Department of Electrical Communication Engineering
Indian Institute of Science

E9 211 Adaptive Signal Processing

28 January 2021, Take-home final assessment

This exam has two questions (20 points).

This is an open book assessment with a turn in time of 24 hrs. Make sure the report is turned in before 9 am, 29 January 2021. This is a hard deadline. Your report should be a single PDF file, legible, scanned/pictured under good lighting conditions with your full name. Use `firstname_lastname.pdf` as the file name. The report should be submitted via MS Teams. Late or plagiarized submissions will not be graded.

Question 1 (10 points)

In decision-directed feedback equalization using Viterbi decoder, the desired signal and thus the error is not available until a number of samples later. Suppose the signal $\mathbf{x}(k)$ is real-valued, we consider an LMS algorithm with the update equation

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu e(k-1)\mathbf{x}(k-1)$$

where the error signal is given by

$$e(k-1) = d(k-1) - y(k-1) = d(k-1) - \mathbf{w}_{k-1}^T \mathbf{x}(k-1).$$

Here, $d(k)$ is the desired signal. This LMS algorithm works with signals delayed by one sample. Without any delay, this would be the conventional LMS algorithm.

(5pts) (a) Find the values of μ for which this LMS algorithm converges in the mean.

Hint: The solution of a second-order difference equation converges to zero when the roots associated with its characteristic equation lie within the unit circle.

(5pts) (b) Determine the slowest decaying mode when the eigenvalues of $\mathbf{R}_x = E\{\mathbf{x}(k)\mathbf{x}(k)\}$ are all one and we choose $\mu = 0.5$. Find the time constant for the LMS algorithm.

Question 2 (10 points)

An autoregressive process of order 1 is described by the difference equation

$$x(n) = 0.5x(n-1) + w(n)$$

where $w(n)$ is zero-mean white noise with a variance $\sigma_w^2 = 0.64$. The observed process $y(n)$ is described by

$$y(n) = x(n) + v(n)$$

where $v(n)$ is zero-mean white noise with a variance of $\sigma_v^2 = 1$.

(2pts) (a) Write the Kalman filtering equations to find the minimum mean-square estimate, $\hat{x}(n|n)$, of $x(n)$ given the observations $y(i), i = 1, \ldots, n$. The initial conditions are $\hat{x}(0|0) = 0$ and $E\{|x(0) - \hat{x}(0|0)|^2\} = 1$.

(3pts) (b) Assuming that the filter reaches a steady state solution, find the steady state covariance and the steady state Kalman gain. In the steady state, $P(n+1|n+1) = P(n|n)$. Find the limiting form of the estimation equation for $\hat{x}(n|n)$.

(5pts) (c) Now consider the autoregressive moving average ARMA(1,1) process

$$y(n) + ay(n-1) = w(n) + bw(n-1).$$

Give the state-state model for the ARMA process and write the Kalman filtering equations.