This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2009 proceedings.

Robust THP Transceiver Designs for Multiuser MIMO Downlink

P. Ubaidulla and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012,INDIA

***Abstract*—In this paper, we present two robust nonlinear transceiver designs for multiuser multi-input multi-output (MIMO) downlink in the presence of imperfections in the channel state information at the transmitter (CSIT). Both the base station (BS) as well as the users are equipped with multiple antennas. The BS employs Tomlinson-Harashima precoding (THP) for inter-user interference pre-cancellation at the transmitter. First, we consider the case where the CSIT error is Gaussian-distributed. In this case, the robust transceiver design seeks to minimize a stochastic function of the sum mean square error (SMSE) under a constraint on the total BS transmit power. We propose an iterative algorithm to solve this problem. Each iteration involves the solution of a second order cone program (SOCP). Next, we consider the case where the CSIT error can be specified by an uncertainty set. In this case, we consider a minimax design for the robust transceiver, where the worst-case SMSE is minimized under a constraint on the total BS transmit power. We show that this design problem can be solved by an iterative algorithm, wherein each iteration involves a pair of semi-definite programs (SDP). Further, we consider an extension of the proposed algorithm to the case with per-antenna power constraints. We illustrate the robustness of the proposed algorithms to imperfections in CSIT through simulations.**

I. Introduction

Multiuser multiple-input multiple-output (MIMO) wireless communication systems have attracted considerable interest due to their potential to offer the benefits of transmit diversity and increased channel capacity [1], [2]. Multiuser interference limits the performance of such multiuser systems. To realize the potential of such systems in practice, it is important to devise methods to reduce the multiuser interference. Transmit-side processing in the form of precoding has been studied widely [2] as a means to reduce the multiuser interference. Several studies on linear and non-linear precoding have been reported in the literature.

Joint precoder and receive filter designs for multiuser MIMO systems with several performance criteria have been widely reported in the literature [3]–[8]. A transceiver design based on minimizing the total BS transmit power under individual user SINR constraints is reported in [9]. Transceiver design with a general QoS constraint is considered in [7]. Another important criterion that has been frequently used in precoder designs for multiuser MIMO downlink is sum mean square error (SMSE). Iterative algorithms that minimize SMSE with a constraint on total transmit power are reported in [4], [5]. These algorithms are not guaranteed to converge to the global minimum. Minimum SMSE transceiver designs based on uplink-downlink duality have been proposed in [6], [7]. These algorithms are guaranteed to converge to the global minimum. Non-linear transceivers, though more complex, result in improved performance compared to the linear transceivers. Studies on non-linear THP transceiver design have been reported in the literature. An iterative THP transceiver design minimizing weighted SMSE has been reported in [8]. A THP transceiver design minimizing total BS transmit power under SINR constraints is reported in [9].

All the studies on transceiver designs mentioned above assume the availability of perfect channel state information at the transmitter (CSIT). However, in practice, the CSIT is usually imperfect due to different factors like estimation error, feedback delay, quantization, etc. Hence, it is of interest to develop transceiver designs that are robust to errors in CSIT. Robust linear and non-linear transceiver designs for multiuser multi-input single-output (MISO) downlink have been widely studied [10]–[13]. Recently, a robust transceiver design for multiuser MIMO downlink minimizing total BS transmit power under individual user MSE constraints has been reported in [14].

In this paper, we consider robust THP transceiver designs for the downlink of a multiuser *MIMO* system in the presence of imperfect CSIT, which has not been reported so far. We consider two different models for the CSIT error, and propose robust THP transceiver designs suitable for these models. First, we consider a stochastic error (SE) model for the CSIT error. In this model, the CSIT error is assumed to be follow a Gaussian distribution. In this case, we adopt a statistical approach to the robust design. The transceiver design is based on minimizing the SMSE averaged over the CSIT error. To solve this robust design problem, we propose an iterative algorithm which involves the solution of a second order cone program (SOCP). Next, we consider a norm-bounded error (NBE) model for the CSIT error, where the CSIT error is specified in terms of uncertainty set of known size. In this case, we adopt a minimax approach to the robust design, and propose an iterative algorithm which involves the solution of semi-definite programs (SDP). We also consider the extension of both the designs to incorporate individual power constraints. We illustrate the robustness of the proposed algorithms to imperfections in CSIT through simulations.

The rest of the paper is organized as follows. The system model and the CSIT error models are presented in Section II. The proposed robust THP transceiver design for SE model of CSIT error is presented in Section III. The proposed robust transceiver design for NBE model of CSIT error is presented in

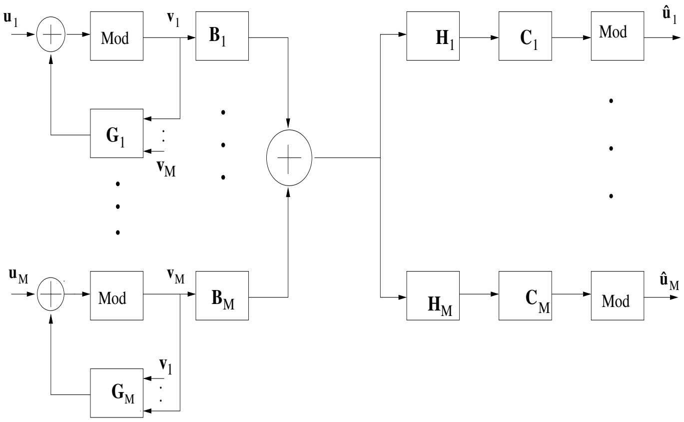

Fig. 1. Multiuser MIMO downlink system model with Tomlinson-Harashima Precoding.

Section IV. Simulation results and comparisons are presented in Section V. Conclusions are presented in Section VI.

II. System Model

We consider a multiuser MIMO downlink, where a base station (BS) communicates with M users on the downlink. The BS employs Tomlinson-Harashima precoding (THP) for inter-user interference pre-cancellation. The BS employs N_t transmit antennas and the kth user is equipped with N_{r_k} receive antennas, $1 \leq k \leq M$. Let $\mathbf{u}_k$ denote[1] the $L_k \times 1$ data symbol vector for the kth user, where L_k, $k = 1, 2, \cdots, M$, is the number of data streams for the kth user. Stacking the data vectors for all the users, we get the global data vector $\mathbf{u} = [\mathbf{u}_1^T, \cdots, \mathbf{u}_M^T]^T$. The output of the kth modulo operator is denoted by $\mathbf{v}_k$. Let $\mathbf{B}_k \in \mathbb{C}^{N_t \times L_k}$ represent the precoding matrix for the kth user. The global precoding matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_M]$. The transmit vector is given by

$$\mathbf{x} \;=\; \mathbf{B}\mathbf{v}, \tag{1}$$

where $\mathbf{v} = [\mathbf{v}_1^T, \cdots, \mathbf{v}_M^T]^T$. The feedback filters

$$\mathbf{G}_k = \left[\underline{\mathbf{G}}_{k,1} \cdots \underline{\mathbf{G}}_{k,k-1} \quad \mathbf{0}_{L_k \times \sum_{j=k}^{M} L_j}\right], \quad 1 \leq k \leq M, \tag{2}$$

where $\underline{\mathbf{G}}_{kj} \in \mathbb{C}^{L_k \times L_j}$, perform the interference pre-subtraction. We consider only inter-user interference pre-subtraction. The vectors $\mathbf{u}_k$ and$\mathbf{v}_k$ are related as

$$\mathbf{v}_k \;=\; \left(\mathbf{u}_k - \sum_{j=1}^{k-1} \underline{\mathbf{G}}_{k,j}\mathbf{v}_j\right) \mod a, \tag{3}$$

where the constant a depends on the constellation. The kth component of the transmit vector $\mathbf{x}$ is transmitted from the kth transmit antenna. Let $\mathbf{H}_k$ denote the $N_{r_k} \times N_t$ channel matrix of the kth user. The overall channel matrix is given by

$$\mathbf{H} \;=\; \left[\mathbf{H}_1^T \quad \mathbf{H}_2^T \cdots \mathbf{H}_M^T\right]^T. \tag{4}$$

The entries of the channel matrices are assumed to be zero-mean, unit-variance complex Gaussian random variables. The received signal vectors are given by

$$\mathbf{y}_k \;=\; \mathbf{H}_k\mathbf{B}\mathbf{v} + \mathbf{n}_k, \quad 1 \leq k \leq M. \tag{5}$$

The kth user estimates its data vector as

$$\begin{aligned} \widehat{\mathbf{u}}_k &= (\mathbf{C}_k\mathbf{y}_k) \mod a \\ &= (\mathbf{C}_k\mathbf{H}_k\mathbf{B}\mathbf{v} + \mathbf{C}_k\mathbf{n}_k) \mod a, \quad 1 \leq k \leq M, \end{aligned} \tag{6}$$

where $\mathbf{C}_k$ is the $L_k \times N_{r_k}$ dimensional receive filter of the kth user, and $\mathbf{n}_k$ is the zero-mean noise vector with $\mathbb{E}\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_n^2\mathbf{I}$. Stacking the estimated vectors of all users, the global estimate vector can be written as

$$\widehat{\mathbf{u}} \;=\; \mathbf{CHBv} + \mathbf{Cn} \mod a, \tag{7}$$

where $\mathbf{C}$ is a block diagonal matrix with $\mathbf{C}_k$, $1 \leq k \leq M$ on the diagonal, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_M^T]^T$. The global receive matrix $\mathbf{C}$ has block diagonal structure as the receivers are non-cooperative. Neglecting the modulo loss, and assuming $\mathbb{E}\{\mathbf{v}_k\mathbf{v}_k^H\} = \mathbf{I}$, we can write MSE between the symbol vector $\mathbf{u}_k$ and the estimate $\widehat{\mathbf{u}}_k$ at the kth user as [8]

$$\begin{aligned} \epsilon_k &= \mathbb{E}\{\|\widehat{\mathbf{u}}_k - \mathbf{u}_k\|^2\} \\ &= \text{tr}\Big[(\mathbf{C}_k\mathbf{H}_k\mathbf{B} - \bar{\mathbf{G}}_k)(\mathbf{C}_k\mathbf{H}_k\mathbf{B} - \bar{\mathbf{G}}_k)^H + \sigma_n^2\mathbf{C}_k\mathbf{C}_k^H\Big], \\ &\quad 1 \leq k \leq M, \end{aligned} \tag{8}$$

where $\bar{\mathbf{G}}_k = \left[\underline{\mathbf{G}}_{k,1} \cdots \underline{\mathbf{G}}_{k,k-1} \quad \mathbf{I}_{L_k,L_k} \quad \mathbf{0}_{L_k \times \sum_{j=k+1}^{M} L_j}\right]$.

A. CSIT Error Models

We consider two models for the CSIT error. In both the models, the true channel matrix of the kth user, $\mathbf{H}_k$, is represented as

$$\mathbf{H}_k \;=\; \widehat{\mathbf{H}}_k + \mathbf{E}_k, \quad 1 \leq k \leq M, \tag{9}$$

where $\widehat{\mathbf{H}}_k$ is the CSIT of the kth user, and $\mathbf{E}_k$ is the CSIT error matrix. The overall channel matrix can be written as

$$\mathbf{H} \;=\; \widehat{\mathbf{H}} + \mathbf{E}, \tag{10}$$

where $\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \quad \widehat{\mathbf{H}}_2^T \cdots \widehat{\mathbf{H}}_M^T]^T$, and $\mathbf{E} = [\mathbf{E}_1^T \quad \mathbf{E}_2^T \cdots \mathbf{E}_M^T]^T$.

In a stochastic error (SE) model, $\mathbf{E}_k$ is the estimation error matrix. The error matrix $\mathbf{E}_k$ is assumed to be Gaussian distributed with zero mean and $\mathbb{E}\{\mathbf{E}_k\mathbf{E}_k^H\} = \sigma_E^2\mathbf{I}_{N_{r_k}N_{r_k}}$. This statistical model is suitable for systems with uplink-downlink reciprocity. We use this model in Sec. III. An alternate error model is a norm-bounded error (NBE) model, where

$$\|\mathbf{E}_k\|_F \;\leq\; \delta_k, \quad 1 \leq k \leq M, \tag{11}$$

or, equivalently, the true channel $\mathbf{H}_k$ belongs to the uncertainty set $\mathcal{R}_k$ given by

[1]Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec(.) operator stacks the columns of the input matrix into one column-vector. $\|\cdot\|_F$ denotes the Frobenius norm, and $\mathbb{E}\{\cdot\}$ denotes expectation operator. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A} - \mathbf{B}$ is positive semi-definite.

$$\mathcal{R}_k = \{\boldsymbol{\zeta} \big| \boldsymbol{\zeta} = \widehat{\mathbf{H}}_k + \mathbf{E}_k, \|\mathbf{E}_k\|_F \le \delta_k\}, \ 1 \le k \le M, \quad (12)$$

where δ_k is the CSIT *uncertainty size*. This model is suitable for systems where quantization of CSIT is involved [13]. We use this model in Sec. IV.

III. ROBUST SMSE TRANSCEIVER DESIGN

In this section, we consider the design of minimum SMSE precoder and receive filter in the presence of CSIT error, which is assumed to follow the SE model. In order to incorporate the CSIT imperfections in the transceiver design to make it robust, we consider an appropriately modified objective function for minimization. Since the distribution of CSIT is known to be Gaussian according the SE model, we consider the SMSE averaged over the CSIT in the robust design. Following this approach, the robust transceiver design problem can be written as

$$\begin{aligned} \min_{\mathbf{B},\mathbf{C},\mathbf{G}} \quad & \mathbb{E}_{\mathbf{E}}\{\texttt{smse}\} \\ \text{subject to} \quad & \text{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \le P_T, \end{aligned} \quad (13)$$

where P_T is the maximum total BS transmit power. Substituting $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$ in (8), the SMSE can be written as

$$\begin{aligned} \texttt{smse}(\mathbf{B},\mathbf{C},\mathbf{G},\mathbf{E}) &= \mathbb{E}\{\|\widehat{\mathbf{u}} - \mathbf{u}\|^2\} \\ = \sum_{k=1}^{M} \text{tr}\Big[&(\mathbf{C}_k(\widehat{\mathbf{H}}_k + \mathbf{E}_k)\mathbf{B} - \bar{\mathbf{G}}_k)(\mathbf{C}_k(\widehat{\mathbf{H}}_k + \mathbf{E}_k)\mathbf{B} - \bar{\mathbf{G}}_k)^H \\ &+ \sigma_n^2 \mathbf{C}_k\mathbf{C}_k^H\Big]. \end{aligned} \quad (14)$$

Averaging the smse over $\mathbf{E}$, we write the new objective function as

$$\begin{aligned} \mu(\mathbf{B},\mathbf{C},\mathbf{G}) &= \mathbb{E}_{\mathbf{E}}\{\texttt{smse}\} \\ &= \sum_{k=1}^{M} \text{tr}\Big[(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B} - \bar{\mathbf{G}}_k)(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B} - \bar{\mathbf{G}}_k)^H \\ &\quad + \left(\sigma_E^2 \text{tr}(\mathbf{B}\mathbf{B}^H) + \sigma_n^2\right)\mathbf{C}_k\mathbf{C}_k^H\Big]. \end{aligned} \quad (15)$$

Using the new objective function μ, the robust transceiver design problem can be written as

$$\begin{aligned} \min_{\mathbf{b},\mathbf{C},\mathbf{G}} \quad & \mu(\mathbf{b},\mathbf{C},\mathbf{G}) \quad (16) \\ \text{subject to} \quad & \|\mathbf{b}\|^2 \le P_T, \end{aligned}$$

where $\mathbf{b} = \text{vec}(\mathbf{B})$. From (15), we observe that μ is not jointly convex in $\mathbf{B}$, $\mathbf{G}$, and $\mathbf{C}$. So, we propose an iterative algorithm in order to solve the problem in (13).

A. Robust Receive and Feedback Filter Design

Here, we consider the design of the receive and feedback filters. For a given $\mathbf{B}$ and $\mathbf{C}_k$, as we can see from (15), the optimum feedback filter $\mathbf{G}_{k,j}$, $1 \le k \le M, j < k$, is given by

$$\underline{\mathbf{G}}_{k,j} = \mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B}_j. \quad (17)$$

In order to compute the optimum receive filter, we differentiate (15) with respect to $\mathbf{C}_k$ after substituting $\mathbf{G}_{k,j}$ given in (17), and set the result to zero. We get

$$\begin{aligned} \mathbf{B}_k^H\widehat{\mathbf{H}}_k^H &= \mathbf{C}_k\left(\widehat{\mathbf{H}}_k\Big(\sum_{j=k+1}^{M}\mathbf{B}_j\mathbf{B}_j^H\Big)\widehat{\mathbf{H}}_k^H + (\sigma_n^2 + \sigma_{\mathbf{E}}^2\|\mathbf{b}\|^2)\mathbf{I}\right), \\ & 1 \le k \le M. \end{aligned} \quad (18)$$

From the above equation, we get

$$\begin{aligned} \mathbf{C}_k &= \mathbf{B}_k^H\mathbf{H}_k^H\left(\widehat{\mathbf{H}}_k\Big(\sum_{j=k+1}^{M}\mathbf{B}_j\mathbf{B}_j^H\Big)\widehat{\mathbf{H}}_k^H + \sigma_n^2\mathbf{I} + \sigma_{\mathbf{E}}^2\|\mathbf{b}\|^2\right)^{-1}, \\ & 1 \le k \le M. \end{aligned} \quad (19)$$

B. Robust Precoder Design

Having described the design of the receive filter $\mathbf{C}$ and the feedback matrix $\mathbf{G}$ for a given precoder matrix $\mathbf{B}$, we now present the design of the robust precoder for the given receive filter and feedback matrices. Towards this end, we express the robust transceiver design problem in (16) as

$$\begin{aligned} \min_{\mathbf{b},\mathbf{c},\mathbf{g}} \quad & \sum_{k=1}^{M}\|\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k\|^2 + (\sigma_E^2\|\mathbf{b}\|^2 + \sigma_n^2)\|\mathbf{c}_k\|^2 \\ \text{Subject to} \quad & \|\mathbf{b}\|^2 \le P_T, \end{aligned} \quad (20)$$

where $\mathbf{D}_k = (\mathbf{B}^T \otimes \mathbf{C}_k)$, $\widehat{\mathbf{h}}_k = \text{vec}(\widehat{\mathbf{H}}_k)$, $\mathbf{c}_k = \text{vec}(\mathbf{C}_k)$, $\bar{\mathbf{g}}_k = \text{vec}(\bar{\mathbf{G}}_k)$, and $\mathbf{h}_k = \text{vec}(\mathbf{H}_k)$. For given $\mathbf{C}$ and $\mathbf{G}$, the problem given above is a convex optimization problem. The robust precoder design problem, given $\mathbf{C}$ and $\mathbf{G}$, can be written as

$$\begin{aligned} \min_{\mathbf{b}} \quad & \sum_{k=1}^{M}\|\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k\|^2 + \sigma_E^2\|\mathbf{b}\|^2\|\mathbf{c}_k\|^2 + \sigma_n^2\|\mathbf{c}_k\|^2 \\ \text{Subject to} \quad & \|\mathbf{b}\|^2 \le P_T. \end{aligned} \quad (21)$$

As the last term in (21) does not affect the optimum value of $\mathbf{b}$, we drop this term in the formulation of the precoder design problem. Dropping this term and introducing the dummy variables t_k, r_k, $1 \le k \le M$, (21), the robust precoder design can be formulated as the following convex optimization problem:

$$\begin{aligned} \min_{\mathbf{b},\{t_i\}_1^M,\{r_i\}_1^M} \quad & \sum_{k=1}^{M} t_k + \sigma_E\|\mathbf{c}_k\|^2 r_k^2 \quad (22) \\ \text{subject to} \quad & \|\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k\|^2 \le t_k, \quad (23) \\ & \|\mathbf{b}\|^2 \le r_k, \\ & r_k \le P_T, \ 1 \le k \le M. \end{aligned}$$

C. Iterative Algorithm for Solving (13)

Here, we propose an iterative algorithm for the minimization of the SMSE averaged over the CSIT error under a constraint on the total BS transmit power. At the $(n+1)$th iteration, the value of $\mathbf{B}$, denoted by $\mathbf{B}^{n+1}$, is the solution to the following problem:

$$\mathbf{B}^{n+1} = \underset{\mathbf{B}:\text{Tr}(\mathbf{B}\mathbf{B}^H)\le P_T}{\text{argmin}} \mu(\mathbf{B},\mathbf{C}^n,\mathbf{G}^n), \quad (24)$$

which is solved in the previous subsection. Having computed $\mathbf{B}^{n+1}$, $\mathbf{C}^{n+1}$ is the solution to the following problem:

$$\mathbf{C}^{n+1} = \underset{\mathbf{C}}{\operatorname{argmin}}\, \mu(\mathbf{B}^{n+1}, \mathbf{C}, \mathbf{G}^n), \quad (25)$$

and its solution is given in (19). Having computed $\mathbf{B}^{n+1}$ and $\mathbf{C}^{n+1}$, $\mathbf{G}^{n+1}$ is the solution to the following problem:

$$\mathbf{G}^{n+1} = \underset{\mathbf{G}}{\operatorname{argmin}}\, \mu(\mathbf{B}^{n+1}, \mathbf{C}^{n+1}, \mathbf{G}), \quad (26)$$

and its solution is given in (17). This iterative optimization over $\{\mathbf{B}\}$, $\{\mathbf{C}\}$, and $\{\mathbf{G}\}$ can be repeated till convergence of the optimization variables. As the objective in (15) is monotonically decreasing after each iteration and is lower bounded, convergence is guaranteed. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached. We note that proposed algorithm is not guaranteed to converge to the global minimum.

IV. Proposed Robust Transceiver Design with NBE Model

In this section, we consider the robust transceiver design when the CSIT follows the NBE model. In this case, we consider a minimax design, wherein the robust transceiver design seeks to minimize the worst case SMSE under a total BS transmit power constraint. This problem can be written as

$$\min_{\mathbf{B},\mathbf{C},\mathbf{G}} \max_{\mathbf{E}_k : \|\mathbf{E}_k\| \leq \delta_k, \forall k} \mathtt{smse}(\mathbf{B}, \mathbf{C}, \mathbf{G}, \mathbf{E}) \quad (27)$$
$$\text{subject to} \quad \operatorname{tr}(\mathbf{B}\mathbf{B}^H) \leq P_T.$$

The above problem deals with case where the true channel, unknown to the transmitter, may lie anywhere in the uncertainty region. In order to ensure, a priori, that MSE constraints are met for the actual channel, the precoder should be so designed that the constraints are met for all members of the uncertainty set. This, in effect, is a semi-infinite optimization problem, which in general is intractable. We show, in the following, that an appropriate transformation makes the problem in (27) tractable. We note that the problem in (27) can be written as

$$\min_{\mathbf{B},\mathbf{C},\mathbf{G},t} \sum_{k=1}^{M} t_k \quad (28)$$
$$\text{Subject to} \quad \|\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k\|^2, \quad (29)$$
$$+\sigma_n^2 \|\mathbf{c}_k\|^2 \leq t_k, \ \forall \|\mathbf{e}_k\| \leq \delta_k,$$
$$\|\mathbf{b}\|^2 \leq P_T,$$

where $\mathbf{e}_k = \operatorname{vec}(\mathbf{E}_k)$. The first constraint in (28) is convex in $\mathbf{B}$ and $\mathbf{G}_k$ for a fixed value of $\mathbf{C}_k$ and vice versa, but not jointly convex in $\mathbf{B}$, $\mathbf{G}_k$ and $\mathbf{C}_k$. Hence, to design the transceiver, we propose an iterative algorithm, wherein the optimization is performed alternately over $\{\mathbf{B}, \mathbf{G}\}$ and $\{\mathbf{C}\}$.

A. Robust Precoder and Feedback Filter Design

For the design of the precoder matrix $\mathbf{B}$ and the feedback filter $\mathbf{G}$ for a fixed value of $\mathbf{C}$, the second term in the left hand side of the first constraint in (28) is not relevant, and hence we drop this term. Invoking the Schur Complement Lemma [15], and dropping the second term, we can write the constraint in (28) as the following linear matrix inequality (LMI):

$$\begin{bmatrix} t_k & \left[\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k\right]^H \\ \left[\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k\right] & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \quad (30)$$

Hence, the robust precoder and feedback filter design problem, for a given value of $\mathbf{C}$, can be represented as

$$\min_{\mathbf{B},\mathbf{G}} \sum_{k=1}^{M} t_k \quad (31)$$
$$\text{Subject to} \quad \begin{bmatrix} t_k & [\mathbf{D}_k\mathbf{h}_k - \bar{\mathbf{g}}_k]^H \\ [\mathbf{D}_k\mathbf{h}_k - \bar{\mathbf{g}}_k] & \mathbf{I} \end{bmatrix} \succeq \mathbf{0},$$
$$\forall \|\mathbf{e}_k\| \leq \delta_k, \quad 1 \leq k \leq M,$$
$$\|\mathbf{b}\| \leq \sqrt{P_T},$$

where $\mathbf{h}_k = \widehat{\mathbf{h}}_k + \mathbf{e}_k$. From (30), the first constraint in (31) can be written as

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \quad (32)$$

where

$$\mathbf{A} = \begin{bmatrix} t_k & \left[\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k\right]^H \\ \left[\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k\right] & \mathbf{I} \end{bmatrix}, \quad (33)$$

$\mathbf{P} = \begin{bmatrix}\mathbf{0} & \mathbf{D}_k^H\end{bmatrix}$, $\mathbf{X} = \mathbf{e}_k$, and $\mathbf{Q} = -\begin{bmatrix}1 & 0\end{bmatrix}$. Having reformulated the constraint as in (32), we can invoke the following Lemma [16] to solve the problem in (31)

Lemma 1: Given matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{A}$ with $\mathbf{A} = \mathbf{A}^H$,

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \quad \forall \mathbf{X} : \|\mathbf{X}\| \leq \rho \quad (34)$$

if and only if $\exists \lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A} - \lambda\mathbf{Q}^H\mathbf{Q} & -\rho\mathbf{P}^H \\ -\rho\mathbf{P} & \lambda\mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \quad (35)$$

Applying Lemma 1, we can formulate the robust precoder design problem as the following convex optimization problem:

$$\min_{\mathbf{B},\mathbf{G},t,\beta} \sum_{k=1}^{M} t_k \quad (36)$$
$$\text{Subject to} \quad \mathbf{M}_k \succeq \mathbf{0}, \ \beta_k \geq 0 \ \forall k,$$
$$\|\mathbf{b}\| \leq \sqrt{P_T},$$

where

$$\mathbf{M}_k = \begin{bmatrix} t_k - \beta_k & (\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k)^H & \mathbf{0} \\ (\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k) & \mathbf{I} & -\rho\mathbf{D}_k \\ \mathbf{0} & -\rho\mathbf{D}_k^H & \beta_k\mathbf{I} \end{bmatrix}. \quad (37)$$

B. Robust Receive Filter Design

In the previous subsection, we considered the design of the precoder and the feedback filter design for a fixed value of $\mathbf{C}$. Here, we consider the design of robust receive filter for a given precoder filter $\mathbf{B}$ and feedback filter $\mathbf{G}$. This design problem can be written as

$$\min_{\mathbf{C},t} \quad \sum_{k=1}^{M} t_k \tag{38}$$

$$\text{Subject to} \quad \|\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k\|^2 + \sigma_n^2\|\mathbf{c}_k\|^2 \leq t_k, \quad \forall \|\mathbf{E}_k\| \leq \delta_k, \quad 1 \leq k \leq M. \tag{39}$$

Applying the Schur Complement Lemma, we can represent the first constraint in (38), as

$$\begin{bmatrix} t_k & \begin{bmatrix} \mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k \\ \mathbf{0} \end{bmatrix}^H \\ \begin{bmatrix} \mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \bar{\mathbf{g}}_k \\ \mathbf{0} \end{bmatrix} & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \tag{40}$$

The second inequality in the above problem, like in the precoder design problem, represents an infinite number of constraints. To make the problem in (38) tractable, we again invoke Lemma 1. Following the same procedure as in the precoder design, starting with (40), we can reformulate the robust receive filter design as the following convex optimization problem:

$$\min_{\mathbf{C},t,\beta} \quad \sum_{k=1}^{M} t_k \tag{41}$$

$$\text{Subject to} \quad \mathbf{N}_k \succeq \mathbf{0}, \ \beta_k \geq 0 \ \forall k, \tag{42}$$

where

$$\mathbf{N}_k = \begin{bmatrix} t_k - \beta_k & \begin{bmatrix} (\mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k \\ \sigma_n \mathbf{c}_k \end{bmatrix}^H & \mathbf{0} \\ \begin{bmatrix} \mathbf{D}_k\widehat{\mathbf{h}}_k - \bar{\mathbf{g}}_k \\ \sigma_n \mathbf{c}_k \end{bmatrix} & \mathbf{I} & -\rho\mathbf{D}_k \\ \mathbf{0} & -\rho\mathbf{D}_k^H & \beta_k\mathbf{I} \end{bmatrix}. \tag{43}$$

C. Iterative Algorithm for Solving (27)

In the previous subsections, we described the design of $\mathbf{B}$ and $\mathbf{G}$ for a fixed value of $\mathbf{C}$ and vice versa. Here, we propose an iterative algorithm for the minimization of the SMSE under a constraint on the total BS transmit power, when the CSIT error follows NBE model. This algorithm iterates over the precoder, feedback filter and receive filter design described earlier. At the $(n+1)$th iteration, the value of $\mathbf{B}$, denoted by $\mathbf{B}^{n+1}$, is the solution to problem (36), and hence satisfies the BS transmit power constraint. Having computed $\mathbf{B}^{n+1}$, $\mathbf{C}^{n+1}$ is the solution to the problem in (41). So $J(\mathbf{B}^{n+1}, \mathbf{C}^{n+1}) \leq J(\mathbf{B}^{n+1}, \mathbf{C}^{n}) \leq J(\mathbf{B}^{n}, \mathbf{C}^{n})$, where

$$J(\mathbf{B}, \mathbf{C}) = \max_{\|\mathbf{E}_k\| < \delta_k, \forall k} \{\mathtt{smse}(\mathbf{B}, \mathbf{C}, \mathbf{G}, \mathbf{E})\}. \tag{44}$$

The monotonically decreasing nature of $J(\mathbf{B}^n, \mathbf{C}^n)$, together with the fact that $J(\mathbf{B}^n, \mathbf{C}^n)$ is lower-bounded, implies that the proposed algorithm converges to a limit as $n \to \infty$. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached. We note that proposed algorithm is not guaranteed to converge to the global minimum.

D. Transceiver Design With Per-Antenna Power Constraints

As each antenna at the base station usually has its own amplifier, it is important to consider transceiver design with constraints on power transmitted from each antenna. A precoder design for multiuser MISO downlink with per-antenna power constraint with perfect CSIT was considered in [17]. Here, we incorporate per-antenna power constraint in the proposed robust transceiver design. For this, only the precoder matrix design (36) has to be modified by including the constraints on power transmitted from each antenna as given below:

$$\min_{\mathbf{b}} \quad \sum_{k=1}^{M} t_k \tag{45}$$

$$\text{Subject to} \quad \mathbf{M}_k \succeq \mathbf{0} \ \forall k, \quad \|\mathbf{S}_k\mathbf{b}_k\|^2 \leq P_k, \quad 1 \leq k \leq M, \tag{46}$$

where $\mathbf{S}_k = [\mathbf{0}_{1\times k-1} \quad 1 \quad \mathbf{0}_{1\times N_t-k}]$ selects the kth row of $\mathbf{B}$. The receive filter can be computed using (41).

V. Simulation Results

In this section, we illustrate the performance of the proposed robust THP transceiver algorithms, evaluated through simulations. We compare the performance of the proposed design with the non-robust transceiver designs reported in the literature. We do not compare the designs in Sec. III and Sec. IV with each other as the CSIT error model for each of them is different. The channel fading is modeled as Rayleigh, with the channel matrices $\mathbf{H}_k$, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each antenna of each user terminal is assumed to be zero-mean complex Gaussian random variable.

In the first experiment, we consider the performance of the robust precoder design with SE model for the CSIT error. We consider a system with the BS transmitting $L = 2$ data streams each to $M = 3$ users. The simulation results are shown in Fig. 2. The SMSE performances of the proposed robust design and the non-robust design proposed in [8] for different numbers of transmit antennas at the BS and receive antennas at the user terminals are evaluated. Specifically we consider three configurations: *i)* $N_t = 6$, $N_r = 2$, *ii)* $N_t = 8$, $N_r = 2$, and *iii)* $N_t = 8$, $N_r = 3$. We use $\sigma_E^2 = 0.1$ for all scenarios. In all the three configurations, the proposed robust design is seen to outperform the non-robust design in [8]. Comparing the results for $N_t = 6$ and $N_t = 8$, we find that the difference between the non-robust design and the proposed robust design decreases when more transmit antennas are provided. A similar effect is observed for increase in number of receive antennas for fixed number of transmit antennas. It is also found that the difference between the performance of these algorithms increase as the SNR increases. This is observable in (15), where the second term shows the effect of the CSIT error variance amplified by the transmit power. For the second experiment, with the same setting as in first experiment, Fig. 3 shows the performance of the proposed robust design and the non-robust design proposed in [8] in

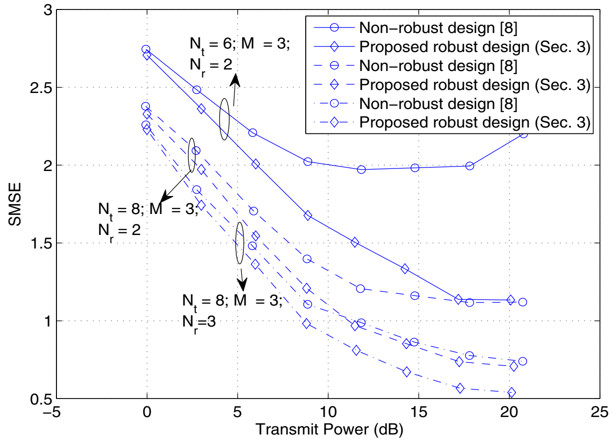

Fig. 2. SMSE versus transmit power $P_{Tr} = \|\mathbf{B}\|_F^2$. $N_t = 8, 6$, $M = 3$, $N_{r_1} = N_{r_2} = N_{r_3} = 2$, $L_1 = L_2 = L_3 = 2$, $\sigma_n^2 = 1$, $\sigma_E^2 = 0.1$.

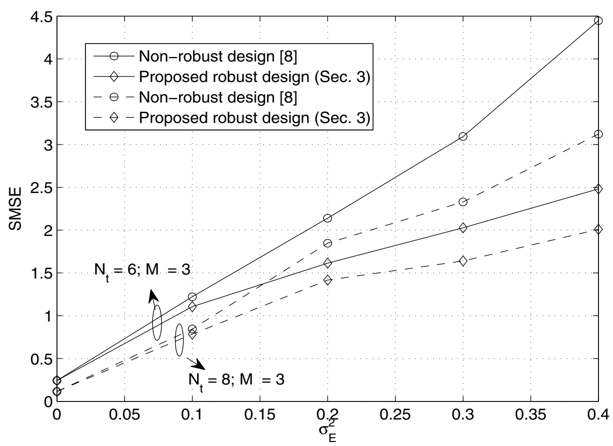

Fig. 3. SMSE versus CSIT error variance σ_E^2. $N_t = 6, 8$, $M = 3$, $N_{r_1} = N_{r_2} = N_{r_3} = 2$, $L_1 = L_2 = L_3 = 2$, $\sigma_n^2 = 1$, $P_T = 15$ dB.

terms of the SMSE for different channel estimation error variances. Here again, we observe that the proposed robust design outperforms the non-robust design in the presence of CSIT error; larger the estimation error variance, higher is the performance improvement due to the robustification in the proposed algorithm. In the third experiment, we study the performance of the robust precoder design with NBE model for the CSIT error. The SMSE performances of the proposed robust design and the non-robust design proposed in [8] for different values of the CSIT error norm are evaluated, and the results are shown in Fig. 4. The proposed robust design is found to outperform the non-robust design.

VI. Conclusions

We proposed two robust THP transceiver designs for multiuser MIMO downlink that minimize the SMSE in the presence of imperfect CSIT. The first design is for the scenario where the CSIT error can be modeled by the SE model and the second design is for the scenario where the CSIT error can be modeled by the NBE model. Through simulation results, we illustrated the superior performance of the proposed robust designs compared to the non-robust designs in the presence of CSIT imperfections.

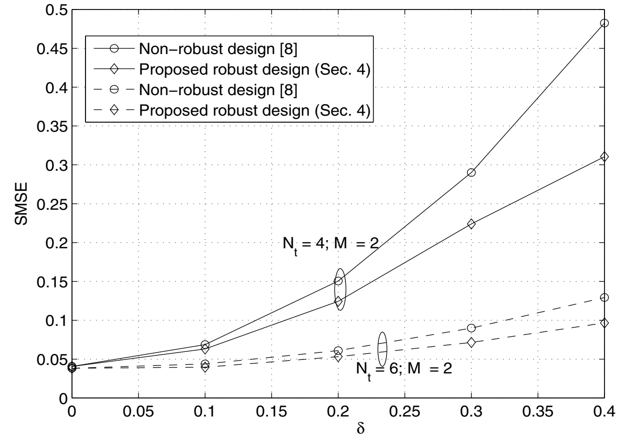

Fig. 4. SMSE versus CSIT error norm $\delta = \delta_1 = \delta_2$. $N_t = 6, 4$, $M = 2$, $N_{r_1} = N_{r_2} = N_{r_3} = 2$, $L_1 = L_2 = L_3 = 2$, $\sigma_n^2 = 0.1$, $P_T = 15$ dB.

References

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2006.

[2] H. Bolcskei, D. Gesbert, C. B. Papadias, and A.-J. van der Veen, *Space-time Wireless Systems: From Array Processing to MIMO Communications*. Cambridge University Press, 2006.

[3] R. Doostnejad, T. J. Lim, and E. Sousa, "Joint precoding and beamforming design for the downlink in a multiuser MIMO system," in *Proc. WiMob'2005*, Aug. 2005, pp. 153–159.

[4] B. Bandemer, M. Haardt, and S. Visuri, "Linear MMSE multi-user MIMO downlink precoding for users with multple antennas," in *Proc. PIMRC'06*, Sep. 2006, pp. 1–5.

[5] J. Zhang, Y. Wu, S. Zhou, and J. Wang, "Joint linear transmitter and receiver design for the downlink of multiuser MIMO systems," *IEEE Commun. Lett.*, vol. 9, pp. 991–993, Nov. 2005.

[6] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE transceiver optimization for multiuser MIMO systems: Duality and sum-MSE minimization," *IEEE Trans. Signal Process.*, vol. 55, pp. 5436–5446, Nov. 2007.

[7] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver design for multi-user MIMO systems," in *Proc. WSA 2006*, Mar. 2006.

[8] ——, "Iterative THP transceiver optimization for multi-user MIMO systems based on weighted sum-MSE minimization," in *Proc. IEEE SPAWC 2006*, Jul. 2006.

[9] R. Doostnejad, T. J. Lim, and E. Sousa, "Precoding for the MIMO broadcast channels with multiple antennas at each receiver," presented at the 2005 Conference on ISS, John Hopkins University, Mar. 2005.

[10] R. Hunger, F. Dietrich, M. Joham, and W. Utschick, "Robust transmit zero-forcing filters," in *Proc. ITG Workshop on Smart Antennas*, Munich, Mar. 2004, pp. 130–137.

[11] M. B. Shenouda and T. N. Davidson, "Linear matrix inequality formulations of robust QoS precoding for broadcast channels," in *Proc. CCECE'2007*, Apr. 2007, pp. 324–328.

[12] M. Biguesh, S. Shahbazpanahi, and A. B. Gershman, "Robust downlink power control in wireless cellular systems," *EURASIP Jl. Wireless Commun. Networking*, vol. 2, pp. 261–272, 2004.

[13] M. Payaro, A. Pascual-Iserte, and M. A. Lagunas, "Robust power allocation designs for multiuser and multiantenna downlink communication systems through convex optimization," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 1392–1401, Sep. 2007.

[14] N. Vucic, H. Boche, and S. Shi, "Robust transceiver optimization in downlink multiuser MIMO systems with channel uncertainty," in *Proc. IEEE ICC'2008*, May 2008.

[15] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[16] Y. C. Eldar, A. Ben-Tal, and A.Nemirovski, "Robust mean-squared error estimation in the presence of model uncertainties," *IEEE Trans. Signal Process.*, vol. 53, pp. 161–176, Jan. 2005.

[17] W. Yu and T. Lan, "Transmitter optimization for multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, pp. 2646–2660, Jun. 2007.