Indian Institute of Science
Department of Electrical Communications Engineering

E9 211: Adaptive Signal Processing

October 2020 - January 2021

Homework 2 (deadline 24 Dec. 2020)

This homework consists of two parts on implementing and studying the adaptive filters: (a) for source separation using an antenna array and (b) for channel estimation and equalization using the convolutive model with a single source and a single receiver.

Make a short report containing the required Matlab/Python files, plots, explanations, and answers, and turn it in by the deadline using Microsoft Teams under your name.

Part A: Antenna beamforming

As in HW1, using `function X = gen_data(M,N,Delta,theta,SNR)` generate the data matrix $\mathbf{X} = \mathbf{A}_\theta \mathbf{S} + \mathbf{N}$. Recall that

$$\mathbf{A}_\theta = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_d)] : M \times d.$$

The source symbols $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_d]^{\mathrm{T}} : d \times N$ are chosen uniformly at random from a QPSK alphabet $\{(\pm 1 \pm j)/\sqrt{2}\}$. The noise matrix $\mathbf{N} : M \times N$ is random zero-mean complex Gaussian matrix.

Consider a system with two sources and take $\boldsymbol{\theta} = [0°, 5°]^{\mathrm{T}}$, $M = 5$, $\Delta = 0.5$, $N = 2000$, SNR $= 20$ dB. Make Matlab subroutines to

1. Compute the beamformer for the first source, i.e., $\mathbf{y} = \hat{\mathbf{s}}_1 = \mathbf{w}^{\mathrm{H}} \mathbf{X}$ using LMS as

   `[y,w]=lms(X,s_ref,mu,w_init)`

   Use $\mathtt{w_{init}} = \mathbf{0}$ and for $\mathtt{s_{ref}}$ use the true source symbols of the source at $0°$. Plot the estimated symbols in the complex plane such that you observe four clusters (use `plot(s_est,'x')`) and compare it with the symbol estimates from the LMMSE receiver from HW1. Also, plot the learning curve for different values of `mu` to show the convergence and divergence of LMS. Compare it the minimum mean-squared error obtained with the LMMSE receiver when the algorithm converges. What do you observe?

2. Construct a blind equalizer using CMA(1,2) algorithm

   `[y,w]=cma12(X,mu,w_init)`

   Use $\mathtt{w_{init}} = [0, 1, \mathbf{0}]^{\mathrm{T}}$ and $\mathtt{mu} = 0.01$. Plot the received symbols before and after beamforming as scatter plots. What do you observe, which source does the beamformer converge to, and why? Also, plot the learning curve `abs(y)` for different values of `mu`, and comment your observations. Now initialize the CMA algorithm using the LMMSE solution; what changes now?

Part B: Channel estimation and equalization

As in HW1, using `function x = gen_data1(h,s,SNR)` generate the data. Make Matlab subroutines to

1. Estimate the channel using pilots using steepest gradient descent method

   ```
   [h]=sd(X,S_pilot,mu,h_init)
   ```

   Plot the channel estimates and compare it to the true channel and the minimum variance unbiased channel estimator from HW1 for $N = 1000$ and $\text{SNR} = \{10, 100\}$ dB. What can you conclude?

2. To estimate the source sequence, you need to construct a blind equalizer with M taps using CMA(2,2).

   ```
   [y,w]=cma22(X,mu,w_init)
   ```

   Use $\mathtt{w_{init}} = [0, 1, \mathbf{0}]^{\mathrm{T}}$ and $\mathtt{mu} = 0.01$. As before, plot the estimated symbols in the complex plane for SNR values of 10 dB and 100 dB, $M = 5$ and $N = 1000$. Which row/delay of $\mathbf{S}$ does the equalizer converege to and why?