the underlying data model and the statistical description of the noise and/or unknowns.

- How to **mathematically formulate** such problems?
- Determine optimal estimators, characterize performance of these estimators, and compute the estimation bound.
- Determine optimal detectors and characterize the performance of these detectors.