Full-rate Precoding in V-BLAST with Angle Parameter Feedback

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***Abstract*— In this paper, we present novel precoder designs for V-BLAST systems which achieve full-rate. We first present a precoding scheme based on algebraic lattices over real number fields. We prove that this full-rate scheme achieves full-diversity, assuming availability of full channel state information at the transmitter. We then propose a precoding scheme which achieves full-rate and high orders of diversity with limited feedback. The proposed scheme involves a precoder codebook design consisting of unitary matrices parametrized by a single angular parameter, thereby requiring less feedback overhead. Our simulation results show that the proposed limited feedback precoding scheme provides a performance improvement of 1.5 dB and 2.5 dB at a bit error rate of 10^{-3} over existing schemes for 2×2 and 4×4 V-BLAST systems, respectively, using 4-QAM.**

Keywords: *MIMO precoding, full-rate, limited feedback, V-BLAST, diversity.*

I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques can provide the benefits of spatial diversity and multiplexing gain [1]-[3]. Spatial multiplexing (V-BLAST) using N_t transmit antennas achieves the full-rate of N_t symbols per channel use. However, full transmit diversity of order N_t is not achieved in V-BLAST. Orthogonal space-time block codes (e.g., 2×2 Alamouti code) achieve full transmit diversity, but suffer from rate loss for increased number of antennas [2]. Achieving full-rate as well as full-diversity simultaneously at low complexities is a desired goal in MIMO communications.

Non-orthogonal space-time block codes (STBC) [4]-[8], offer the full-rate of N_t symbols per channel use by having N_t^2 symbols in one $N_t \times N_t$ STBC matrix, and full-diversity under maximum likelihood (ML) decoding. A drawback with non-orthogonal STBCs is their high decoding complexities, because ML decoding of these STBCs involves joint decoding of N_t^2 symbols. ML decoding in V-BLAST, on the other hand, involves joint decoding of only N_t symbols. The inability of V-BLAST to achieve transmit diversity can be overcome through the use of precoding at the transmitter [9]-[14]. Precoding based on knowledge of full channel state information at the transmitter (CSIT) and first-/second-order statistics of the channel have been studied widely [9]-[11]. Limited feedback (LFB) precoding schemes in V-BLAST are of interest in practice. Full-diversity is achieved in limited feedback precoding in V-BLAST, but with some loss in rate [12]-[14]. For e.g., the precoding scheme in [12] is based on Grassmannian subspace packing, which does not allow simultaneous transmission of more than N_t-1 streams (i.e., achievable rate is $\leq N_t - 1$ symbols per channel use). In a 2×2 MIMO system, this means a rate loss of 50%. The same is true with any other Grassmannian subspace packing based scheme or transmit antenna selection based scheme [14]. Precoding schemes of [15]-[16] achieve full-rate, but require feedback of real values, thereby increasing feedback overhead.

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In this paper, firstly, we propose a precoding scheme based on algebraic lattices over totally real number fields [17]. Based on the maximal diversity property of these lattices, we prove, through analysis, that our full-rate precoding scheme achieves full-diversity, given full CSIT. Simulation results confirm that the lattice-based precoder scheme achieves full-diversity for the cases of 2×2 and 4×4 V-BLAST. Secondly, we propose a full-rate limited feedback precoding scheme that achieves high diversity orders. The proposed precoding scheme involves a precoder codebook design consisting of unitary matrices parametrized by only a single angular variable, thereby requiring less number of feedback bits. Since the proposed scheme is not based on subspace packing or antenna selection, there is no loss in rate. We evaluate the performance of the proposed LFB precoding scheme and compare it with that of the existing full-rate precoding schemes presented in [15] and [16]. Our simulation results show that the proposed LFB precoding scheme for a 2×2 V-BLAST system using 4-QAM outperforms Orthogonalized Spatial Multiplexing schemes in [15] by 1.5 dB at a bit error rate (BER) of 10^{-3}. Similarly, for a 4×4 V-BLAST system with 4-QAM, the proposed LFB precoding scheme shows a gain of 2.5 dB at BER of 10^{-3} over the scheme presented in [16].

The rest of this paper is organized as follows. In Section II, we present the proposed lattice based precoder with full CSIT and prove that it achieves full-diversity. In Section III, we present the proposed full-rate limited feedback precoding scheme and its BER performance. Conclusions are presented in Section IV.

II. LATTICE BASED PRECODER

In this section, we consider a precoded V-BLAST system with full channel knowledge at the receiver as well as the transmitter. Let N_t and N_r denote the number of transmit and receive antennas, respectively. Let $\mathbf{H} \in \mathbb{C}^{N_r\times N_t}$ denote the channel gain matrix, whose entries are assumed to be i.i.d and $\mathcal{CN}(0,1)$. Using $\mathcal{R}\{.\}$ and $\mathcal{I}\{.\}$ to denote the real and imaginary parts of a complex argument, the equivalent real-valued model for $\mathbf{H}$ can be written as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathcal{R}\{\mathbf{H}\} & -\mathcal{I}\{\mathbf{H}\} \\ \mathcal{I}\{\mathbf{H}\} & \mathcal{R}\{\mathbf{H}\} \end{bmatrix}. \quad (1)$$

Singular value decomposition (SVD) of $\tilde{\mathbf{H}}$ is given by

$$\tilde{\mathbf{H}} = \mathbf{U\Sigma V}^T, \quad (2)$$

where $\mathbf{U} \in \mathbb{R}^{2N_r \times 2N_r}$ and $\mathbf{V} \in \mathbb{R}^{2N_t \times 2N_t}$ are orthogonal matrices and $\mathbf{\Sigma}$ is a $2N_r \times 2N_t$ diagonal matrix with real entries $\sigma_1, \sigma_2, \cdots, \sigma_r$, where r is the rank of $\tilde{\mathbf{H}}$.

Let $\mathbf{u} \in \mathbb{A}^{N_t}$ denote the complex data symbol vector at the transmitter, where $\mathbb{A}$ is the modulation alphabet. Let $\tilde{\mathbf{u}} = [\mathcal{R}\{\mathbf{u}^T\}\ \mathcal{I}\{\mathbf{u}^T\}]^T \in \mathbb{S}^{2N_t}$, where $\mathbb{S}$ is the real PAM constellation corresponding to $\mathbb{A}$. Let $\mathbf{R}$ be the generator matrix of the rotated $\mathbb{Z}^n$ lattice of dimension $2N_t$, constructed over real number field [17], where $\mathbf{R} \in \mathbb{R}^{2N_t \times 2N_t}$. Then, $\mathbf{R}\tilde{\mathbf{u}} \in \mathbb{R}^{2N_t}$ represents a point in the real lattice defined by $\mathbf{R}$. This vector is precoded with the right singular matrix $\mathbf{V}$ of $\tilde{\mathbf{H}}$ to give

$$\tilde{\mathbf{x}} = \mathbf{V}\mathbf{R}\tilde{\mathbf{u}}. \quad (3)$$

The transmitted symbol vector is given by $\mathbf{x} \in \mathbb{C}^{N_t}$, such that $\tilde{\mathbf{x}} = [\mathcal{R}\{\mathbf{x}^T\}\ \mathcal{I}\{\mathbf{x}^T\}]^T$. The received signal vector, $\mathbf{y} \in \mathbb{C}^{N_r}$, at the receiver is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (4)$$

where $\mathbf{n} \in \mathbb{C}^{N_r}$ is the noise vector whose entries are i.i.d $\mathcal{CN}\big(0, \sigma^2 = \frac{N_t E_s}{\gamma}\big)$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna.

A. Diversity Analysis

The probability of error of the ML decision in the lattice-precoded system depends on the minimum distance, d_{min}, which is given by

$$d^2_{min}(\mathbf{H}) = \min_{\tilde{\mathbf{u}}_j, \tilde{\mathbf{u}}_k \in \mathbb{S}^{2N_t},\, \tilde{\mathbf{u}}_j \neq \tilde{\mathbf{u}}_k} \|\tilde{\mathbf{H}}\mathbf{V}\mathbf{R}(\tilde{\mathbf{u}}_j - \tilde{\mathbf{u}}_k)\|_2^2. \quad (5)$$

At high SNR values, P_e (symbol error probability) has a tight upper bound in the form of union bound, which depends on the minimum distance $d_{min}(\mathbf{H})$:

$$P_e \leq e^{-\frac{E_s}{4\sigma^2} d^2_{min}(\mathbf{H})}. \quad (6)$$

Let $\mathbf{z} \triangleq \mathbf{R}\tilde{\mathbf{u}}$ and z_i^k be the kth entry of $\mathbf{z}_i, \forall k = 1, \cdots, 2N_t$. Also, let, $\mathbf{z}_{(i,j)} \triangleq \mathbf{R}(\tilde{\mathbf{u}}_i - \tilde{\mathbf{u}}_j),\ \forall i,j,\ i \neq j$ and $z^k_{(i,j)}$ be the kth entry of $\mathbf{z}_{(i,j)}$. For any two real data vectors $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{u}}_j,\ i \neq j$, using (3), we define,

$$\begin{aligned} d^2(\tilde{\mathbf{u}}_i, \tilde{\mathbf{u}}_j, \tilde{\mathbf{H}}) &\triangleq ||\tilde{\mathbf{H}}(\tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j)||_2^2 \\ &= ||\tilde{\mathbf{H}}\mathbf{V}\mathbf{R}(\tilde{\mathbf{u}}_i - \tilde{\mathbf{u}}_j)||_2^2 \quad (7) \\ &= ||\mathbf{U}\mathbf{\Sigma}\mathbf{R}(\tilde{\mathbf{u}}_i - \tilde{\mathbf{u}}_j)||_2^2 \\ &= ||\mathbf{\Sigma}(\mathbf{z}_i - \mathbf{z}_j)||_2^2 \\ &= \sum_{k=1}^{r} \sigma_k^2 |z_i^k - z_j^k|^2. \quad (8) \end{aligned}$$

We further define

$$\delta_{(i,j)} \triangleq \min_{k=1,\cdots,2N_t} |z^k_{(i,j)}|, \quad i \neq j \quad (9)$$

and

$$\delta \triangleq \min_{i \neq j} \delta_{(i,j)}. \quad (10)$$

The rotated $\mathbb{Z}^n$ lattice $\mathbf{R}$ exhibits full-diversity property [18], i.e., $\forall \mathbf{u} \neq \mathbf{0}$, the vector $\mathbf{Ru}$ does not have any zero entries. Therefore,, $|z^k_{(i,j)}| \neq 0\ \forall k = 1, \ldots, 2N_t$. Hence, $\delta_{(i,j)} \neq 0\ \forall i \neq j$ and therefore $\delta \neq 0$.

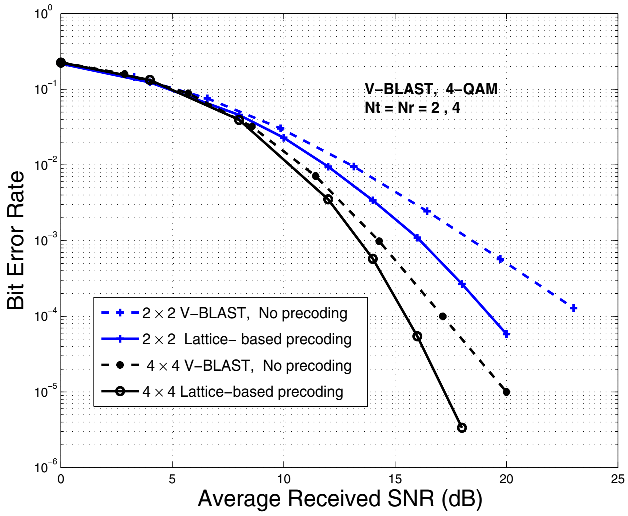

Fig. 1. BER performance of the proposed lattice-based precoder for 2×2 and 4×4 V-BLAST, 4-QAM.

Using (9) and (10), we can rewrite (8) as

$$\begin{aligned} d^2(\tilde{\mathbf{u}}_i, \tilde{\mathbf{u}}_j, \tilde{\mathbf{H}}) &= \sum_{k=1}^{r} \sigma_k^2 |z^k_{(i,j)}|^2 \\ &> \delta^2 \sum_{k=1}^{r} \sigma_k^2 \\ &= \delta^2 ||\tilde{\mathbf{H}}||_F^2 \\ &= 2\delta^2 ||\mathbf{H}||_F^2. \quad (11) \end{aligned}$$

Using the definition of $d^2(\tilde{\mathbf{u}}_i, \tilde{\mathbf{u}}_j, \tilde{\mathbf{H}})$ in (7), we can rewrite (5) as

$$d^2_{min}(\mathbf{H}) = \min_{\tilde{\mathbf{u}}_i, \tilde{\mathbf{u}}_j \in \mathbb{S}^{2N_t}, \tilde{\mathbf{u}}_i \neq \tilde{\mathbf{u}}_j} d^2(\tilde{\mathbf{u}}_i, \tilde{\mathbf{u}}_j, \tilde{\mathbf{H}}). \quad (12)$$

From (11) and (12), it follows that

$$d^2_{min}(\mathbf{H}) > 2\delta^2 ||\mathbf{H}||_F^2. \quad (13)$$

The average probability of symbol error is thus given by

$$\overline{P}_e < \mathbb{E}_{\mathbf{H}} \left[e^{-\frac{E_s}{2\sigma^2} \delta^2 ||\mathbf{H}||_F^2} \right], \quad (14)$$

where $\mathbb{E}(.)$ denotes the expectation operator. $||\mathbf{H}||_F^2$ is distributed as a Chi-square $\chi^2_{2N_r N_t}$ variable with $2N_r N_t$ degrees of freedom. Hence, the above lattice precoded scheme achieves the full-diversity order of $N_r N_t$.

B. Simulation Results

We evaluated the BER performance of the proposed lattice-based precoding scheme for the cases of 2×2 and 4×4 V-BLAST systems as a function of the average received SNR, γ, and the results are shown in Fig 1. Sphere decoding algorithm is used for detection at the receiver. The BER versus SNR plots of lattice-based precoders for both $N_t = N_r$ = 2, 4 have steeper slopes compared to that of V-BLAST with no precoding, thus indicating that the proposed scheme has significant diversity improvement. The diversity achieved by 2×2 and 4×4 systems using the proposed precoding scheme are 4 and 16 respectively, as compared to 2 and 4 for V-BLAST with no precoding.

We note that, although the proposed full-rate lattice-based precoder achieves full-diversity, the scheme requires the full knowledge of CSIT. However, in practical MIMO systems, perfect CSIT is often not available. Hence, we propose a full-rate precoding scheme for V-BLAST with limited feedback, based on a single angular parameter, which achieves high diversity orders without requiring full CSIT.

III. Proposed Full-rate Precoding Scheme with Angle Parameter Feedback

In this section, we present the proposed full-rate limited feedback based precoding scheme. Let $\mathcal{F} = \{\mathbf{F}_0, \mathbf{F}_1, \cdots, \mathbf{F}_{N-1}\}$ denote the precoder codebook of size N, where the $\mathbf{F}_n$'s, $n = 0, 1, \cdots, N-1$, are $N_t \times N_t$ unitary precoding matrices. This codebook is known to both the transmitter and the receiver. For a given channel $\mathbf{H}$, the receiver chooses the precoding matrix from $\mathcal{F}$ that maximizes the minimum distance with ML decoding, and sends the corresponding index to the transmitter. Let $B = \lceil \log_2 N \rceil$ denote the number of feedback bits needed to represent this index. Given this index, k, the transmitter uses the corresponding precoding matrix, denoted by $\mathbf{F} = \mathbf{F}_k$. Let $\mathbf{u} \in \mathbb{A}^{N_t}$ denote the complex data symbol vector at the transmitter, where $\mathbb{A}$ is the modulation alphabet. The transmitted signal vector, $\mathbf{x} \in \mathbb{C}^{N_t}$ is given by $\mathbf{x} = \mathbf{F}\mathbf{u}$. The received signal vector, $\mathbf{y} \in \mathbb{C}^{N_r}$, at the receiver is

$$\mathbf{y} \;=\; \mathbf{H}\mathbf{F}\mathbf{u} + \mathbf{n}, \tag{15}$$

where $\mathbf{n} \in \mathbb{C}^{N_r}$ is the noise vector whose entries are i.i.d $\mathcal{CN}\big(0, \sigma^2 = \frac{N_t E_s}{\gamma}\big)$.

For a non-preceded system (i.e., for $\mathbf{F} = \mathbf{I}_{N_t}$), the ML decision is given by

$$\widehat{\mathbf{x}} \;=\; \arg\min_{\mathbf{x} \in \mathbb{A}^{N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2. \tag{16}$$

The probability of error in the decision depends on the minimum distance, d_{min}, which is given by

$$d_{min} \;=\; \min_{\mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}, \mathbf{x}_j \neq \mathbf{x}_k} \|\mathbf{H}(\mathbf{x}_j - \mathbf{x}_k)\|_2. \tag{17}$$

It is known that precoding at the transmitter improves d_{min} [3]. We illustrate this point using the following example and Fig. 2. Assume a 2×2 system with $\mathbf{H} = \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$ and PAM modulation. As can be seen from Fig. 2, $d_{min}(\mathbf{H}) = d_1 = 1.414$. Now, consider unitary precoding with $\mathbf{F} = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}$. The new effective channel matrix is given by $\mathbf{H}' = \mathbf{H}\mathbf{F}$. From Fig. 2, it can be seen that $d_{min}(\mathbf{H}') = d_2 = 2.875 > d_{min}(\mathbf{H})$.

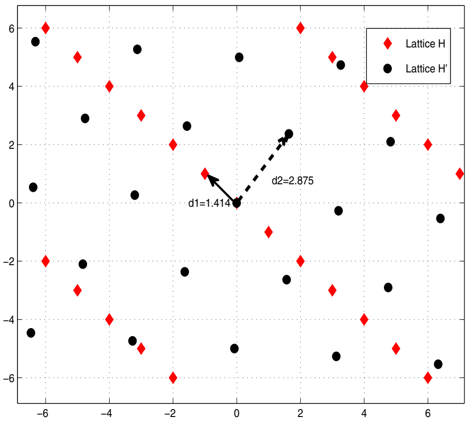

Fig. 2. Illustration of d_{min} improvement of a 2-dimensional lattice through transformation by a unitary matrix.

A. Codebook Design

Let $\mathcal{U}_{N_t}(\theta) = \{$all $N_t \times N_t$ unitary matrices parametrized by a single angular variable, $\theta\}$. With each matrix $\mathbf{U}_{N_t}(\theta) \in \mathcal{U}_{N_t}(\theta)$, we associate an infinite size codebook, $\zeta_\infty(\mathbf{U}_{N_t}(\theta)) = \{\mathbf{U}_{N_t}(\theta)|_{\theta=\alpha}, \ \forall \alpha \in (0, 2\pi)\}$. To define a finite set precoder, we select a finite subset of size $N = 2^B$ from $\zeta_\infty(\mathbf{U}_{N_t}(\theta))$, where B is the number of feedback bits. Specifically, we define the finite precoding codebook as $\zeta_N(\mathbf{U}_{N_t}(\theta)) = \{\mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi i}{N}}, \ i = 0, 1, \cdots, N-1)\}$. For e.g., for $N_t = 2$, $B = 2$ and $N = 4$, with $\mathbf{U}_2(\theta)$ given by

$$\mathbf{U}_2(\theta) \;=\; \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-j\theta/2} & e^{-j\theta} \\ e^{j3\theta/2} & -e^{j\theta} \end{bmatrix}. \tag{18}$$

the finite precoding codebook is given by $\zeta_4(\mathbf{U}_2(\theta)) = \left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1-i}{2} & \frac{-i}{\sqrt{2}} \\ \frac{-1+i}{2} & \frac{-i}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{-1-i}{2} & \frac{i}{\sqrt{2}} \\ \frac{1+i}{2} & \frac{i}{\sqrt{2}} \end{bmatrix} \right\}$.

The performance of the codebook is dependent on the choice of $\mathbf{U}_{N_t}(\theta)$ and N. We, therefore, come up with the following performance indicator for a given codebook:

$$\mu(\zeta_N(\mathbf{U}_{N_t}(\theta))) \;=\; \mathbb{E}_{\mathbf{H}} \left[\frac{\max\limits_{\theta_i = \frac{2\pi i}{N}, i \in \{0, \cdots, N-1\}} \ \min\limits_{\mathbf{x}_j \neq \mathbf{x}_k, \mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}} \|\mathbf{H}\mathbf{U}_{N_t}(\theta_i)(\mathbf{x}_j - \mathbf{x}_k)\|_2^2}{\min\limits_{\mathbf{x}_j \neq \mathbf{x}_k, \mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}} \|\mathbf{H}(\mathbf{x}_j - \mathbf{x}_k)\|_2^2} \right]. \tag{19}$$

In words, $\mu(\zeta_N(\mathbf{U}_{N_t}(\theta)))$ is the expected ratio of the maximum squared d_{min} with precoding to that without precoding. Hence, this performance indicator gives us the improvement in the d_{min} value of the precoded lattice and thereby, relates directly to the BER performance improvement achieved by precoding (eqn. 6). Then, the optimal finite precoding codebook with $B = \log_2 N$ feedback bits is $\zeta_N(\mathbf{U}_{N_t}^{opt}(\theta))$, where $\mathbf{U}_{N_t}^{opt}(\theta)$ is given by

$$\mathbf{U}_{N_t}^{opt}(\theta) \;=\; \underset{\mathbf{U}_{N_t}(\theta) \in \mathcal{U}_{N_t}(\theta)}{\text{argmax}} \ \mu(\zeta_N(\mathbf{U}_{N_t}(\theta))). \tag{20}$$

Obtaining an exact solution for $\mathbf{U}_{N_t}^{opt}(\theta)$ analytically is difficult. In the absence of a solution to the above problem, we tried out several $\mathbf{U}_{N_t}(\theta)$ matrices for small values of N_t (e.g., $N_t = 2, 3, 4$, which are of interest in practical MIMO systems), and found that the following designs for $N_t = 2, 3, 4$ work very well[1] in the proposed scheme:

$$\mathbf{U}_{N_t=2}(\theta) \;=\; \frac{1}{\sqrt{2}} \begin{bmatrix} e^{j\theta} & 1 \\ -1 & e^{-j\theta} \end{bmatrix}, \tag{21}$$

$$\mathbf{U}_{N_t=3}(\theta) \;=\; \frac{1}{3} \begin{bmatrix} 2e^{j\theta} & -2 & e^{j\theta} \\ e^{j\frac{\theta}{2}} & 2e^{-j\frac{\theta}{2}} & 2e^{j\frac{\theta}{2}} \\ 2 & e^{-j\theta} & -2 \end{bmatrix}, \tag{22}$$

[1]Our computer simulations show that these designs for $N_t = 2, 3, 4$ achieve very good BER performance (as we will see in Sec. III-D).

$$\mathbf{U}_{N_t=2^{m+1}}(\theta) = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbf{U}_{2^m}(\theta) & \mathbf{I}_{2^m} \\ -\mathbf{I}_{2^m} & \mathbf{U}_{2^m}^H(\theta) \end{pmatrix}. \quad (23)$$

B. Precoding Matrix Selection

At the receiver, given the knowledge of $\mathbf{H}$, we define $d_{min}(\mathbf{H}, i)$, $i = 0, 1, \cdots, N-1$, as

$$d_{min}(\mathbf{H}, i) \triangleq \min_{\mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}, \mathbf{x}_j \neq \mathbf{x}_k} \left\| \mathbf{H}\mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi i}{N}}(\mathbf{x}_j - \mathbf{x}_k)\right\|_2. \quad (24)$$

The receiver sends to the transmitter the index p, given by

$$p = \arg\max_{i \in \{0,1,..,N-1\}} d_{min}(\mathbf{H}, i), \quad (25)$$

using B bits of feedback. Hence, the optimum precoding matrix chosen is given by

$$\mathbf{F}_p = \mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi p}{N}}. \quad (26)$$

C. Feedback Computation

In the following, we present the computation of $d_{min}(\mathbf{H}, i)$ in (24) for $i = 0, 1, \cdots N-1$.

We can rewrite the system model equation (15) for the precoded system, when precoded with the ith precoding matrix as

$$\mathbf{y} = \mathbf{H}_i\mathbf{x} + \mathbf{n}, \quad (27)$$

where $\mathbf{H}_i \triangleq \mathbf{H}\mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi i}{N}}$. The above equation can be transformed into an equivalent real-valued model as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}_i\tilde{\mathbf{x}} + \tilde{\mathbf{n}}, \quad (28)$$

where $\tilde{\mathbf{y}} = [\mathcal{R}\{\mathbf{y}^T\} \;\; \mathcal{I}\{\mathbf{y}^T\}]^T$, $\tilde{\mathbf{x}} = [\mathcal{R}\{\mathbf{x}^T\} \;\; \mathcal{I}\{\mathbf{x}^T\}]^T$, $\tilde{\mathbf{n}} = [\mathcal{R}\{\mathbf{n}^T\} \;\; \mathcal{I}\{\mathbf{n}^T\}]^T$, and

$$\tilde{\mathbf{H}}_i = \begin{bmatrix} \mathcal{R}\{\mathbf{H}_i\} & -\mathcal{I}\{\mathbf{H}_i\} \\ \mathcal{I}\{\mathbf{H}_i\} & \mathcal{R}\{\mathbf{H}_i\} \end{bmatrix}. \quad (29)$$

Here, $\tilde{\mathbf{y}} \in \mathbb{R}^{2N_r \times 1}$, $\tilde{\mathbf{x}} \in \mathbb{S}^{2N_t \times 1}$, $\tilde{\mathbf{n}} \in \mathbb{R}^{2N_r \times 1}$ and $\tilde{\mathbf{H}}_i \in \mathbb{R}^{2N_r \times 2N_t}$. Also, $\mathbb{S}$ is the real PAM constellation corresponding to $\mathbb{A}$. Henceforth, we shall work with the real-valued system in (28). With the new system model in (28), we can re-write (24) as

$$\begin{aligned} d_{min}(\mathbf{H}, i) &\triangleq \min_{\mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}, \mathbf{x}_j \neq \mathbf{x}_k} \left\| \mathbf{H}\mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi i}{N}}(\mathbf{x}_j - \mathbf{x}_k)\right\|_2 \\ &= \min_{\tilde{\mathbf{x}}_j, \tilde{\mathbf{x}}_k \in \mathbb{S}^{2N_t}, \tilde{\mathbf{x}}_j \neq \tilde{\mathbf{x}}_k} \left\| \tilde{\mathbf{H}}_i(\tilde{\mathbf{x}}_j - \tilde{\mathbf{x}}_k)\right\|_2 \\ &= \min_{\mathbf{z} \in \mathbb{D}^{2N_t}, \mathbf{z} \neq \mathbf{0}} \left\| \tilde{\mathbf{H}}_i\mathbf{z}\right\|_2, \end{aligned} \quad (30)$$

where $\mathbb{D}$ is the difference constellation of $\mathbb{S}$. For example, for square M-QAM modulation, $\mathbb{S}$ is given by $\{\pm1, \pm3, .., \pm(\sqrt{M}-1)\}$ and $\mathbb{D}$ is $\{0, \pm2, \pm4, .., \pm2(\sqrt{M}-1)\}$. Since the factor of 2 can be neglected for each $\mathbf{z}$ for the minimization of $||\tilde{\mathbf{H}}_i\mathbf{z}||_2$, $\mathbb{D}$ can be taken to be $\mathbb{D} = \{s : s \in \mathbb{Z}, \; |s| \; \leq (\sqrt{M}-1)\}$.

Geometrically, $\tilde{\mathbf{H}}_i$ defines a $2N_t$ dimensional lattice in $\mathbb{R}^{2N_r}$, denoted as $\Lambda = \{\tilde{\mathbf{H}}_i\mathbf{z} : \mathbf{z} \in \mathbb{Z}^{2N_t}\}$ and a finite subset of Λ is $\tilde{\Lambda} = \{\tilde{\mathbf{H}}_i\mathbf{z} : \mathbf{z} \in \mathbb{D}^{2N_t}\}$. For the receiver to find the optimal precoding matrix, it needs to evaluate $d_{min}(\mathbf{H}, i)$ using (30) for all $i = 0, 1, \cdots N-1$. Given the geometrical interpretation it is easy to see that calculation of (30) is equivalent to the problem of finding the shortest vector in a finite subset of the lattice Λ. This problem is a constrained version of the well known shortest vector problem (SVP) for any arbitrary lattice. Unconstrained SVP can be solved by appropriately modifying the closest lattice point search algorithm, as discussed in [19]. We solve the constrained SVP problem by restricting the search to be within the finite subset $\tilde{\Lambda}$.

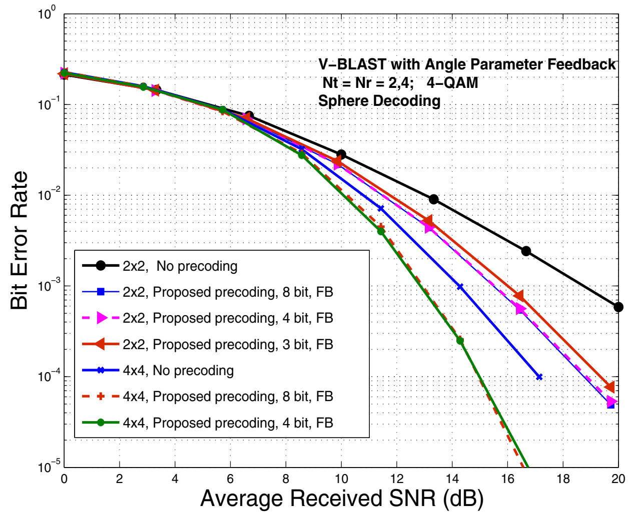

Fig. 3. BER performance of the proposed limited feedback precoded 2×2 and 4×4 V-BLAST scheme. 4-QAM, # feedback bits, $B = 8, 4, 3$.

D. Simulation Results

We evaluated the BER performance of the proposed angle feedback based precoding scheme as a function of average received SNR per receive antenna, γ, through simulations for $N_t = N_r = 2$ and 4. Feedback bits are assumed to be available error-free at the transmitter.

Figure 3 shows the simulation results for 2×2 and 4×4 V-BLAST with and without the proposed precoding, for 4-QAM using the sphere decoding algorithm in [20] for detection at the receiver. BER performance plots for different levels of quantization requiring different number of feedback bits ($B = 8, 4, 3$ bits) are shown. The interesting observation in Fig. 3 is the effect of feedback bits on the BER performance. It can be seen that precoded system with $B = 4$ performs very close to that with $B = 8$, showing that the performance in the proposed scheme remains robust even with a nominal quantization of using 4 bits. Also, from Fig. 3, it is observed that the proposed precoding scheme achieves significantly better performance compared to 'no precoding' scheme.

Next, in Figs. 4 to 5, we compare the BER performances of the proposed angle feedback based precoding scheme with those of the existing full-rate precoding schemes in [15] and [16]. Sphere decoding algorithm in [20] is used for detection at the receiver. Both precoding schemes presented in [15] and [16] are based on Orthogonalized Spatial Multiplexing (OSM) and requires the receiver to send back to the transmitter more than one real feedback parameter. On the other hand, the proposed scheme is based on a single angular parameter which can be quantized with small number of bits. Hence, the

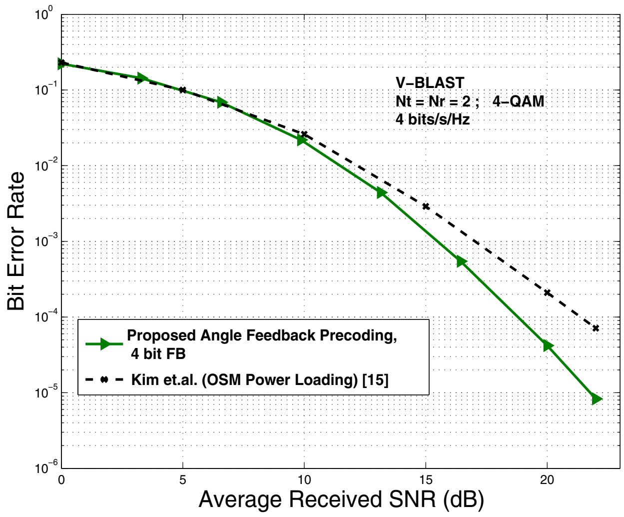

Fig. 4. BER performance of the proposed limited feedback precoded 2×2 V-BLAST scheme versus OSM power allocation schemes [15]. 4-QAM, # feedback bits for proposed scheme, $B = 4$.

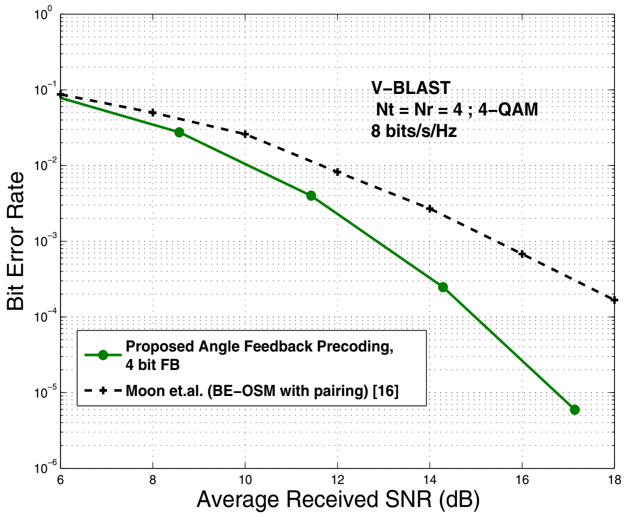

Fig. 5. BER performance of the proposed limited feedback precoded 4×4 V-BLAST scheme versus BE-OSM scheme (with pairing) [16]. 4-QAM, # feedback bits for proposed scheme, $B = 4$.

feedback overhead in the proposed scheme is less compared to that of [15] and [16].

The BER comparisons in Fig. 4 and 5 show that the proposed angle feedback based scheme achieves significantly higher diversity orders than the schemes of [15] and [16], as indicated by distinct difference in the slopes of the BER versus SNR plots for both 2×2 and 4×4 V-BLAST systems. Furthermore, Fig. 4 shows that our precoder scheme outperforms the precoder in [15] by 1.5 dB at a BER of 10^{-3}. Also, Fig. 5 shows that our scheme is better by a margin of 2.5 dB at a BER of 10^{-3} compared to the scheme adopted in [16].

IV. Conclusions

In this paper, we first presented a full-rate lattice-based precoder scheme for V-BLAST assuming full CSIT. We showed that this scheme achieves full transmit diversity. We then presented a simple, single angle parameter based codebook design for limited feedback precoding in V-BLAST. The proposed precoding scheme achieves full-rate by design. Also, the design, being based on a single angular parameter only, requires less number of quantization bits. Our simulation results showed that in both 2×2 and 4×4 V-BLAST, the proposed angle parameter based precoding outperformed the other full-rate precoding schemes in the literature by good margins.

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