Joint Clock Synchronization and Ranging: Asymmetrical Time-Stamping and Passive Listening

Sundeep Prabhakar Chepuri, *Student Member, IEEE*, Raj Thilak Rajan, *Student Member, IEEE*, Geert Leus, *Fellow, IEEE*, and Alle-Jan van der Veen, *Fellow, IEEE*

***Abstract*—A fully asynchronous network with one sensor and M anchors (nodes with known locations) is considered in this letter. We propose a novel asymmetrical time-stamping and passive listening (ATPL) protocol for joint clock synchronization and ranging. The ATPL protocol exploits broadcast to not only reduce the number of active transmissions between the nodes, but also to obtain more information. This is used in a simple estimator based on least-squares (LS) to jointly estimate all the unknown clock-skews, clock-offsets, and pairwise distances of the sensor to each anchor. The Cramér–Rao lower bound (CRLB) is derived for the considered problem. The proposed estimator is shown to be asymptotically efficient, meets the CRLB, and also performs better than the available clock synchronization algorithms.**

***Index Terms*—Clock synchronization, clock-offset, clock-skew, wireless sensor networks.**

I. Introduction

Clock synchronization among different nodes each having its own autonomous clock is a key component of a wireless sensor network (WSN). A WSN enables coordinated functions such as data sampling, data fusion, time-based channel sharing and scheduling, sleep and wake-up coordination, and other time-based tasks [1]. These tasks demand a common time frame for the entire network. Individual clocks in a WSN drift from each other due to imperfections in the oscillator, aging and other environmental variations, and it is essential to determine these drifts. Sensor nodes are usually powered with just a battery. Thus, all the tasks of a WSN, including synchronization, should be carefully performed to ensure longer operating lifetime. For synchronization, this means to minimize the number of transmissions between nodes during which the time-stamps are recorded.

Early research on synchronization focussed on protocol design [1], e.g., hierarchical protocols like the network time protocol (NTP), which can be used for applications where the requirements on the synchronization accuracies are not too stringent. For better synchronization (below the order of a ms), a plethora of algorithms based on the exchange of time-stamps has been proposed [1]–[4], which could operate via two-way time-stamp exchange [2] or pairwise broadcast synchronization (PBS) [3]. In a WSN, assuming one of the nodes as a reference, the unknown clock-skew and clock-offset of the other nodes could be estimated using a pairwise least-squares (PLS) estimator [4]. Extending from a pair of nodes to a network, a global least-squares (GLS) estimator based on the two-way time-stamp exchange was also proposed in [4], for joint synchronization and ranging. The GLS algorithm exploits the redundancy achieved due to all possible pairwise links in the network.

In this letter, we propose a novel scheme for joint clock synchronization and ranging in energy-efficient WSNs, in which we harness the broadcast property of the wireless medium to significantly reduce the number of active transmissions between the nodes and at the same time we aim to increase both the synchronization and ranging accuracies.

More specifically, we propose an asymmetrical time-stamping and passive listening (ATPL) protocol for joint clock synchronization and ranging. The ATPL protocol presumes the protocol proposed in [5] and [3]. The main goal of [3] was synchronization and did not focus on ranging. The algorithm proposed in [5] also exploits the broadcast property and focussed on localization of a target node in an asynchronous network, however, estimation of the clock parameters was not specifically considered. In the ATPL protocol, during communication between a pair of nodes, time-stamps are recorded and exchanged. Besides this, the remaining nodes in the network also passively listen and record the time-stamps with their respective clocks, in a cooperative way. However, they do not respond back to either of the active node pair. In addition, the protocol does not put any constraint on the sequence of transmissions, and this together with passive listening results in asymmetrical links, and hence, asymmetrical time-stamps. The ATPL protocol is energy-efficient in the sense that we obtain more information just by passive listening, and reception usually consumes less power than transmission.

For a fully asynchronous network with one sensor and M anchors, we propose a novel estimator based on the ATPL protocol for jointly estimating all the unknown clock-skews, clock-offsets, and pairwise distances of the sensor to each anchor, which is the main contribution of this work.

Notation

Upper (lower) bold face letters are used for matrices (column vectors); $(\cdot)^T$ denotes transposition; $\odot$ ($\oslash$) denotes the element-wise matrix or vector product (division); $(.)^{\odot 2}$ denotes the element-wise matrix or vector squaring; $\mathrm{bdiag}(.)$ a block diagonal matrix with the matrices in its argument on the main diagonal;

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S. P. Chepuri, G. Leus, and A.-J. van der Veen are with the Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Delft, The Netherlands (e-mail: s.p.chepuri@tudelft.nl; g.j.t.leus@tudelft.nl;a.j.vanderveen@tudelft.nl.

R. T. Rajan is with the Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, The Netherlands, and also with Netherlands Institute for Radio Astronomy (ASTRON), Dwingeloo, The Netherlands. (e-mail: rajan@astron.nl).

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$\mathbf{1}_N$ ($\mathbf{0}_N$) denotes the $N \times 1$ vector of ones (zeros); $\mathbf{I}_N$ is an identity matrix of size N; $\mathbb{E}(.)$ denotes the expectation operation; $\otimes$ is the Kronecker product.

II. System Model

We consider a fully asynchronous network with M anchors and one sensor (*node* 0). We assume that one of the nodes has a relatively stable clock oscillator and is used as a clock reference. All the other nodes suffer from clock-skews and clock-offsets. The network model considered here is a special case of the model in [4], as the pairwise distances of certain nodes (anchors) are now assumed to be known.

The distance between the ith and the jth node is denoted by $d_{i,j} = d_{j,i}$. The distance between the sensor and the ith anchor is denoted by $d_{0,i} = d_{i,0}$, and is unknown. Let t_i be the local time at the ith node and t be the reference time. We assume that the relation between the local time and the reference time can be given by a first order affine clock model [4],

$$t_i = \omega_i t + \phi_i \quad \Leftrightarrow \quad t = \alpha_i t_i + \beta_i \tag{1}$$

where $\omega_i \in \mathbb{R}_+$ is the clock-skew, $\phi_i \in \mathbb{R}$ is the clock-offset, $\alpha_i = \omega_i^{-1}$ and $\beta_i = -\omega_i^{-1}\phi_i$ are the synchronization parameters of the ith node. Without loss of generality, we use anchor M as absolute time reference, i.e., $[\omega_M, \phi_M] = [1, 0]$. The unknown synchronization parameters are collected in $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \ldots, \alpha_{M-1}]^T$ and $\boldsymbol{\beta} = [\beta_0, \beta_1, \ldots, \beta_{M-1}]^T$. The unknown clock-skews and clock-offsets are respectively given by

$$\boldsymbol{\omega} = \mathbf{1}_M \oslash \boldsymbol{\alpha} \quad \text{and} \quad \boldsymbol{\phi} = -\boldsymbol{\beta} \oslash \boldsymbol{\alpha}. \tag{2}$$

The transmission and reception time-stamps are recorded both during the forward link (ith active anchor to the sensor) and the reverse link (sensor to the ith active anchor). The time-stamp recorded at the ith node when the kth iteration message departs is denoted by $T_i^{(k)}$, and on arrival of the corresponding message, the jth node records the time-stamp $R_{i,j}^{(k)}$. Note that the time-stamps recorded at the sensor will be either $T_0^{(k)}$ or $R_{i,0}^{(k)}$.

III. The ATPL Protocol

In the two-way time-stamp exchange protocol between the ith anchor and the sensor, the remaining nodes of the network are idle. In the ATPL protocol, we propose that all the remaining $M-1$ anchors passively listen to the communication between the ith anchor and the sensor, and record the time-stamps $R_{i,j\neq 0}^{(k)}$ of their respective local clocks. By doing so, we obtain more information with extra equations corresponding to transmissions between a) active anchor and other passive anchors, and b) sensor and remaining passive anchors. This is additional to the equations corresponding to the active anchor sensor pair as compared to the two-way time-stamp exchange. The ATPL protocol initiated by the ith anchor is illustrated in Fig. 1. An illustration of the sequence of time-stamps recorded during the ATPL protocol is shown in Fig. 2.

In the proposed protocol, we do not put any constraints on the sequence of forward links and reverse links [4], i.e., the reverse link need not always follow the forward link as in [1]–[3]. This means that the sensor need not respond to the request from the anchor immediately. Therefore the processing time at the sensor typically considered in clock synchronization algorithms [1]–[3], [5] need not be taken into account as long as the clock parameters are stable within certain reasonable limits.

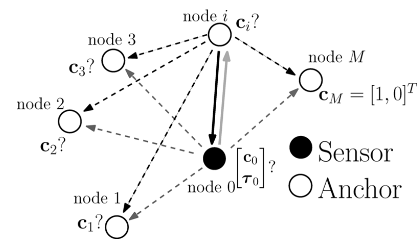

Fig. 1. The ATPL protocol with the ith anchor transmitting. (Solid (dotted) lines refer to the active (passive) links. Dark (light) shaded lines refer to the forward (reverse) link.).

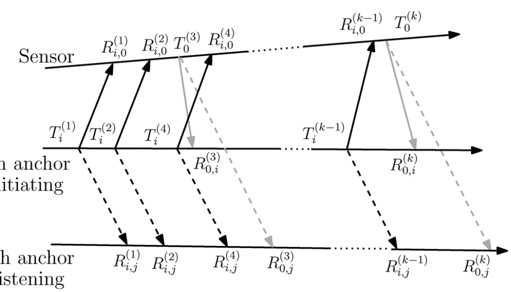

Fig. 2. An example sequence of the recorded time-stamps. (Solid (dotted) lines refer to the active (passive) links. Dark (light) shaded lines refer to the forward (reverse) link.

Remark 1: (Protocol modes): Possible ways of executing the ATPL protocol are **mode a)** Anchor node i *makes* K_i *transmissions and the sensor replies back with* K_0 *messages, with* K_0 *not necessarily equal to* K_i *and the transmissions need not be sequential. This is repeated by all the remaining anchors;* **mode b)** each anchor node i *makes* K_i *transmissions, and, in the end the sensor replies only once with* K_0 *messages; and* **mode c)** m*-out-of-*M *anchors* ($m \leq M$) *make transmissions, and the sensor replies as described in either mode a or mode b.*

A suitable protocol mode can be adopted depending on the performance requirement and the energy constraint per node.

Remark 2: (Centralized or distributed): The computation can be done in a centralized way in a fusion center (FC). However, there is an involved communication load in transmitting the time-stamps recorded at each node to a FC.

An FC based approach can be avoided by including the time-stamps $R_{i,0}^{(k)}$, $k = 1, 2, \ldots, K_i$ *in the payload when the sensor responds to the* i*th anchor. However, additional broadcast messages to distribute the time-stamps a)* $R_{0,i}^{(k)}$*, b)* $R_{i,j}^{(k)}$ *and* $R_{0,j}^{(k)}$ *are still required. This approach would avoid transmission of the computed unknown parameters to the nodes, that is required with an FC based approach. Moreover, it allows each node to independently perform computations in a distributed fashion.*

IV. Proposed Joint Estimator

The time-of-flight for a line-of-sight (LOS) transmission between the ith and the jth node can be defined as $\tau_{i,j} = \nu^{-1} d_{i,j}$, where ν denotes the speed of a wave (electromagnetic or acoustic) in a medium. Using (1), $\tau_{i,j}$ can be written in terms of the time-stamps recorded using respective local clocks of the ith and jth node as

$$\tau_{i,j} = (\alpha_j R_{i,j}^{(k)} + \beta_j) - (\alpha_i T_i^{(k)} + \beta_i) + n_{i,j}^{(k)} \tag{3}$$

where $n_{i,j}^{(k)}$ denotes the aggregate measurement error on the time-stamps.

The transmission and reception time-stamps recorded at the ith and the jth node are respectively collected in vectors $\mathbf{t}_i = [T_i^{(1)}, T_i^{(2)}, \ldots, T_i^{(K_i)}]^T \in \mathbb{R}^{K_i \times 1}$ and $\mathbf{r}_{i,j} = [R_{i,j}^{(1)}, R_{i,j}^{(2)}, \ldots, R_{i,j}^{(K_i)}]^T \in \mathbb{R}^{K_i \times 1}$, where K_i is the number of transmissions made by the ith node. The error vector is denoted by $\mathbf{n}_{i,j} = [n_{i,j}^{(1)}, n_{i,j}^{(2)}, \ldots, n_{i,j}^{(K_i)}]^T \in \mathbb{R}^{K_i \times 1}$.

For the sake of exposition, we consider a network with one sensor (*node* 0) and $M = 2$ anchors (*node* 1 and *node* 2) and the following example protocol: (i) *node* 1 makes K_1 transmissions, *node* 0 and *node* 2 passively listen, (ii) *node* 2 makes K_2 transmissions, *node* 0 and *node* 1 passively listen, and finally (iii) sensor *node* 0 responds with K_0 messages and *node* 1 and *node* 2 passively listen. This is an example of protocol *mode b* that was described earlier.

Collecting the clock parameters of the ith node in a vector $\mathbf{c}_i = [\alpha_i, \beta_i]^T$, we can now write the equations of the form given in (3), obtained for all the $K = K_0 + K_1 + K_2$ time-stamps recorded in a matrix-vector form as shown in (4) at the bottom of the page. The ordering of the rows of the system matrix $\mathbf{A}$ is arbitrary and does not imply the order of transmission. The columns of $\mathbf{A}$ corresponding to $\tau_{0,1}$, $\tau_{0,2}$, and $\tau_{1,2}$ have two non-zero sub-vectors each as $\tau_{i,j} = \tau_{j,i}$.

We define the vector $\boldsymbol{\tau}_0 = [\tau_{0,1}, \tau_{0,2}, \ldots, \tau_{0,M}]^T \in \mathbb{R}^{M \times 1}$, where the entries of $\boldsymbol{\tau}_0$ are not known. Note that $\tau_{1,2} = \nu^{-1} d_{1,2}$ corresponds to the distance between the nodes 1 and 2, and is known.

Remark 3: (Rank-deficiency): The linear model in (4) does not have a unique solution, unless we impose certain linear constraints. Here, we do that by assigning node 2 as the clock-reference, i.e., $\mathbf{c}_2 = [1, 0]^T$.

Moving all the knowns (columns corresponding to $\mathbf{c}_2$ and $\tau_{1,2}$) to one side, (4) simplifies to the generalized linear model given in (5) at the bottom of the page.

The generalization of the data model (5) for any $M > 2$ is straightforward and can be easily derived along the same lines. The generalized linear model based on the ATPL protocol is given by

$$\bar{\mathbf{A}}\boldsymbol{\theta} = \mathbf{x} + \mathbf{n} \tag{6}$$

where $\bar{\mathbf{A}} \in \mathbb{R}^{KM \times 3M}$, $\boldsymbol{\theta} \in \mathbb{R}^{3M \times 1}$, $\mathbf{x} \in \mathbb{R}^{KM \times 1}$ and $\mathbf{n} \in \mathbb{R}^{KM \times 1}$, all having a similar structure as that of (5).

Remark 4: (Correlated error vector): In case of broadcasting, the entries of the error vector $\mathbf{n}$ *are not uncorrelated due to a common error on the transmit time-stamp* $T_i^{(k)}$.

We assume that the aggregate error $n_{i,j}^{(k)}$ in (3) is due to the additive stochastic noise components on the time-stamps, $T_i^{(k)}$ denoted by $\epsilon_i^{(k)}$ and the time-stamps, $R_{i,j}^{(k)}$ denoted by $\epsilon_{i,j}^{(k)}$. We model the aggregate error in (3) as

$$n_{i,j}^{(k)} = \epsilon_i^{(k)} + \epsilon_{i,j}^{(k)} \tag{7}$$

where both $\epsilon_i^{(k)}$ and $\epsilon_{i,j}^{(k)}$ are modeled as zero mean i.i.d. Gaussian [6] with variance $0.5\sigma^2$, such that, $\mathbb{E}(\epsilon_i^{(k)}\epsilon_{i,j}^{(k)}) = 0$ for $i \neq j$. (This is a simplified noise model and more accurate models could be considered.)

We can compute the covariance matrix as $\boldsymbol{\Sigma} = \mathrm{bdiag}(\boldsymbol{\Sigma}_0, \boldsymbol{\Sigma}_1, \ldots, \boldsymbol{\Sigma}_M) \in \mathbb{R}^{MK \times MK}$, where $\boldsymbol{\Sigma}_i = \mathbb{E}(\mathbf{n}_i \mathbf{n}_i^T)$. For $M = 2$, we find

$$\boldsymbol{\Sigma}_i = \begin{bmatrix} \sigma^2 \mathbf{I}_{K_i} & 0.5\sigma^2 \mathbf{I}_{K_i} \\ 0.5\sigma^2 \mathbf{I}_{K_i} & \sigma^2 \mathbf{I}_{K_i} \end{bmatrix} \in \mathbb{R}^{2K_i \times 2K_i}. \tag{8}$$

The structure of $\boldsymbol{\Sigma}$ can be generalized for any $M > 2$ in a similar way, leading to $\boldsymbol{\Sigma}_i = 0.5\sigma^2(\mathbf{1}_M \mathbf{1}_M^T + \mathbf{I}_M) \otimes \mathbf{I}_{K_i} \in \mathbb{R}^{MK_i \times MK_i}$.

We can now prewhiten the observation model in (5) by forming $\mathbf{A}' = \boldsymbol{\Sigma}^{-1/2}\bar{\mathbf{A}}$ and $\mathbf{x}' = \boldsymbol{\Sigma}^{-1/2}\mathbf{x}$. For $K \geq 3$, $\bar{\mathbf{A}}$ is tall and is left-invertible. Hence, the unknown parameters in $\boldsymbol{\theta}$ can be estimated using LS, i.e.,

$$\hat{\boldsymbol{\theta}} = (\mathbf{A}'^T \mathbf{A}')^{-1} \mathbf{A}'^T \mathbf{x}' = (\bar{\mathbf{A}}^T \boldsymbol{\Sigma}^{-1} \bar{\mathbf{A}})^{-1} \bar{\mathbf{A}}^T \boldsymbol{\Sigma}^{-1} \mathbf{x}. \tag{9}$$

Subsequently, the clock-skews $\boldsymbol{\omega}$, clock-offsets $\boldsymbol{\phi}$ can be obtained using the relation in (2), and the pairwise distances of the sensor to each anchor using the relation $\mathbf{d}_0 = \nu \boldsymbol{\tau}_0$.

$$\begin{array}{ll} \text{Node 0 responds} & j=1 \\ (i=0) & j=2 \\ \hdashline \text{Node 1 transmits} & j=0 \\ (i=1) & j=2 \\ \hdashline \text{Node 2 transmits} & j=0 \\ (i=2) & j=1 \end{array}
\overbrace{\left[\begin{array}{cc|cc|cc|c|c|c}
\mathbf{t}_0 & \mathbf{1}_{K_0} & -\mathbf{r}_{0,1} & -\mathbf{1}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} & \mathbf{1}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} \\
\mathbf{t}_0 & \mathbf{1}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} & -\mathbf{r}_{0,2} & -\mathbf{1}_{K_0} & \mathbf{0}_{K_0} & \mathbf{1}_{K_0} & \mathbf{0}_{K_0} \\ \hdashline
-\mathbf{r}_{1,0} & -\mathbf{1}_{K_1} & \mathbf{t}_1 & \mathbf{1}_{K_1} & \mathbf{0}_{K_1} & \mathbf{0}_{K_1} & \mathbf{1}_{K_1} & \mathbf{0}_{K_1} & \mathbf{0}_{K_1} \\
\mathbf{0}_{K_1} & \mathbf{0}_{K_1} & \mathbf{t}_1 & \mathbf{1}_{K_1} & -\mathbf{r}_{1,2} & -\mathbf{1}_{K_1} & \mathbf{0}_{K_1} & \mathbf{0}_{K_1} & \mathbf{1}_{K_1} \\ \hdashline
-\mathbf{r}_{2,0} & -\mathbf{1}_{K_2} & \mathbf{0}_{K_2} & \mathbf{0}_{K_2} & \mathbf{t}_2 & \mathbf{1}_{K_2} & \mathbf{0}_{K_2} & \mathbf{1}_{K_2} & \mathbf{0}_{K_2} \\
\mathbf{0}_{K_2} & \mathbf{0}_{K_2} & -\mathbf{r}_{2,1} & -\mathbf{1}_{K_2} & \mathbf{t}_2 & \mathbf{1}_{K_2} & \mathbf{0}_{K_2} & \mathbf{0}_{K_2} & \mathbf{1}_{K_2}
\end{array}\right]}^{\mathbf{A} \in \mathbb{R}^{2K \times 9},\, K = K_0 + K_1 + K_2}
\begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \tau_{0,1} \\ \tau_{0,2} \\ \tau_{1,2} \end{bmatrix}
= \overbrace{\left[\begin{array}{c} \mathbf{n}_{0,1} \\ \mathbf{n}_{0,2} \\ \hdashline \mathbf{n}_{1,0} \\ \mathbf{n}_{1,2} \\ \hdashline \mathbf{n}_{2,0} \\ \mathbf{n}_{2,1} \end{array}\right]}^{\mathbf{n} \in \mathbb{R}^{2K \times 1}}
\begin{array}{l} \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\}\mathbf{n}_0 \\ \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\}\mathbf{n}_1 \\ \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\}\mathbf{n}_2 \end{array}. \tag{4}$$

$$\begin{array}{ll} \text{Node 0 responds} & j=1 \\ (i=0) & j=2 \\ \hdashline \text{Node 1 transmits} & j=0 \\ (i=1) & j=2 \\ \hdashline \text{Node 2 transmits} & j=0 \\ (i=2) & j=1 \end{array}
\overbrace{\left[\begin{array}{cc|cc|cc}
\mathbf{t}_0 & \mathbf{1}_{K_0} & -\mathbf{r}_{0,1} & -\mathbf{1}_{K_0} & \mathbf{1}_{K_0} & \mathbf{0}_{K_0} \\
\mathbf{t}_0 & \mathbf{1}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} & \mathbf{1}_{K_0} \\ \hdashline
-\mathbf{r}_{1,0} & -\mathbf{1}_{K_1} & \mathbf{t}_1 & \mathbf{1}_{K_1} & \mathbf{1}_{K_1} & \mathbf{0}_{K_1} \\
\mathbf{0}_{K_1} & \mathbf{0}_{K_1} & \mathbf{t}_1 & \mathbf{1}_{K_1} & \mathbf{0}_{K_1} & \mathbf{0}_{K_1} \\ \hdashline
-\mathbf{r}_{2,0} & -\mathbf{1}_{K_2} & \mathbf{0}_{K_2} & \mathbf{0}_{K_2} & \mathbf{0}_{K_2} & \mathbf{1}_{K_2} \\
\mathbf{0}_{K_2} & \mathbf{0}_{K_2} & -\mathbf{r}_{2,1} & -\mathbf{1}_{K_2} & \mathbf{0}_{K_2} & \mathbf{0}_{K_2}
\end{array}\right]}^{\bar{\mathbf{A}} \in \mathbb{R}^{2K \times 6}}
\overbrace{\begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \boldsymbol{\tau}_0 \end{bmatrix}}^{\boldsymbol{\theta} \in \mathbb{R}^{6 \times 1}}
= \overbrace{\left[\begin{array}{cc|c}
\mathbf{0}_{K_0} & \mathbf{0}_{K_0} & \mathbf{0}_{K_0} \\
-\mathbf{r}_{0,2} & -\mathbf{1}_{K_0} & \mathbf{0}_{K_0} \\ \hdashline
\mathbf{0}_{K_1} & \mathbf{0}_{K_1} & \mathbf{0}_{K_1} \\
-\mathbf{r}_{1,2} & -\mathbf{1}_{K_1} & \mathbf{1}_{K_1} \\ \hdashline
\mathbf{t}_2 & \mathbf{1}_{K_2} & \mathbf{0}_{K_2} \\
\mathbf{t}_2 & \mathbf{1}_{K_2} & \mathbf{1}_{K_2}
\end{array}\right]
\begin{bmatrix} \mathbf{c}_2 \\ \tau_{1,2} \end{bmatrix}}^{\mathbf{x} \in \mathbb{R}^{2K \times 1}}
+ \mathbf{n}. \tag{5}$$

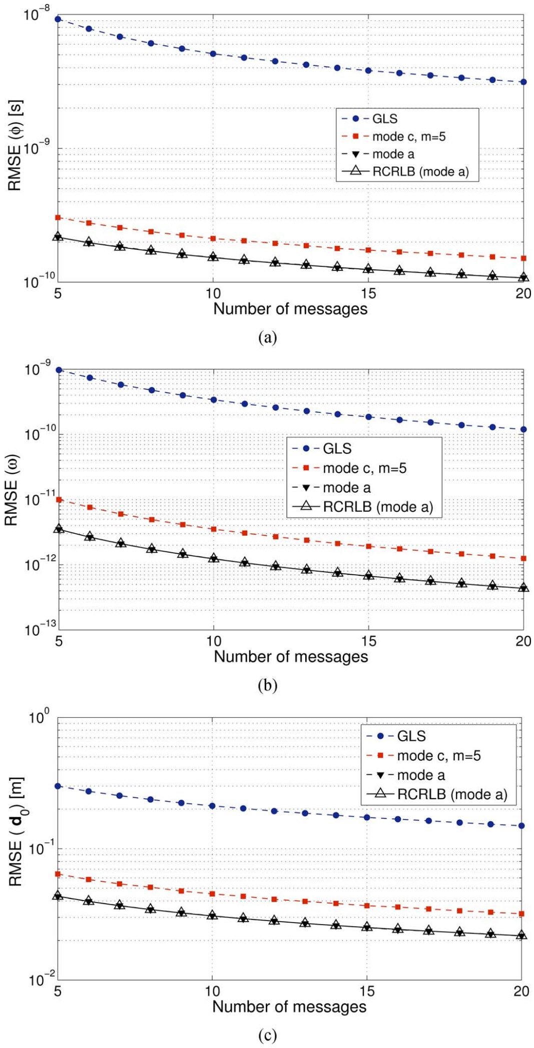

Fig. 3. RMSE of the estimated unknown parameters. (a) clock-offsets $\boldsymbol{\phi}$. (b) clock-skews $\boldsymbol{\omega}$. (c) pairwise distances $\mathbf{d}_0$.

Remark 5: (Sensor does not respond): When only one of the nodes transmits, say node 1, $\bar{\mathbf{A}}$ *in (5) will not have rows corresponding to transmissions of node 0 and node 2, and it is rank-deficient as the columns two and five are dependent. This also holds when only either node 2 or node 0 transmits.*

If only anchor nodes transmit, and the sensor does not respond, then $\bar{\mathbf{A}}$ *will not have rows corresponding to transmissions of node 0. In that case,* $\bar{\mathbf{A}}$ *will be again rank-deficient, as column two is a linear combination of columns five and six. Therefore, for (6) to have a unique solution, the sensor should respond at least once.* $K_0 = 1$ *is possible if* $K \geq 3$ *is satisfied.*

The possibility that the sensor node responds with only one message in the end makes the protocol energy-efficient.

The proposed algorithm can be seen as a specialized version of GLS [4], taking into account: (1) known distances between anchors which are hence not estimated, (2) the broadcast property, which results in additional observations and a correlated error as compared to the pairwise transmissions.

V. Cramér–Rao Lower Bound

For an unbiased estimator $\hat{\bar{\boldsymbol{\theta}}}$ it follows from the CRLB theorem that $\mathbb{E}(\hat{\bar{\boldsymbol{\theta}}}\hat{\bar{\boldsymbol{\theta}}}^T) \geq \mathbf{F}^{-1}$, where $\bar{\boldsymbol{\theta}} = [\boldsymbol{\omega}^T, \boldsymbol{\phi}^T, \mathbf{d}_0^T]^T$ and $\mathbf{F}$ is the Fisher information matrix. If the error $\mathbf{n}$ is zero-mean Gaussian, then $\mathbf{F} \in \mathbb{R}^{3M\times 3M}$ can be computed as $\mathbf{F} = \mathbf{J}^T\boldsymbol{\Sigma}^{-1}\mathbf{J}$, where $\mathbf{J}$ is a Jacobian matrix. The Jacobian matrix is given by [4]

$$\mathbf{J} = \frac{\partial(\bar{\mathbf{A}}\boldsymbol{\theta} - \mathbf{x})}{\partial\bar{\boldsymbol{\theta}}^T} = [\mathbf{J}_{\boldsymbol{\omega}} \quad \mathbf{J}_{\boldsymbol{\phi}} \quad \mathbf{J}_{\mathbf{d}_0}] \in \mathbb{R}^{KM\times 3M} \tag{10}$$

with sub-blocks

$$\begin{aligned}
\mathbf{J}_{\boldsymbol{\omega}} &= \frac{\partial(\bar{\mathbf{A}}\boldsymbol{\theta} - \mathbf{x})}{\partial\boldsymbol{\omega}^T} \\
&= -\left(\bar{\mathbf{A}}\mathbf{S}_{\boldsymbol{\alpha}} - \bar{\mathbf{A}}\mathbf{S}_{\boldsymbol{\beta}} \odot \mathbf{1}_{KM}\boldsymbol{\phi}^T\right) \oslash \left(\mathbf{1}_{KM}\boldsymbol{\omega}^T\right)^{\odot 2}, \\
\mathbf{J}_{\boldsymbol{\phi}} &= \frac{\partial(\bar{\mathbf{A}}\boldsymbol{\theta} - \mathbf{x})}{\partial\boldsymbol{\phi}^T} = -\bar{\mathbf{A}}\mathbf{S}_{\boldsymbol{\beta}} \oslash \mathbf{1}_{KM}\boldsymbol{\omega}^T, \\
\mathbf{J}_{\mathbf{d}_0} &= \frac{\partial(\bar{\mathbf{A}}\boldsymbol{\theta} - \mathbf{x})}{\partial\mathbf{d}_0^T} = \nu^{-1}\bar{\mathbf{A}}\mathbf{S}_{\boldsymbol{\tau}_0}
\end{aligned}$$

where $\mathbf{S}_{\boldsymbol{\alpha}}$, $\mathbf{S}_{\boldsymbol{\beta}}$, and $\mathbf{S}_{\boldsymbol{\tau}_0}$ are selection matrices to select the columns of $\bar{\mathbf{A}}$ corresponding to $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\tau}_0$, respectively.

VI. Simulations

A network with one sensor and 10 anchors is considered for simulations. Both the sensor and the anchor nodes are deployed randomly within a range of 100 m. Clock-skews $\boldsymbol{\omega}$ and clock-offsets $\boldsymbol{\phi}$ are uniformly distributed in the range $[1 - 100\ \mathrm{ppm}, 1 + 100\ \mathrm{ppm}]$ and $[-1\ \mathrm{s}, 1\ \mathrm{s}]$, respectively. We use an observation interval of 100 s during which the clock-parameters are fixed and $\nu = 3 \times 10^8\ \mathrm{m/s}$. The time-stamps are corrupted with an i.i.d. Gaussian process having a standard deviation $\sigma = 1\ \mathrm{ns}$[6].

The proposed estimator based on the ATPL protocol is compared with the GLS algorithm proposed in [4, Fig. 3(c)], as it is already shown to outperform other existing synchronization algorithms. We apply the GLS algorithm based on two-way communication between each sensor anchor pair. Fig. 3 shows the root mean square error (RMSE) of the estimates $\boldsymbol{\phi}$ and $\boldsymbol{\omega}$, and $\mathbf{d}_0$ for different number of messages, K. We show simulations for *mode a* and *mode c* of the ATPL protocol described in Section III. It can be seen from the figures that the proposed algorithm performs better than GLS in both the considered scenarios due the additional links obtained from passive listening. The proposed algorithm also achieves the theoretical root CRLB (RCRLB).

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