Full-Rate Full-Diversity Achieving MIMO Precoding with Partial CSIT

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***Abstract*— In this paper, we consider a slow-fading $n_t \times n_r$ multiple-input multiple-output (MIMO) channel subjected to block fading. Reliability (in terms of achieved diversity order) and rate (in number of symbols transmitted per channel use) are of interest in such channels. We propose a new precoding scheme which achieves both full diversity ($n_t n_r$th order diversity) as well as full rate (n_t symbols per channel use) using partial channel state information at the transmitter (CSIT). The proposed scheme achieves full diversity and improved coding gain through an optimization over the choice of constellation sets. The optimization maximizes d^2_{min} for our precoding scheme subject to an energy constraint. The scheme requires feedback of $n_t - 1$ angle parameter values, compared to $2n_t n_r$ real coefficients in case of full CSIT. Further, for the case of $n_t \times 1$ system, we prove that the capacity achieved by the proposed scheme is same as that achieved with full CSIT. Error rate performance results for $n_t = 3, 4, 8$ show that the proposed scheme performs better than other precoding schemes in the literature; the better performance is due to the choice of the signal sets and the feedback angles in the proposed scheme.**

***Keywords:*MIMO precoding, constellation sets, full-diversity, full-rate, partial CSIT.**

I. Introduction

Multiple-input multiple-output (MIMO) techniques can achieve high data rates and spatial diversity in wireless communications over fading channels [1],[2]. Spatial multiplexing (V-BLAST) with n_t antennas at the transmitter achieves the full rate of n_t symbols per channel use, but does not achieve transmit diversity. Space constraints in user terminals like mobile/portable receivers make asymmetric MIMO configuration with $n_r < n_t$ antennas at the receiver to be a preferred choice. Precoding techniques can improve performance through the use of channel state information at the transmitter (CSIT). Several precoding methods use CSIT and achieve diversity benefits, but compromise on the achieved rate; e.g., only one symbol per channel use is achieved in transmit beamforming [3]. It is desirable that precoding methods achieve high rates (preferably the full rate of n_t symbols per channel use as in V-BLAST) and high diversity orders (preferably the full diversity order of $n_t n_r$)[1] with partial CSIT. Our work reported in this paper addresses this problem.

A vast body of research in the literature has addressed the problem of MIMO precoding. In particular, precoding using partial CSIT/limited feedback has been of interest because providing full CSIT (which refers to the full knowledge of all the channel gains between transmit and receive antennas) through feedback can be too expensive [4]-[7].

Precoding in spatial multiplexing (V-BLAST) systems has been considered for achieving high rates and transmit diversity [9]-[14]. Precoding schemes in [10]-[12] incur some loss in rate. For example, the Grassmannian subspace packing based precoding in [10] does not allow simultaneous transmission of more than $n_t - 1$ streams (i.e., achievable rate is $\leq n_t - 1$ symbols per channel use). The Precoding scheme in [13] achieves full rate asymptotically using D-BLAST architecture with partial CSIT, but achieves full diversity only for $\min(n_t, n_r) = 2$ and less diversity orders for other cases. Other recent works have proposed to improve the diversity gain of singular value decomposition (SVD) precoding by pairing good and bad subchannels, and jointly coding information across each pair of subchannels [15],[16]. Though high diversity orders are achieved at low decoding complexity in these schemes, full diversity is not guaranteed. Also, these schemes use full CSIT. In addition, the number of symbols transmitted per channel use $n_s \leq n_r$, and so for $n_r < n_t$ full rate is not achieved. The precoding scheme in [14] uses partial CSIT in the form of a single angle parameter feedback. Though this scheme achieves full rate and high diversity, it failed to achieve full diversity.

In the above context, the significance of our contribution in this paper is that we propose a novel precoding scheme that achieves both *full rate of n_t symbols per channel use* as well as *full diversity of $n_t n_r$* using *partial CSIT*. The scheme requires the feedback of only $n_t - 1$ angle parameter values, compared to $2n_t n_r$ real coefficients in case of full CSIT. The proposed scheme achieves full diversity and coding gain through an optimization over the choice of constellation sets. The optimization maximizes d^2_{min} for our precoding scheme subject to an energy constraint. We present codeword error performance results for $n_t = 3, 4, 8$. We analytically prove the full diversity of the proposed scheme, and simulation results are shown to validate the result. It is further shown that the proposed scheme performs better than other existing schemes. This better performance is attributed to the choice of signal sets and feedback angles in the proposed scheme.

Notation: Vectors are denoted by lower case boldface letters, and matrices are denoted by upper case boldface letters. $(.)^*$, $(.)^H$, $(.)^T$ and tr$(.)$ denote conjugation, hermitian, transpose and trace operators, respectively. $\mathbf{A}_{xy}$ will be used

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[1]In this paper, 'full rate' is defined as n_t symbols per channel use regardless of the number of receive antennas, and 'full diversity' is defined as $n_t n_r$th diversity.

to denote the (x, y)th entry of matrix $\mathbf{A}$.

The rest of the paper is organized as follows. The system model is presented in Section II. The proposed precoding scheme is presented in Section III. Capacity of the proposed scheme for $n_t \times 1$ system is presented in Section IV. Simulation results are presented in Section V. Conclusions are given in Section VI.

II. System Model

Consider a precoded V-BLAST system with n_t antennas at the transmitter and n_r antennas at the receiver. Let $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix, whose entries h_{pq} are distributed i.i.d $\mathcal{CN}(0,1)$, $\forall p = 1, \cdots, n_r$, $\forall q = 1, \cdots, n_t$. Let $\mathbf{F}$ denote the precoder matrix of size $n_t \times n_t$, which is known to both the transmitter and receiver. For a given channel matrix $\mathbf{H}$, the receiver computes $n_t - 1$ number of real feedback parameters and sends them to the transmitter. Given the feedback parameters, the transmitter forms the precoding matrix $\mathbf{F}$. Let $\mathbf{x}$ denote the $n_t \times 1$ complex data symbol vector of the form

$$\mathbf{x} = \left[\, x_1 \; x_2 \quad \cdots \quad x_{n_t} \,\right]^T, \tag{1}$$

where x_i is transmitted by the ith antenna. x_i's will take values from a suitable complex constellation, the choice of which will be discussed later. The received signal vector, $\mathbf{y} \in \mathbb{C}^{n_r \times 1}$, at the receiver, is given by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ is the noise vector with its entries distributed as i.i.d. $\mathcal{CN}\big(0, \sigma^2\big)$.

III. Proposed Precoding Scheme

Consider the precoded V-BLAST system described in Section II. Let

$$\mathbf{F} = \left[\, \mathbf{a} \; \mathbf{a} \quad \cdots \quad \mathbf{a} \,\right], \tag{3}$$

where the column vector $\mathbf{a} = [a_1 \; \cdots \; a_{n_t}]^T \in \mathbb{C}^{n_t \times 1}$ and $|a_i| = 1, \forall i = 1, \cdots, n_t$. At high SNRs, symbol error probability P_e decays as $\text{SNR}^{-n_t n_r}$, provided the received distance squared between each pair of codewords $\mathbf{x}_k, \mathbf{x}_l$ $(k \neq l)$, denoted by $d^2_{k,l}$, is a Chi-square $\chi^2_{2n_t n_r}$ distributed variable with $2n_t n_r$ degrees of freedom [14]. In general,

$$d^2_{k,l} = \|\mathbf{H}\mathbf{F}\Delta\mathbf{x}\|^2, \tag{4}$$

where $\Delta\mathbf{x} = \mathbf{x}_k - \mathbf{x}_l$, $k \neq l$. Now, (4) can be written as

$$\begin{aligned} d^2_{k,l} &= \text{tr}(\mathbf{H}\mathbf{F}\Delta\mathbf{x}\Delta\mathbf{x}^H\mathbf{F}^H\mathbf{H}^H) \\ &= \text{tr}(\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F}\Delta\mathbf{x}\Delta\mathbf{x}^H). \end{aligned} \tag{5}$$

It can be shown that the (p, q)th element of $\mathbf{H}^H\mathbf{H}\mathbf{F}$ is given by

$$(\mathbf{H}^H\mathbf{H}\mathbf{F})_{pq} = \sum_{m=1}^{n_t} a_m \Big(\sum_{o=1}^{n_r} h^*_{op} h_{om} \Big). \tag{6}$$

It may be noted that all the elements of the pth row of the matrix $\mathbf{H}^H\mathbf{H}\mathbf{F}$ are identical.

$$\begin{aligned} \Rightarrow (\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F})_{pq} &= \sum_{n=1}^{n_t} a^*_n \Big(\sum_{m=1}^{n_t} a_m \Big(\sum_{o=1}^{n_r} h^*_{on} h_{om} \Big)\Big) \\ &= \sum_{n=m=1}^{n_t} |a_n|^2 \Big(\sum_{o=1}^{n_r} |h_{on}|^2 \Big) \\ &\quad + \sum_{n \neq m}^{n_t} a^*_n a_m \Big(\sum_{o=1}^{n_r} h^*_{on} h_{om} \Big). \end{aligned} \tag{7}$$

Moreover, it is clear that all the entries of the matrix $\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F}$ are identical. So it amounts to saying that

$$\begin{aligned} \mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F} &= \Bigg(\sum_{n=m=1}^{n_t} |a_n|^2 \Big(\sum_{o=1}^{n_r} |h_{on}|^2 \Big) \\ &\quad + 2\mathcal{R}\Big(\sum_{m,n,n>m}^{n_t} a^*_n a_m \sum_{o=1}^{n_r} h^*_{on} h_{om} \Big) \Bigg) \mathbf{B}, \end{aligned} \tag{8}$$

where $\mathbf{B}_{pq} = 1$. In order to ensure that $d^2_{k,l}$ is distributed as a Chi-square $\chi^2_{2n_t n_r}$ variable, we require that in (8),

$$\mathcal{R}\Big(\sum_{m,n,n>m}^{n_t} a^*_n a_m \Big(\sum_{o=1}^{n_r} h^*_{on} h_{om} \Big)\Big) = 0. \tag{9}$$

Let $a_i = |a_i| e^{j\theta_i}$, $\sum_{o=1}^{n_r} h^*_{on} h_{om} = |\sum_{o=1}^{n_r} h^*_{on} h_{om}| e^{j\alpha_{nm}}$, where $j = \sqrt{-1}$. Then, with $|a_i| = 1$, $\forall i = 1, \cdots, n_t$, (9) becomes

$$\sum_{m,n,n>m}^{n_t} \Big| \sum_{o=1}^{n_r} h^*_{on} h_{om} \Big| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0. \tag{10}$$

For a given realization of $\mathbf{H}$, the receiver needs to compute θ_m's to satisfy (10). Again, (10) can be rewritten as

$$\sum_{n=2}^{n_t} \sum_{m=1}^{n-1} \Big| \sum_{o=1}^{n_r} h^*_{on} h_{om} \Big| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0. \tag{11}$$

Let the inner summation of (11) with index m be equal to 0 for each n. For $n = 2$,

$$\cos(\theta_1 - \theta_2 + \alpha_{21}) = 0. \tag{12}$$

Let $\theta_1 = 0$. Hence, $\theta_2 = \alpha_{21} - \pi/2$. Next, for any $n > 2$,

$$\sum_{m=1}^{n-1} \Big| \sum_{o=1}^{n_r} h^*_{on} h_{om} \Big| \cos(\theta_m - \theta_n + \alpha_{nm}) = 0 \tag{13}$$

$$\begin{aligned} \Rightarrow \sum_{m=1}^{n-1} \Big| \sum_{o=1}^{n_r} h^*_{on} h_{om} \Big| (&\cos(\theta_m + \alpha_{nm}) \cos\theta_n \\ &+ \sin(\theta_m + \alpha_{nm}) \sin\theta_n) = 0. \end{aligned}$$

$$\therefore \theta_n = \tan^{-1}\left(\frac{-\sum_{m=1}^{n-1} |\sum_{o=1}^{n_r} h^*_{on} h_{om}| (\cos(\theta_m + \alpha_{nm})}{\sum_{m=1}^{n-1} |\sum_{o=1}^{n_r} h^*_{on} h_{om}| \sin(\theta_m + \alpha_{nm})} \right). \tag{14}$$

Using (14), θ_n can be recursively calculated, given the values of θ_1 to θ_{n-1}. From (5),(8),(9) and (14), after simplification, we have

$$d_{k,l}^2 = \Big(\sum_{n=1}^{n_t} \sum_{o=1}^{n_r} |h_{on}|^2 \Big) \left| \sum_{i=1}^{n_t} \Delta x_i \right|^2, \tag{15}$$

where

$$\Delta \mathbf{x} = [\ \Delta x_1\ \ \Delta x_2\ \ \cdots\ \ \Delta x_{n_t}\ ]^T. \tag{16}$$

A. Choice Of Constellation Sets

From (15), it is clear that full diversity is guaranteed if, for each codeword pair $\mathbf{x}_k, \mathbf{x}_l$, $k \neq l$ in the codebook,

$$\sum_{i=1}^{n_t} \Delta x_i \neq 0. \tag{17}$$

Let x_i, $\forall i = 1, \cdots, n_t$, take values from some set $C_i \subseteq a_i\mathbb{Z}[j] = \{a_i z : z \in \mathbb{Z}[j]\}$, where $\mathbb{Z}[j] = u + jv$, u and v are integers, i.e., regular QAM constellations scaled by a_i. Clearly then, Δx_i belongs to some other set $D_i \subseteq a_i\mathbb{Z}[j]$ for each i. It may be noted that $-\Delta x_i \in D_i$. Now, let $a_1 = 1$ and choose a_2 such that

$$D_1 \cap D_2 = \{0\}. \tag{18}$$

Again, define $D_1 + D_2 \stackrel{\triangle}{=} \{\Delta x_1 + \Delta x_2 \text{ s.t } \forall \Delta x_1 \in D_1, \Delta x_2 \in D_2\}$. Next, choose a_3 such that

$$D_3 \cap (D_1 + D_2) = \{0\}. \tag{19}$$

Proceeding in this way, a_i is chosen such that

$$D_i \cap (D_1 + \cdots + D_{i-1}) = \{0\}. \tag{20}$$

This implies that for such choice of a_i's and $\Delta x_i \in D_i$, (17) is satisfied.

A trivial choice of a_i's could be $\sqrt{p_i}$, where p_i's are distinct prime numbers in $\mathbb{Z}$. This shows that the solution set of a_i's satisfying (20) is non-empty. For these values of a_i's, (17) holds true for all codeword pairs in the codebook ensuring full diversity. Since the number of elements of C_i is finite, the cardinality of D_i is also finite. This enables us to exhaustively enumerate all the elements of D_i as functions of a_i. Next, we have to choose a_i's $\in \mathbb{C}$ such that (20) is satisfied $\forall i = 1, \cdots, n_t$. Thus, there exist scaled QAM constellations for which the proposed precoding scheme guarantees full diversity. A parallel argument can be made for PSK constellation to ensure full diversity.

Now, the d_{min}^2 parameter of a codebook determines its coding gain as given by the error probability expression at high SNR. Furthermore, d_{min}^2 is maximized if $min(\sum_{i=1}^{n_t} \Delta x_i)$ is maximized in (15). Hence, after scaling the constellation sets C_i's by appropriate a_i's, we propose to rotate/scale each C_i by angles (ϕ_i's) and scaling real numbers (b_i's), ensuring that the average constellation power never exceeds the total transmit power constraint and also the condition (20) holds true $\forall i = 1, \cdots, n_t$. The corresponding $\min(\sum_{i=1}^{n_t} \Delta x_i)$ is computed over all C_i's. The optimum rotation angle ($\phi_{i,opt}$) and scaling real number ($b_{i,opt}$) for each C_i are chosen by computer search for which $\min(\Sigma_{i=1}^{n_t} \Delta x_i)$ is maximum. This fixes the choice of constellation sets C_i's that guarantee full transmit diversity as well maximizes d_{min}^2 subject to transmit power constraint.

B. A Simplified Approach to Choose Constellation Sets

The treatment described in the previous subsection (Section III-A) is feasible for constellations of small cardinality. The computational complexity involved in choosing large-sized constellation sets makes the choice cumbersome. In this subsection, we illustrate a suboptimal mechanism to choose large constellations. Our choice of $\mathbf{F}$ in (3) ensures that the effective transmitted vector $\mathbf{Fx}$ becomes

$$\Big(\sum_{i=1}^{n_t} x_i \Big) [\ a_1\ \ a_2\ \ \cdots\ \ a_{n_t}\ ]^T. \tag{21}$$

It then implies that each of the n_t antennas effectively transmits the symbol $\sum_{i=1}^{n_t} x_i$. Now, for a given energy constraint, it is known that square QAM constellations (i.e., 4^f sized constellation sets, where f is a positive integer) achieve better d_{min}^2 over PAM and PSK constellations. Hence, we propose to choose constellation sets for each x_i in such a way that $\sum_{i=1}^{n_t} x_i$ takes values from QAM constellation. Such a choice guarantees full diversity since for each codeword $\mathbf{x}$ transmitted the corresponding effective symbol $\sum_{i=1}^{n_t} x_i$ maps to a unique point in some QAM constellation satisfying condition (17).

C. Constellation Sets Obtained for $n_t = 3, 4, 8$

Based on the treatment described in the above subsections (Sections III-A, III-B), we carried out a computer search to obtain full diversity achieving constellation sets for various n_t. The obtained constellation sets for $n_t = 3, 4, 8$ are given in Table-I.

IV. Capacity Achieved by the Proposed Scheme

In this section, we analyze the capacity of the proposed precoding scheme for the special case of $n_t \times 1$ system, and compare it with corresponding capacity in the full CSIT scenario. The capacity of a $n_t \times n_r$ MIMO channel [2] is

$$\log_2 \det \left(\mathbf{I}_{n_r \times n_r} + \frac{1}{\sigma^2} \mathbf{HKH}^H \right), \tag{22}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{K}$ denotes the $n_t \times n_t$ optimal capacity achieving input covariance matrix. With full CSIT and $n_r = 1$, the optimal strategy is to transmit only along the single eigen mode λ of the channel $\mathbf{H}$. Now, for a $n_t \times 1$ channel $\mathbf{H}$, $|\lambda|^2 = \|\mathbf{H}\|_F^2$. Hence, given the transmit power constraint P, the full CSIT capacity with water-filling

n_t	Modulation (1 bit/symbol)	Modulation (2 bits/symbol)
3	$x_1 \in \{\pm 1\}$	$x_1 \in \{\pm\sqrt{2}e^{j\frac{\pi}{4}}, \pm\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_2 \in \{\pm j\}$	$x_2 \in \{\pm 0.5\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.5\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_3 \in \{\pm 0.675e^{j\frac{\pi}{4}}\}$	$x_3 \in \{\pm 0.25\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.25\sqrt{2}e^{j3\frac{\pi}{4}}\}$
4	$x_1 \in \{\pm 1\}$	$x_1 \in \{\pm\sqrt{2}e^{j\frac{\pi}{4}}, \pm\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_2 \in \{\pm j\}$	$x_2 \in \{\pm 0.5\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.5\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_3 \in \{\pm 0.5\}$	$x_3 \in \{\pm 0.25\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.25\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_4 \in \{\pm 0.5j\}$	$x_4 \in \{\pm 0.125\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.125\sqrt{2}e^{j3\frac{\pi}{4}}\}$
8	$x_1 \in \{\pm 1\}$	$x_1 \in \{\pm\sqrt{2}e^{j\frac{\pi}{4}}, \pm\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_2 \in \{\pm j\}$	$x_2 \in \{\pm 0.5\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.5\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_3 \in \{\pm 0.5\}$	$x_3 \in \{\pm 0.25\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.25\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_4 \in \{\pm 0.5j\}$	$x_4 \in \{\pm 0.125\sqrt{2}e^{j\frac{\pi}{4}}, \pm 0.125\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_5 \in \{\pm 0.25\}$	$x_5 \in \{\pm .0625\sqrt{2}e^{j\frac{\pi}{4}}, \pm .0625\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_6 \in \{\pm 0.25j\}$	$x_6 \in \{\pm .03125\sqrt{2}e^{j\frac{\pi}{4}}, \pm .03125\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_7 \in \{\pm 0.125\}$	$x_7 \in \{\pm .015625\sqrt{2}e^{j\frac{\pi}{4}}, \pm .015625\sqrt{2}e^{j3\frac{\pi}{4}}\}$
	$x_8 \in \{\pm 0.125j\}$	$x_8 \in \{\pm .0078125\sqrt{2}e^{j\frac{\pi}{4}}, \pm .0078125\sqrt{2}e^{j3\frac{\pi}{4}}\}$

TABLE I

FULL DIVERSITY ACHIEVING CONSTELLATION SETS FOR $n_t = 3, 4, 8$ OBTAINED BY COMPUTER SEARCH.

is given by

$$\begin{aligned} C_{CSIT} &= \log_2\left(1 + \frac{|\lambda|^2 P}{\sigma^2}\right) \\ &= \log_2\left(1 + \frac{\|\mathbf{H}\|_F^2 P}{\sigma^2}\right). \end{aligned} \tag{23}$$

For the proposed precoding scheme, let us consider a covariance matrix of the transmitted vector $\mathbf{x}$ of the form

$$\mathbf{K} = \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & P_{n_t} \end{pmatrix},$$

where P_i is the power allocated in transmitting x_i. According to (2), the effective channel matrix can taken as $\mathbf{HF}$. Now, by Sylvester's determinant theorem

$$\begin{aligned} &\det\left(\mathbf{I}_{1\times 1} + \frac{1}{\sigma^2}\mathbf{HFKF}^H\mathbf{H}^H\right) \\ &= \det\left(\mathbf{I}_{n_t\times n_t} + \frac{1}{\sigma^2}\mathbf{F}^H\mathbf{H}^H\mathbf{HFK}\right). \end{aligned} \tag{24}$$

Using (8) and (9), (24) becomes

$$\det\begin{pmatrix} \frac{1}{\sigma^2}P_1\|\mathbf{H}\|_F^2 + 1 & \frac{1}{\sigma^2}P_2\|\mathbf{H}\|_F^2 & \cdots & \frac{1}{\sigma^2}P_{n_t}\|\mathbf{H}\|_F^2 \\ \frac{1}{\sigma^2}P_1\|\mathbf{H}\|_F^2 & \frac{1}{\sigma^2}P_2\|\mathbf{H}\|_F^2 + 1 & \cdots & \frac{1}{\sigma^2}P_{n_t}\|\mathbf{H}\|_F^2 \\ \vdots & & & \\ \frac{1}{\sigma^2}P_1\|\mathbf{H}\|_F^2 & \frac{1}{\sigma^2}P_2\|\mathbf{H}\|_F^2 & \cdots & \frac{1}{\sigma^2}P_{n_t}\|\mathbf{H}\|_F^2 + 1 \end{pmatrix} \tag{25}$$

Using elementary row and column operations for determinant, the above form can be simplified to

$$\det\begin{pmatrix} 1 & 0 & \cdots & -1 \\ 0 & 1 & \cdots & -1 \\ \vdots & & & \\ \frac{1}{\sigma^2}P_1\|\mathbf{H}\|_F^2 & \frac{1}{\sigma^2}P_2\|\mathbf{H}\|_F^2 & \cdots & \frac{1}{\sigma^2}P_{n_t}\|\mathbf{H}\|_F^2 + 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ \frac{1}{\sigma^2}P_1\|\mathbf{H}\|_F^2 & \frac{1}{\sigma^2}P_2\|\mathbf{H}\|_F^2 & \cdots & \frac{1}{\sigma^2}\Sigma_{i=1}^{n_t}P_i\|\mathbf{H}\|_F^2 + 1 \end{pmatrix}$$

$$= 1 + \frac{\|\mathbf{H}\|_F^2 P}{\sigma^2}. \tag{26}$$

So the capacity achieved by the proposed scheme in a MISO setting is

$$C_{Proposed} = \log_2\left(1 + \frac{\|\mathbf{H}\|_F^2 P}{\sigma^2}\right), \tag{27}$$

which is equal to C_{CSIT} in (23) that is achieved with full CSIT and water-filling.

V. RESULTS AND DISCUSSIONS

We evaluated the codeword error rate (CER) performance of the proposed precoding scheme for the cases of 3×1, 4×1 and 8×1 full-rate MIMO systems as a function of the average received SNR through simulations. ML decoding is used. The results obtained are shown in Figs. 1, 2, and 3, respectively. Plots of two performance measures, namely, $i)$ CER and $ii)$ pdf of the normalized d_{min}^2, are shown in two separate sub-figures in each of Figs. 1, 2, 3. For comparison purposes, we have plotted the CER performance and d_{min}^2 pdf of the full-rate precoding scheme in [14]. In Fig. 1, CER plots for the rates of 3 and 6 bits/channel use are shown. For 3 bits/cu, the constellation points used in the simulation for the proposed scheme are as given in the entries of $n_t = 3$ and 1 bit/symbol modulation in Table-I. Likewise, for 6 bits/cu, the constellation points used are from the $n_t = 3$ and 2 bits/symbol modulation in Table-I. Appropriate constellations from Table-I are used for the plots of different rates shown in Figs. 2 and 3. Since constellation optimization is not done in the precoding scheme in [14], we have used regular

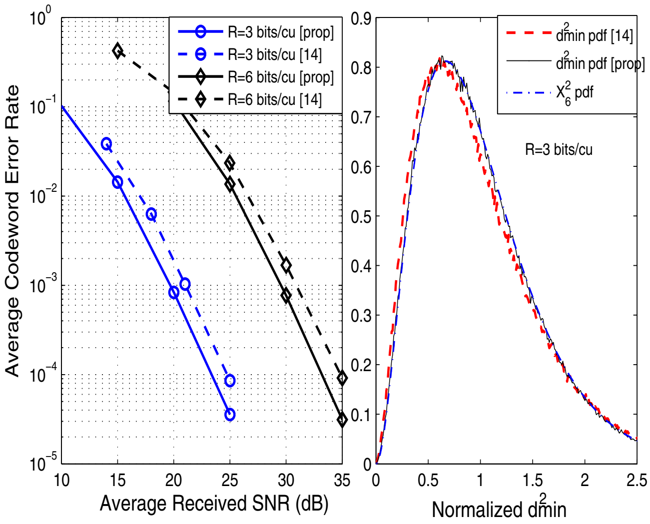

Fig. 1. Comparison of *a*) CER performance and *b*) pdf of the normalized d^2_{min} of the proposed precoder with that in [14] for 3×1 MIMO system.

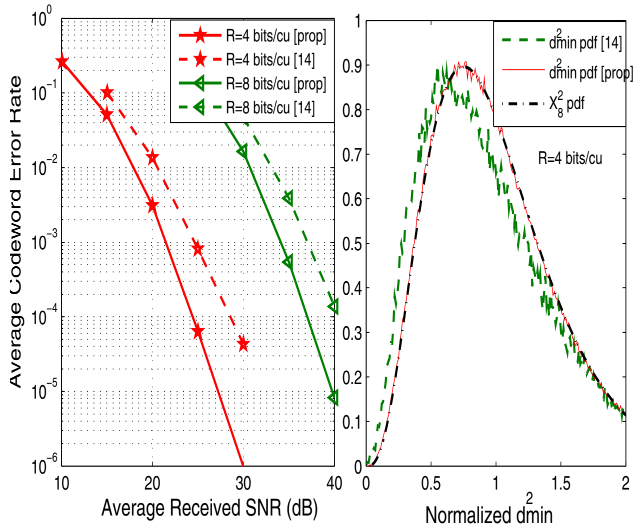

Fig. 2. Comparison of *a*) CER performance and *b*) pdf of the normalized d^2_{min} of the proposed precoder with that in [14] for 4×1 MIMO system.

modulation alphabets (e.g., BPSK, 4-QAM) and matched the bits/cu in both schemes for fair comparison. For example, 6 bits/cu plot in Fig. 1 for the scheme in [14] is simulated using 4-QAM.

From the CER plots of in Figs. 1 to 3, we can see that the proposed precoding achieves better diversity orders compared to the precoding scheme in [14]. In fact, the proposed scheme achieves the full diversity of $n_t n_r = 3, 4, 8$ in Figs. 1, 2 and 3, respectively. This can be verified by observing that pdfs of the d^2_{min} in the proposed scheme match with the theoretical $\chi^2_{2n_t n_r}$ pdf (Chi square distribution with $2n_t n_r$ degrees of freedom). For example, in Fig. 2, the proposed scheme's pdf matches with that of the χ^2_8 pdf, verifying the achievability of $n_t n_r = 4$th order diversity. Similar matches with Chi square distribution are observed in other figures as well. Thus, the analytical claim of full diversity of $n_t n_r$ in the proposed precoding scheme is validated through simulations as well. It is further noted that the pdfs in the scheme in [14] do not match with those of the theoretical $\chi^2_{2n_t n_r}$ pdfs, indicating that the scheme in [14] does not achieve full diversity. Finally, Fig. 4 shows the performance of the proposed scheme in a 3×2 MIMO system with 3 and 6 bits/cu, where the full $n_t n_r = 6$th order diversity is found to be achieved.

VI. Conclusion

We presented a *partial CSIT* based precoding scheme which achieved both *full diversity* ($n_t n_r$th diversity) as well as *full rate* (n_t symbols per channel use) with a feedback requirement of only $n_t - 1$ real angular parameters. The full diversity was achieved by choosing optimized constellation sets. Through analysis and simulations we established the full diversity achievability in the proposed scheme.

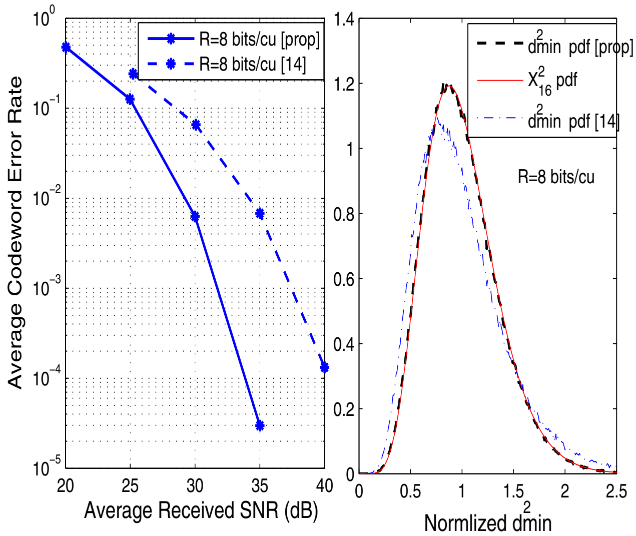

Fig. 3. Comparison of *a*) CER performance and *b*) pdf of the normalized d^2_{min} of the proposed precoder with that in [14] for 8×1 MIMO system.

References

[1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2003.

[2] H. Bolcskei, D. Gesbert, C. B. Papadias, and Alle-Jan van der Veen, Ed., *Space-Time Wireless Systems: From Array Processing to MIMO Communications,* Cambridge University Press, 2006.

[3] D. J. Love and R. W. Heath, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2735-2747, October 2003.

[4] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple antenna systems," *IEEE Trans. Inf. Theory,* vol. 49, no. 10, pp. 2562-2579, October 2003.

[5] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beam-forming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.,* vol. 16, no. 8, pp. 1437-1450, October 1998.

[6] V. K. N. Lau, Y. Liu, and T. A. Chen, "On the design of MIMO block fading channels with feedback-link capacity constraint," *IEEE Trans. Commun.,* vol. 52, no. 1, pp. 62-70, January 2004.

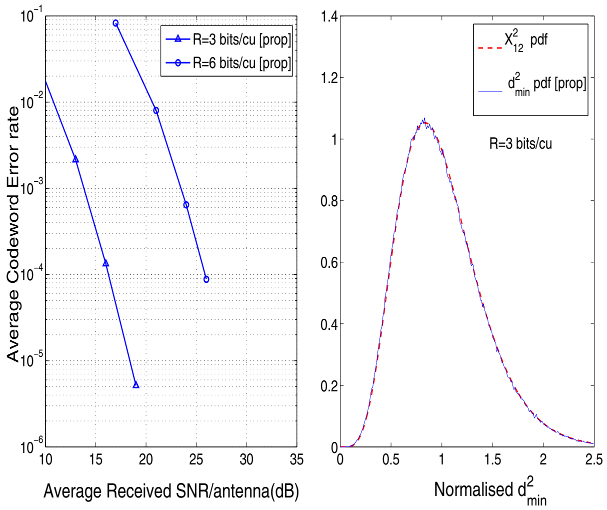

Fig. 4. CER performance and pdf of the normalized d^2_{min} of the proposed precoder in a 3×2 MIMO system.

[7] J. C Roh and B. D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: A VQ-based approach," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 1101-1112, March 2006.

[8] H. Sampath and A. Paulraj, "Linear precoding for space-time coded systems with known fading correlations," *IEEE Commun. Lett.*, vol. 6, no. 6, pp. 239-241, June 2002.

[9] L. Collin, O. Berder, P. Rostaing, and G. Burel, "Optimal minimum distance-based precoder for MIMO spatial multiplexing systems," *IEEE Trans. Signal Process.,* vol. 52, no. 3, pp. 617627, Mar. 2004.

[10] D. J. Love and R. W. Heath, Jr., "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. on Inform. Theory*, vol. 51, No. 8, 2967-2976, 2005.

[11] C. Simon and G. Leus, "Feedback quantization for linear precoded spatial multiplexing," *EURASIP Jl. on Advances in Sig. Proc.*, 2008, Article ID 683030.

[12] R. W. Heath, Jr., S. Sandhu, and A. Paulraj, "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Commun. Lett.*, vol. 5, no. 4, pp. 142-144, April 2001.

[13] K. V. Srinivas, K. Giridhar, and R. D. Koilpillai, "A new diagonally layered spatial multiplexing scheme with partial channel knowledge," *Proc. IEEE GLOBECOM'2008,* December 2008.

[14] S. Barik, S. K. Mohammed, A. Chockalingam, and B. S. Rajan, "Full-rate precoding in V-BLAST with angle parameter feedback," *Proc. IEEE PIMRC'09*, September 2009.

[15] B. Vrigneau, J. Letessier, P. Rostaing, L. Collin, and G. Burel, "Extension of the MIMO precoder based on the minimum Euclidean distance: A cross-form matrix," *IEEE Jl. Sel. Topics in Signal Processing*, vol. 2, no. 2, pp. 135-146, April 2008.

[16] Saif K. Mohammed, E. Viterbo, Y. Hong, and A. Chockalingam, "MIMO precoding with X- and Y-Codes," to appear *IEEE Trans. Inf. Theory*. Also in *Proc. IEEE ISIT'2010*, Austin, June 2010.