Factor Graph Based Joint Detection/Decoding for LDPC Coded Large-MIMO Systems

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***Abstract*—In this paper, we employ message passing algorithms over graphical models to jointly detect and decode symbols transmitted over large multiple-input multiple-output (MIMO) channels with low density parity check (LDPC) coded bits. We adopt a factor graph based technique to integrate the detection and decoding operations. A Gaussian approximation of spatial interference is used for detection. This serves as a low complexity joint detection/decoding approach for large dimensional MIMO systems coded with LDPC codes of large block lengths. This joint processing achieves significantly better performance than the individual detection and decoding scheme.**

Keywords *– Large-MIMO, LDPC, joint detection/decoding, factor graphs, belief propagation, message passing.*

I. Introduction

Large-scale MIMO systems with tens to hundreds of antennas have attracted much interest recently. The motivation to use such large-scale MIMO systems is the potential to practically achieve the theoretically predicted benefits of MIMO, in terms of very high spectral efficiencies/sum rates, increased reliability and power efficiency, through the exploitation of large spatial dimensions. Use of large number of antennas is getting recognized to be a good approach to fulfill the increased throughput requirements in future wireless systems.

Low-complexity receiver processing (detection, channel estimation and decoding) is crucial for the practical realization of large-MIMO systems. Recently, there has been encouraging progress in the development of low-complexity near-optimal MIMO receiver algorithms that can scale well for large dimensions. These algorithms are based on techniques from local neighborhood search including tabu search [1]-[5], and message passing on graphical models including factor graphs and Markov random fields [6]-[8]. In this paper, we propose to *jointly* process both MIMO detection as well as LDPC decoding using message passing on a graph combining both LDPC code constraints as well as MIMO constraints. The motivation is to achieve improved performance compared to individual detection/decoding and scalability for large number of antennas.

Message passing techniques (also referred to as belief propagation) on graphical models are known for their simplicity, low complexity, and near-optimal performance in large dimensions [9]. They are typically used to marginalize probability distributions from a joint distribution defined over a certain graph. Several communication problems have been benefited by message passing techniques; e.g., decoding of turbo codes and LDPC codes [10], [11], multiuser detection in CDMA [12], and detection/equalization.

Often, detection and decoding in communications receivers are carried out as two independent functions. However, processing of detection and decoding functions jointly can lead to improved performance [13]. Also, turbo equalization that performs detection and decoding in an iterative manner is known to give coded performance at low complexities [14]. In [14], a receiver that performs detection and decoding i) independently is referred to as Type-B receiver, ii) iteratively (between detection and decoding) is referred to as Type-C receiver, and iii) jointly (optimal) is referred to as Type-A receiver. Algorithms studied are predominantly of the Type-C kind that iterate extrinsic information between a detector and a decoder. Our contribution in this paper is a *joint detection/decoding scheme that passes messages on an integrated factor graph* (FG) that scales very well for large number of antennas, and, in addition, achieves very good performance. The formulated FG has two sets of factor nodes representing the received vector over MIMO channel and the LDPC check equations. The marginalization of the joint probabilities is done through message passing from the different set of nodes and appropriately combining them at the variable nodes.

II. System Model

Consider a V-BLAST MIMO system with n_t transmit and n_r receive antennas. A sequence $\mathbf{u}$ of k information bits is encoded by an LDPC code into a codeword $\mathbf{b}$ of n coded bits. The coded bits are then BPSK modulated. n_t modulated symbols are sent in one channel use on n_t transmit antennas using spatial multiplexing. So, the number of channel uses needed to send all n coded bits is $M = \lceil n/n_t \rceil$. Let $\mathbf{x}^{(m)}$, $m = 1, \cdots, M$, denote the $n_t \times 1$ sized modulated symbol vector sent in the mth channel use. Let $\mathbf{H}^{(m)} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix in the mth channel use, whose entries are assumed to be i.i.d. Gaussian with zero mean and unit variance. The received vector in the mth channel use, $\mathbf{y}^{(m)}$, is given by

$$\mathbf{y}^{(m)} = \mathbf{H}^{(m)}\mathbf{x}^{(m)} + \mathbf{w}^{(m)}, \qquad (1)$$

where $\mathbf{w}^{(m)}$ is the noise vector whose entries are modeled as i.i.d. $\mathbb{C}\mathcal{N}(0, \sigma^2)$. Perfect knowledge of the channel gains at the receiver is assumed. Detection is done on a per-channel use basis, and decoding is done over all n coded

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bits from M channel uses. For MIMO detection, we need to compute the maximum a posteriori probability (MAP), given by $p(\mathbf{x}^{(m)} \mid \mathbf{y}^{(m)}, \mathbf{H}^{(m)}) \propto p(\mathbf{y}^{(m)} \mid \mathbf{x}^{(m)}, \mathbf{H}^{(m)}) p(\mathbf{x}^{(m)})$, whose exact computation requires exponential complexity in n_t. Here, in an attempt to achieve good performance at low complexities, we focus on *joint* processing of MIMO detection and LDPC decoding based on message passing on an integrated graphical model that defines both LDPC code constraints as well as MIMO constraints.

III. Individual Detection and Decoding

In this section, we present message passing algorithms that can be applied independently for MIMO detection and LDPC decoding. These preliminaries facilitate the formulation of the joint graph and message passing in Section IV.

A. MIMO Detection using Message Passing

Towards describing the detection algorithm, for simplicity and convenience, we drop the channel use index m in the vectors and matrices in (1). The factor graph on which MIMO detection is performed is a bipartite graph with two sets of nodes. There are n_r observation nodes and n_t variable nodes. It is a fully connected bipartite graph, i.e., each observation node is connected to all the variable nodes. The received signal on the ith receive antenna, y_i, can be written as

$$y_i = h_{il} x_l + z_{il}, \tag{2}$$

where x_l is the symbol sent on lth transmit antenna, h_{il} is the entry in ith row and lth column of $\mathbf{H}$ (i.e., h_{il} is the channel gain from lth transmit antenna to ith receive antenna), z_{il} is the interference-plus-noise term given by

$$z_{il} = \sum_{j=1, j\neq l}^{n_t} h_{ij} x_j + w_i, \tag{3}$$

and w_i is the noise variable in ith receive antenna. The z_{il} is approximated to have Gaussian distribution [6] with mean

$$\mu_{il} = \sum_{j=1, j\neq l}^{n_t} h_{ij} \mathbb{E}(x_j), \tag{4}$$

and variance

$$\sigma_{il}^2 = \sum_{j=1, j\neq l}^{n_t} h_{ij}^2 \operatorname{var}(x_j) + \sigma^2. \tag{5}$$

The message from the ith observation node to the lth variable node is the log-likelihood ratio (LLR), given by

$$\begin{aligned} \Lambda_{i\to l} &= \ln\left(\frac{\Pr(y_i \mid \mathbf{H}, x_l = +1)}{\Pr(y_i \mid \mathbf{H}, x_l = -1)}\right) \\ &= \frac{4}{\sigma_{il}^2} \Re\left(h_{il}^*(y_i - \mu_{il})\right). \end{aligned} \tag{6}$$

The message from the lth variable node to the ith observation node is the probability, given by

$$P_i(x_l = +1 \mid \mathbf{y}) = \frac{\exp(\sum_{r\neq i} \Lambda_{r\to l})}{1 + \exp(\sum_{r\neq i} \Lambda_{r\to l})}. \tag{7}$$

The messages are exchanged for a certain number of iterations. In uncoded systems, symbol decision is made as $\hat{x}_l = \operatorname{sgn}\big(\sum_{i=1}^{n_r} \Lambda_{i\to l}\big)$. In coded systems, $\Lambda_{i\to l}$'s can be fed as soft decision inputs to the decoder. This algorithm has a per-symbol computational complexity of just $O(n_t)$, which scales very well for large n_t.

B. LDPC Decoding

The decoding algorithm for LDPC is a message passing algorithm, which is known as the sum product algorithm (SPA). The algorithm gives the a posteriori probabilities (APP) of the coded bits. The LDPC decoder graph is described by the parity check matrix, $\mathbf{F}$, of dimension $n \times (n-k)$. If $\mathbf{b}$ is a valid codeword, then $\mathbf{bF} = \mathbf{0}$. The graph over which the messages are passed is a bipartite graph, consisting of n variable nodes corresponding to the coded bits in a block and $n-k$ check nodes corresponding to the check equations. The message passing algorithm can be briefly described as follows.

- *Initialization:* Initialize the variable node to check node messages V_{lj}'s, $l = 1, \cdots, n$, $j = 1, \cdots, n-k$ with initial probabilities $\Pr(b_l = a)$, $a \in \{0, 1\}$, where b_l is the lth element in the coded bit vector $\mathbf{b}$.
- *Step 1:* Compute check node to variable node messages $U_{jl} = \Pr(S_j \mid b_l = a)$, where S_j is the event of jth check equation being satisfied. U_{jl} is a function of all V_{rj}, $r \in \mathcal{N}(j) \setminus l$, where $\mathcal{N}(j)$ denotes the set of all variable nodes connected to check node j.
- *Step 2:* Compute $V_{lj} \propto \Pr(b_l = a \mid S_{\mathcal{N}(l)\setminus j}, \mathbf{y}) \Pr(b_l = a)$, where $S_{\mathcal{N}(l)\setminus j}$ is the event that all check equations which involve bit b_l except the jth check equation are satisfied. V_{lj} is a function of all U_{rl}, $r \in \mathcal{N}(l) \setminus j$.
- *Step 3:* Compute $\Pr(b_l = a \mid S, \mathbf{y})$, where S is the event of all check equations being satisfied. Steps 1 and 2 are repeated until $\mathbf{bF} = \mathbf{0}$ or a certain number of iterations are completed. At the end, decisions on the bits are made based on the probabilities in Step 3.

IV. Joint Detection and Decoding

In this section, we present a message passing algorithm to jointly detect and decode the received symbols. When jointly performing both detection and decoding operations, there is a transfer of extrinsic information from the decoder to the detector and vice versa, which results in efficient usage of information at the receiver to marginalize the joint probability of the received symbols. In a joint detection and decoding scenario, the objective is to compute

$$\begin{aligned} \Pr(\mathbf{x} \mid S, \mathbf{y}) &\propto \Pr(\mathbf{x}, S, \mathbf{y}) \\ &= \Pr(S \mid \mathbf{x}) \Pr(\mathbf{y} \mid \mathbf{x}) \Pr(\mathbf{x}), \end{aligned} \tag{8}$$

where

$$\Pr(S \mid \mathbf{x}) = \prod_{j=1}^{n-k} \Pr(S_j \mid \mathbf{x}). \tag{9}$$

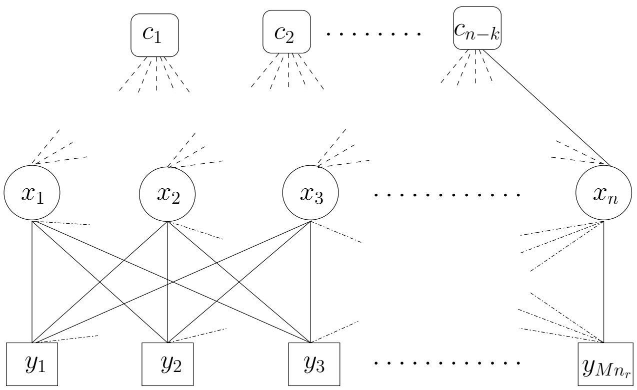

Fig. 1. Illustration of the joint graph with observation nodes y_i's, variable nodes x_l's, and check nodes x_j's.

Hence, we formulate a graph whose joint probability factorizes according to the above equation, and that upon marginalization gives the probability of the received symbols.

The constructed graph consists of three sets of nodes, namely, variable nodes set, observation nodes set, and check nodes set. There are Mn_r observation nodes corresponding to the received vectors, $Mn_t = n$ variable nodes corresponding to the transmitted coded symbol vectors over M channel uses, and $n-k$ check nodes corresponding to the check equations of the LDPC code. Figures 1, 2, and 3 illustrate the joint graph and the messages passed over it.

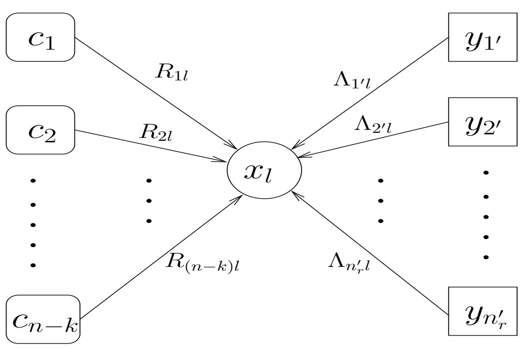

Fig. 2. Messages passed to variable node x_l.

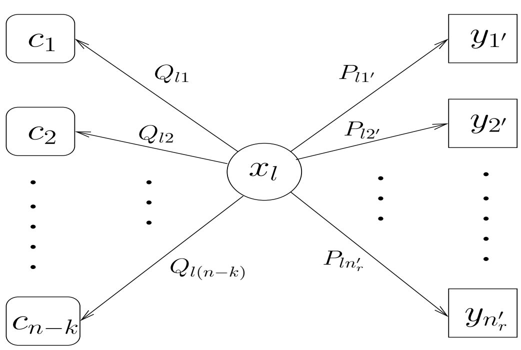

Fig. 3. Messages passed from variable node x_l.

The messages passed over the graph are $i)$ P_{li}: message from variable node x_l to observation node y_i, $ii)$ Q_{lj}: message from variable node x_l to check node c_j, $iii)$ R_{jl}: message from check node c_j to variable node x_l, and $iv)$ Λ_{il}: message from observation node y_i to variable node x_l, where $l = 1, \cdots, n, i = 1, \cdots, Mn_r, j = 1, \cdots, n-k, m = 1, \cdots, M$, and n is chosen such that $n = Mn_t$. It can be observed that $m = \lceil \frac{i}{n_r} \rceil = \lceil \frac{l}{n_t} \rceil$. Λ_{il} is computed as in (6) with only the corresponding $\mathbf{H}^{(m)}$ and $\mathbf{x}^{(m)}$. Thus, Λ_{il} is a function of all P_{ri}, where $r \in \{l' \mid \lceil \frac{l'}{n_t} \rceil = m\} \setminus l$. The various message are given by

$$\begin{aligned} R_{jl} &= \ln\left(\frac{\Pr(S_j \mid x_l = +1)}{\Pr(S_j \mid x_l = -1)}\right) \\ &= \ln\left(\frac{1 + \prod_{r\in\mathcal{N}_v(j)\setminus l}(1 - 2\Pr(b_r = 1))}{1 - \prod_{r\in\mathcal{N}_v(j)\setminus l}(1 - 2\Pr(b_r = 1))}\right), \quad (10) \end{aligned}$$

$$\begin{aligned} P_{li} &= \Pr(x_l = +1 \mid \mathbf{y}_{\mathcal{N}_o(l)\setminus i}, S_{\mathcal{N}_c(l)}) \\ &= \frac{\exp(\sum_{r\in\mathcal{N}_o(l)\setminus i}\Lambda_{rl} + \sum_{r\in\mathcal{N}_c(l)} R_{rl})}{1 + \exp(\sum_{r\in\mathcal{N}_o(l)\setminus i}\Lambda_{rl} + \sum_{r\in\mathcal{N}_c(l)} R_{rl})}, \quad (11) \end{aligned}$$

$$\begin{aligned} Q_{lj} &= \ln\left(\frac{\Pr(x_l = +1 \mid \mathbf{y}_{\mathcal{N}_o(l)}, S_{\mathcal{N}_c(l)\setminus j})}{\Pr(x_l = -1 \mid \mathbf{y}_{\mathcal{N}_o(l)}, S_{\mathcal{N}_c(l)\setminus j})}\right) \\ &= \sum_{r\in\mathcal{N}_o(l)} \Lambda_{rl} + \sum_{r\in\mathcal{N}_c(l)\setminus j} R_{rl}, \quad (12) \end{aligned}$$

where $\mathcal{N}_c(l)$ is the set of check nodes connected to x_l, $\mathcal{N}_v(j)$ is the set of variable nodes connected to c_j and $\mathcal{N}_o(l)$ is the set of observation nodes connected to x_l, $\mathcal{N}_o(l) = \{i' \mid \lceil \frac{i'}{n_r} \rceil = m\}$. In typical LDPC decoding, the computation of R_{jl} is simplified by the use of $\tanh(.)$ function. The messages P_{li} and Q_{lj} are computed as given by (8) and only the extrinsic information from one set of nodes being passed to the other. The LLR or the beliefs of the symbols at the end of an iteration is given by

$$L_l = \sum_{i\in\mathcal{N}_o(l)} \Lambda_{il} + \sum_{j\in\mathcal{N}_c(l)} R_{jl}. \quad (13)$$

The iterations are continued till $\mathbf{bF} = \mathbf{0}$ or a certain number of iterations are completed, upon which bit decisions are made using the final LLRs. The convergence of the algorithm can be improved by damping of beliefs, where, in each iteration, the beliefs are taken to be a weighted average of the beliefs in the previous and current iterations [15].

A. Scheduling of messages

Scheduling (i.e., the order of computation and message passing at each node) can affect the convergence of the joint detection/decoding algorithm. We consider three possible methods of scheduling the messages.

1. Flooding : This is the simplest scheduling scheme. As the name suggests the messages are flooded from each node sets as described above and as shown in Figs. 2 and 3. The flooding scheme requires the least number of computations.

2. Selective schedule : In this scheme, for every set of messages passed between the observation nodes and the variable nodes (Λ_{il}'s and P_{li}'s), multiple set of messages are passed between the check nodes and the variable nodes (R_{jl}'s and Q_{lj}'s) [16]. Thus, the messages that help in error correction are computed multiple times in one iteration.

3. Delay optimal schedule: For the above two schemes described, the detection/decoding procedure starts only after M channel uses. Hence the detection/decoding happens only after a delay of $M-1$ channel uses. In the delay optimal scheme, we start the message passing as soon as the reception starts. After the mth channel use, the observation nodes, $y_1, y_2, \cdots, y_{mn_r}$ are initialized with $\mathbf{y}^1, \mathbf{y}^2, \cdots, \mathbf{y}^m$ and the messages are computed as before. After every channel use, the observation nodes are updated with the new set of received symbols and the previously computed messages. Since the detection/decoding starts from $m = 1$ channel use, the delay is minimal in this scheme. As M increases, the number of computations performed in this scheme increases.

V. Results and Discussions

We evaluated the coded bit error rate (BER) performance of the joint message passing algorithm for detection/decoding presented in the previous section through simulations. The average received SNR is $\gamma = \frac{n_t E_s}{\sigma^2}$, where E_s is the average symbol energy. The simulations are performed for V-BLAST MIMO configuration. Figure 4 shows the coded BER performance in 64×64 V-BLAST MIMO system with an LDPC code of $n = 2640$ and rate 1/2. In addition to the performance of joint detection/decoding, for comparison purposes, we have plotted the performance of the LDPC coded system with individual detection and decoding (i.e., Type-B receiver), and iterative detection and decoding (i.e., Type-C receiver, also called as the turbo equalizer). For Type-B receiver, the number of local iterations in detection and decoding, respectively, are 5 and 10. For Type-C receiver, the number of local iterations in detection and decoding, respectively, are 1 and 10, with 2 global (turbo) iterations between detection and decoding. For the detection/decoding, the number of iterations is 10 and the flooding schedule is used. Belief damping with a damping factor of 0.2 is used.

From Fig. 4, we observe that the turbo equalizer and the joint detection/decoding perform significantly better than the individual detection/decoding scheme. Also, the joint detection/decoding performance is better than the turbo equalizer. Fig. 6 shows the performance of the joint message passing algorithm for different LDPC code rates with block length $n = 4000$. As expected, the performance improves as the LDPC code rate decreases. Finally, in Fig. 7, we plot the average SNR required to achieve a coded BER of 10^{-4} in V-BLAST MIMO, with an LDPC code of length $n = 4000$ and rate 1/2, employing the joint detection and decoding for different number of antennas ($n_t = n_r$). We see that the performance gets increasingly closer to the unfaded AWGN performance for increasing $n_t = n_r$. All the above simulation results highlight the attractiveness of the joint message passing approach in terms of both scalability for large n_t as well as good performance.

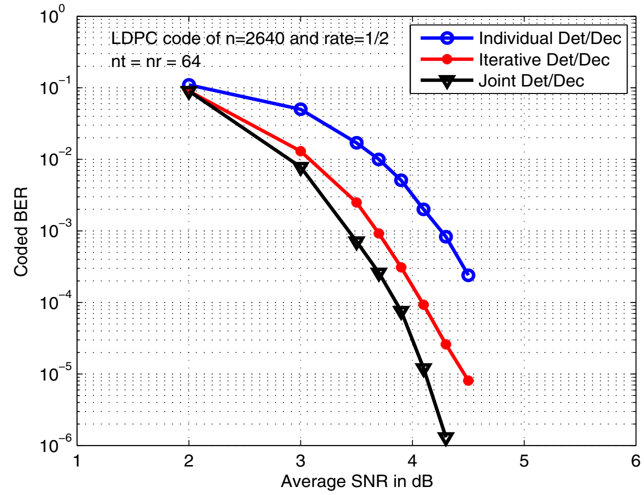

Fig. 4. Coded BER performance of joint message passing based detection/decoding in 64×64 V-BLAST MIMO system with LDPC code block size, $n = 2640$ bits.

The total complexity of the factor graph based detection scheme is $O(n_t n_r)$ for both the computation of the messages at observation nodes and at variable nodes [6]. The complexity of the LDPC decoding algorithm requires $O(cn)$ additions for variable node messages and $O(r(n-k))$ multiplications for check node messages, where c and r are the column and row weights of the parity check matrix, respectively. The joint message passing algorithm requires the same complexity for the computation of messages at observation nodes and check nodes. The complexity of variable node messages computation is $O(n_t M(n_r M + c))$. This clearly is a reduction over the total complexity of individual and iterative type detection/decoding.

The three scheduling schemes considered are found to have similar performance and complexity. From Fig. 8, we observe that the selective scheduling scheme converges relatively faster than the flooding schedule in 64×64 V-BLAST MIMO with an LDPC code of $n = 4000$ and rate 1/2, at 3 dB SNR.

VI. Conclusions

We considered LDPC coded large-MIMO systems with tens of antennas, and applied ideas of message passing to achieve good detection and decoding performance at low complexities. In particular, we formulate a factor graph over which detection and decoding were performed jointly. The presented joint message passing scheme can be extended to remove the effects of spatial interference as well as inter-symbol interference (ISI) in large dimension MIMO-ISI channels with severe delay spreads.

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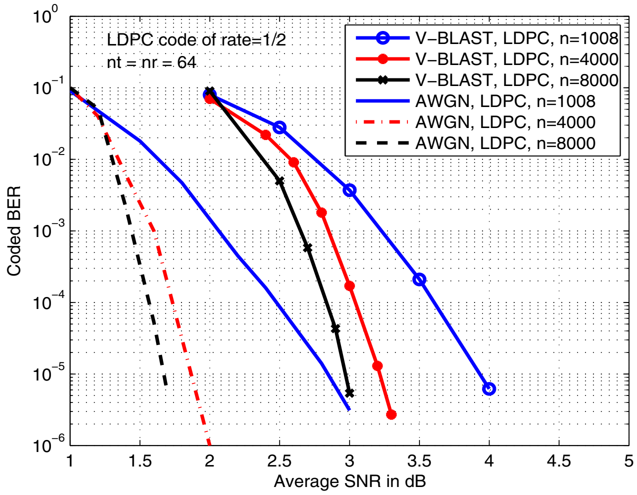

Fig. 5. Coded BER performance of joint message passing based detection/decoding in 64×64 V-BLAST MIMO system for different LDPC code block sizes.

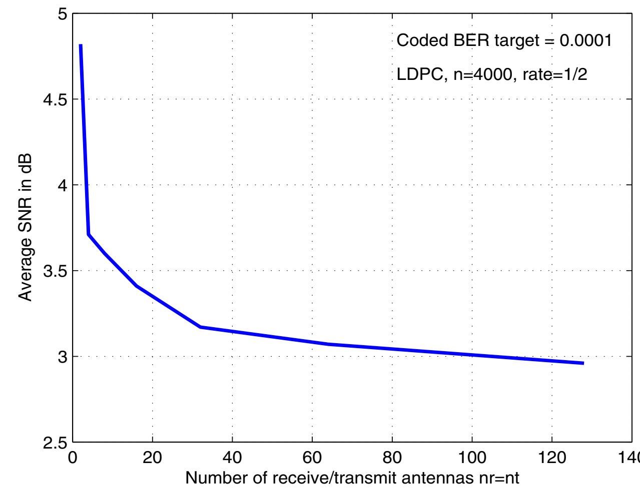

Fig. 7. Average SNR required by joint message passing based detection/decoding for different number of antennas (n_t=n_r) in V-BLAST MIMO system with LDPC code block size $n = 4000$ bits, to achieve 10^{-4} BER.

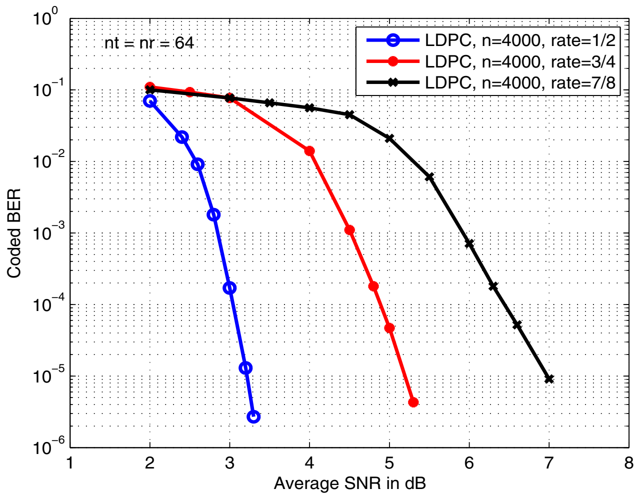

Fig. 6. Coded BER performance of joint message passing based detection/decoding in 64×64 V-BLAST MIMO system for different LDPC code rates with block size, $n = 4000$ bits.

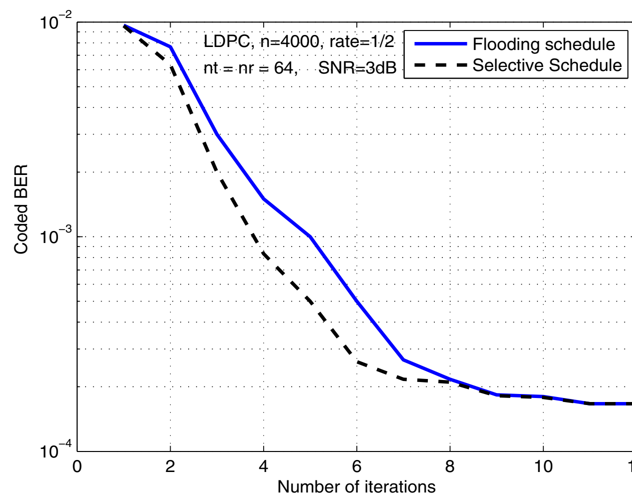

Fig. 8. BER achieved by different scheduling schemes of joint message passing based detection/decoding at different number of iterations, in 64×64 V-BLAST MIMO with LDPC code block size $n = 4000$ bits at SNR = 3dB.

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