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Improved Linear Parallel Interference Cancellers

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Abstract— **In this paper, taking the view that a linear parallel interference canceller (LPIC) can be seen as a linear matrix filter, we propose new linear matrix filters that can result in improved bit error performance compared to other LPICs in the literature. The motivation for the proposed filters arises from the possibility of avoiding the generation of certain interference and noise terms in a given stage that would have been present in a conventional LPIC (CLPIC). In the proposed filters, we achieve such avoidance of the generation of interference and noise terms in a given stage by simply making the diagonal elements of a certain matrix in that stage equal to zero. Hence, the proposed filters do not require additional complexity compared to the CLPIC, and they can allow achieving a certain error performance using fewer LPIC stages.**

Keywords – *Multiuser detection, linear parallel interference cancellation, linear matrix filters, decorrelating detector, MMSE detector.*

I. Introduction

Linear parallel interference cancellers (LPIC) have the advantages of implementation simplicity, analytical tractability, and good performance [1]-[10]. The conventional way to realize LPIC schemes is to use unscaled values of the previous stage soft outputs of different users for multiple access interference (MAI) estimation. In [3], Guo *et al* described and analyzed LPIC schemes for CDMA using a matrix-algebraic approach. They pointed out that an LPIC can be viewed as a *linear matrix filter* applied directly to the chip matched filter (MF) output vector. While the matrix filter corresponding to the conventional LPIC (CLPIC) converges to the decorrelating (DC) detector, they also proposed a modified matrix filter which converges to a minimum mean square detector (MMSE) detector. This was done by exploiting the equivalence of the LPIC to a steepest descent optimization method for minimizing the mean square error. For this optimization, they obtained optimum step sizes for different stages that remove the excess mean square error in K stages (where K is the number of users), leaving only the MSE in stages greater than K. The condition for this convergence has been shown to be that the maximum eigenvalue of the correlation matrix must be less than two.

Our contribution in this paper is that we propose new linear matrix filters that can perform better than the matrix filters studied in [3]. The motivation for the proposed filters arises from the possibility of avoiding the generation of certain interference and noise terms in a given stage that would have been present in the CLPIC. In the proposed filters, we achieve such avoidance of the generation of interference and noise terms in a given stage by simply making the diagonal elements of a certain matrix in that stage equal to zero. Hence, the proposed filters do not require additional complexity compared to the CLPIC. We show that the proposed matrix filters can achieve better performance compared to other filters in the literature. This, in turn, can allow achieving a certain error performance using fewer LPIC stages. We also propose filters that use different step sizes for different stages (but the same step size for all users at a given stage). In addition, we propose filters that use different weights for different users in different stages, where we also obtain closed-form expressions for the optimum weights that maximize the output-average SINR in a given stage.

The rest of the paper is organized as follows. In Sec. II, we present the system model. The proposed filters are presented in Sec. III. Performance results and comparisons are presented in Sec. IV. Conclusions are presented in Sec. V.

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II. System Model

We consider a K-user synchronous single-carrier CDMA system. The received signal is given by

$$y(t) = \sum_{k=1}^{K} A_k h_k b_k s_k(t) + n(t), \qquad t \in [0, T], \qquad (1)$$

where $b_k \in \{+1, -1\}$ is the bit transmitted by the kth user, T_b is one bit duration, A_k is the transmit amplitude of the kth user's signal, h_k is the complex channel fade coefficient corresponding to the kth user, $s_k(t)$ is a unit energy spreading waveform of the kth user defined in the interval $[0, T_b]$, i.e., $\int_0^{T_b} s_k^2(t)dt = 1$, and $n(t)$ is the AWGN with zero mean and variance σ^2. The fade coefficients h_k's are assumed to be i.i.d complex Gaussian r.v's with zero mean and variance 0.5 per dimension. The channel fades are assumed to remain constant over one bit interval.

Consider a multistage LPIC at the receiver. The first stage is a conventional matched filter (MF), which is a bank of K correlators, each matched to a different user's spreading waveform. The received vector $\mathbf{y}^{(1)}$ at the output of the MF stage (the superscript (1) in $\mathbf{y}^{(1)}$ denotes the first stage) is given by

$$\mathbf{y}^{(1)} = \left[y_1^{(1)}, y_2^{(1)}, \cdots, y_K^{(1)}\right]^T = \mathbf{R}\mathbf{x} + \mathbf{n}, \qquad (2)$$

where $[.]^T$ denotes transpose operation, and $y_k^{(1)}$ is the kth user's MF output, given by

$$y_k^{(1)} = \underbrace{A_k h_k b_k}_{\text{desired signal}} + \underbrace{\sum_{j=1,\, j\neq k}^{K} \rho_{jk} A_j h_j b_j}_{\text{MAI}} + \underbrace{n_k}_{\text{noise}}. \qquad (3)$$

The vectors $\mathbf{x}$, $\mathbf{n}$ and correlation matrix $\mathbf{R}$ are given by

$$\mathbf{x} = [A_1 h_1 b_1,\ A_2 h_2 b_2,\ \cdots,\ A_K h_K b_K]^T, \qquad (4)$$

$$\mathbf{n} = [n_1, n_2, \cdots, n_K]^T, \qquad (5)$$

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$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{bmatrix}, \quad (6)$$

where $\rho_{jk} = \rho_{kj}$ is the normalized cross-correlation coefficient between the spreading waveforms of users j and k, given by $\rho_{jk} = \int_0^{T_b} s_j(t)s_k(t)dt$, $|\rho_{jk}| \leq 1$, and n_k's are complex Gaussian with zero mean and $E[n_j n_k^*] = \sigma^2$ when $j = k$ and $E[n_j n_k^*] = \sigma^2 \rho_{jk}$ when $j \neq k$. Throughout the paper, we denote vectors by boldface lowercase letters, matrices by boldface uppercase letters. $[.]^T$ and $[.]^H$ denote transpose and conjugate transpose operations, respectively.

A. Conventional LPIC

In conventional LPIC (CLPIC), an estimate of the MAI for the desired user in the current stage is obtained using all the other users' soft outputs from the previous stage for cancellation in the current stage. The mth stage output of the desired user k, $y_k^{(m)}$, in CLPIC is given by [9]

$$y_k^{(m)} = y_k^{(1)} - \underbrace{\sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(m-1)}}_{\text{MAI estimate}}. \quad (7)$$

The kth user's bit decision after MAI cancellation in the mth stage, $\widehat{b}_k^{(m)}$, is obtained as

$$\widehat{b}_k^{(m)} = \text{sgn}\left(\text{Re}\left(h_k^* y_k^{(m)}\right)\right). \quad (8)$$

The CLPIC output in (7) can be written in matrix form as [3]

$$\mathbf{y}^{(m)} = \left[\mathbf{I} + (\mathbf{I}-\mathbf{R}) + (\mathbf{I}-\mathbf{R})^2 + \cdots + (\mathbf{I}-\mathbf{R})^{m-1}\right]\mathbf{y}^{(1)} \quad (9)$$

$$= \underbrace{\sum_{j=1}^{m} (\mathbf{I}-\mathbf{R})^{j-1}}_{\mathbf{G}^{(m)}} \ \mathbf{y}^{(1)}, \quad (10)$$

where $\mathbf{y}^{(m)} = \left[y_1^{(m)}, y_2^{(m)}, \cdots, y_K^{(m)}\right]^T$. The $\mathbf{G}^{(m)}$ filter in (10) can be viewed as an equivalent one-shot linear matrix filter for the mth stage of the CLPIC.

III. Proposed Linear Matrix Filters

In this section, we propose new linear matrix filters that can outperform the matrix filter $\mathbf{G}^{(m)}$ in (10).

A. Proposed Matrix Filter ${\mathbf{G_p}}^{(m)}$

We first propose a new linear matrix filter, denoted by ${\mathbf{G_p}}^{(m)}$. The motivation for the ${\mathbf{G_p}}^{(m)}$ filter is explained as follows.

What does the matrix filter $\mathbf{G}^{(m)}$ do?: It is noted that the behavior of the $\mathbf{G}^{(m)}$ filter in (10) (i.e., CLPIC) at a given stage $m \geq 2$ is characterized by a) interference removal, b) generation of new interference terms, c) desired signal loss/gain, d) desired signal recovery/removal, and e) noise enhancement. For example, the cancellation operation in the 2nd stage (i.e., $m = 2$) results in i) interference removal, ii) generation of new interference terms, iii) desired signal loss, and iv) noise enhancement. This can be seen by observing the 2nd stage output expression for the desired user k, which can be written, using (7) and (3), as

$$y_k^{(2)} = y_k^{(1)} - \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(1)}$$

$$= \left(x_k + \sum_{i=1, i\neq k}^{K} \rho_{ki} x_i + n_k\right) - \sum_{j=1, j\neq k}^{K} \rho_{jk}\left(x_j + \underbrace{\sum_{l=1, l\neq j}^{K}}_{l \text{ can be } k \text{ here}} \rho_{jl} x_l + n_j\right)$$

$$= x_k - \underbrace{x_k \sum_{j=1, j\neq k}^{K} \rho_{jk}^2}_{\text{desired signal loss}} - \underbrace{\sum_{j=1, j\neq k}^{K} \rho_{jk} \sum_{l=1, l\neq j,k}^{K} \rho_{jl} x_l}_{\text{new interference terms}} + n_k - \underbrace{\sum_{j=1, j\neq k}^{K} \rho_{jk} n_j}_{\text{additional noise terms}}, \quad (11)$$

where $x_k = A_k h_k b_k$, $k = 1, 2, \cdots, K$. Comparing the expression at the MF output, $y_k^{(1)}$, in (3) and the expression for the 2nd stage output, $y_k^{(2)}$, in (11), it can be seen that the cancellation operation in the 2nd stage results in the following at the 2nd stage output.

- The interference terms, $\sum_{j\neq k} \rho_{jk} x_j$, that were present in the MF output in (3) are removed. In the process, i) new interference terms proportional to ρ^2, i.e., $\sum_{j\neq k} \rho_{jk} \sum_{l\neq j,k} \rho_{jl} x_l$ in (11), get generated, ii) a fraction $\sum_{j\neq k} \rho_{jk}^2$ of the desired signal component gets lost, and iii) additional noise terms proportional to ρ, i.e., $\sum_{j\neq k} \rho_{jk} n_j$ in (11), get introduced.

In Appendix A, we present the expression for the 3rd stage output in an expanded form. From (35) in Appendix A, we can make the following observations which result from the cancellation operation in the 3rd stage.

- The desired signal loss that occurred in the 2nd stage is recovered (see the two $\boxed{A}$ terms cancelling each other in (35)). In the process, new interference terms proportional to ρ^3 (see the $\boxed{B_I}$ term in (35)) as well as additional noise terms proportional to ρ^2 (see the $\boxed{B_N}$ term in (35)) get generated.
- Interference terms generated in the 2nd stage are removed (see the two $\boxed{C}$ terms cancelling each other in (35)). In the process, i) further desired signal loss/gain[1] proportional to ρ^3 occurs (see the $\boxed{D}$ term in (35)), and ii), new interference terms proportional to ρ^3 (see the $\boxed{E_I}$ term in (35)) as well as additional noise terms proportional to ρ^2 (see the $\boxed{E_N}$ term in (35)) get generated.

Similar observations can be made on the expanded form of the equations for the subsequent stages of the CLPIC[2]. For $m \rightarrow \infty$, the CLPIC is known to converge to the decorrelating detector, provided the eigenvalues of the $\mathbf{R}$ matrix are less than two [3].

What is proposed to be achieved using the ${\mathbf{G_p}}^{(m)}$ filter?: As explained above, in the $\mathbf{G}^{(m)}$ filter, new interference and

[1] Depending on ρ's being +ve or -ve, the term $\boxed{D}$ in (35) can be +ve or -ve, because of which there can be a desired signal gain or loss.

[2] The general expression for the mth stage output in expanded form, for any $m \geq 3$, and the corresponding observations, are given in [11].

noise terms get generated in the process of interference removal and recovery/removal of desired signal loss/gain. We seek to avoid the generation of some of these new interference and noise terms. For example, as will be explained regarding the proposed ${\mathbf{G_p}}^{(m)}$ filter in the following subsection, the generation of the $\boxed{B_I}$ and $\boxed{B_N}$ terms at the 3rd stage output in (35) can be avoided by simply making the diagonal elements of a certain matrix in the cancellation operation carried out in the 3rd stage equal to zero. This, as we will see later, can result in improved error performance compared to the $\mathbf{G}^{(m)}$ filter.

Proposed ${\mathbf{G_p}}^{(m)}$ *filter:* We propose to avoid the generation of new interference and noise terms in T_3 in Eqn. (5.4) of [11], caused in the process of recovery/removal of desired signal loss/gain in the previous stage. Since there is no desired signal loss/gain in the 1st stage, the 2nd stage of the proposed filter is the same as that of the $\mathbf{G}^{(m)}$ filter, i.e., ${\mathbf{G_p}}^{(2)} = \mathbf{G}^{(2)}$. For stages greater than two, i.e., for $m \geq 3$, the mth stage output of the proposed filter ${\mathbf{G_p}}^{(m)}$, denoted by $y_{k,\mathbf{p}}^{(m)}$, is written as

$$y_{k,\mathbf{p}}^{(m)} = y_{k,\mathbf{p}}^{(m-1)} + (-1)^{m+1} \sum_{k_1 \neq k}^{K} \sum_{k_2 \neq k, k_1}^{K} \sum_{k_3 \neq k, k_2}^{K} \cdots \sum_{k_{m-2} \neq k, k_{m-3}}^{K} \sum_{k_{m-1} \neq k, k_{m-2}}^{K} \rho_{kk_{m-1}} \rho_{k_{m-1}k_{m-2}} \cdots \rho_{k_2 k_1} \, y_{k_1}^{(1)}. \quad (12)$$

We note that the above expression is obtained by i) dropping T_3 and ii) modifying T_4 in Eqn. (5.4) in [11] such that all the summations in it exclude the desired user index k. The above two modifications ensure that the proposed filter removes the previous stage interference while avoiding the recovery/removal of the desired signal loss/gain[3]. Also, because of these modifications, the interference and noise terms in a given stage of the proposed filter will be a subset of the interference and noise terms in the same stage of the $\mathbf{G}^{(m)}$ filter. Equation (12) can be written in the following form:

$$\begin{aligned} y_{k,\mathbf{p}}^{(m)} = \; & y_k^{(1)} - \sum_{k_1 \neq k}^{K} \Bigg(\rho_{kk_1} - \sum_{k_2 \neq k, k_1}^{K} \rho_{kk_2}\rho_{k_2k_1} \\ & + \sum_{k_2 \neq k, k_1}^{K} \sum_{k_3 \neq k, k_2}^{K} \rho_{kk_3}\rho_{k_3k_2}\rho_{k_2k_1} - \cdots \\ & + (-1)^m \sum_{k_2 \neq k, k_1}^{K} \sum_{k_3 \neq k, k_2}^{K} \cdots \sum_{k_{m-2} \neq k, k_{m-3}}^{K} \sum_{k_{m-1} \neq k, k_{m-2}}^{K} \\ & \rho_{kk_{m-1}}\rho_{k_{m-1}k_{m-2}} \cdots \rho_{k_3k_2}\rho_{k_2k_1} \Bigg) y_{k_1}^{(1)}, \end{aligned} \quad (13)$$

which, in turn, can be expressed in matrix form as

$$\mathbf{y}_{\mathbf{p}}^{(m)} = \underbrace{\left(\sum_{j=0}^{m-1} \mathbf{B}_j \right)}_{{\mathbf{G_p}}^{(m)}} \mathbf{y}^{(1)}, \quad (14)$$

where
$$\mathbf{B}_n = \Big[\mathbf{B}_{n-1} (\mathbf{I} - \mathbf{R}) \Big]^{\odot}, \quad (15)$$

$[\mathbf{M}]^{\odot}$ denotes the matrix $\mathbf{M}$ with its diagonal elements made equal to zero, and $\mathbf{A}_0 = \mathbf{I}$. Note that, since (14) is structurally the same as (9), and the $[.]^{\odot}$ operation in (15) does not add to complexity, the proposed ${\mathbf{G_p}}^{(m)}$ filter has the same complexity as the $\mathbf{G}^{(m)}$ filter.

[3] Although possible signal loss recovery is avoided in the process, the net effect can still be beneficial (we will see this in Sec. IV).

The $\mathbf{G}^{(m)}$ filter is known to converge to the decorrelating detector for $m \to \infty$, provided the maximum eigenvalue of the $\mathbf{R}$ matrix is less than two [3]. That is, $\mathbf{G}^{(\infty)} = \mathbf{R}^{-1}$, which results in the output vector

$$\Big(\mathbf{y}^{(\infty)}\Big)_{\mathbf{G}} = \mathbf{R}^{-1}\mathbf{y}^{(1)} = \mathbf{x} + \mathbf{R}^{-1}\mathbf{n}. \quad (16)$$

As with $\mathbf{G}^{(m)}$, all the interference terms in ${\mathbf{G_p}}^{(m)}$ also go to zero for $m \to \infty$. This can be shown as follows. From (14) and (15), ${\mathbf{G_p}}^{(\infty)}$ can be written in the form

$$\begin{aligned} {\mathbf{G_p}}^{(\infty)} = \; & \underbrace{\mathbf{I}}_{\mathbf{B}_0} + \underbrace{[(\mathbf{I}-\mathbf{R}) - \mathbf{D}_1]}_{\mathbf{B}_1} \\ & + \underbrace{\{[(\mathbf{I}-\mathbf{R}) - \mathbf{D}_1](\mathbf{I}-\mathbf{R}) - \mathbf{D}_2\}}_{\mathbf{B}_2} + \cdots \end{aligned} \quad (17)$$

where $\mathbf{D}_n$ is a diagonal matrix with the diagonal elements the same as those in the matrix $\mathbf{B}_{n-1}(\mathbf{I}-\mathbf{R})$. Equation (17) can be written as

$$\begin{aligned} {\mathbf{G_p}}^{(\infty)} = \; & \underbrace{\left(\mathbf{I} + (\mathbf{I}-\mathbf{R}) + (\mathbf{I}-\mathbf{R})^2 + \cdots\right)}_{\mathbf{R}^{-1}} \\ & - \mathbf{D}_1 \left(\mathbf{I} + (\mathbf{I}-\mathbf{R}) + (\mathbf{I}-\mathbf{R})^2 + \cdots\right) \\ & - \mathbf{D}_2 \left(\mathbf{I} + (\mathbf{I}-\mathbf{R}) + (\mathbf{I}-\mathbf{R})^2 + \cdots\right) - \cdots \\ = \; & \underbrace{\left(\mathbf{I} - \mathbf{D}_1 - \mathbf{D}_2 - \cdots\right)}_{\triangleq \mathbf{F}} \mathbf{R}^{-1}. \end{aligned} \quad (18)$$

Hence, the output vector for $m \to \infty$ is given by

$$\Big(\mathbf{y}^{(\infty)}\Big)_{\mathbf{G_p}} = \mathbf{F}\mathbf{R}^{-1}\mathbf{y}^{(1)} = \mathbf{F}\mathbf{x} + \mathbf{F}\mathbf{R}^{-1}\mathbf{n}. \quad (19)$$

The diagonal matrix $\mathbf{F}$ defined in (18) can be written as

$$\mathbf{F} = diag\Big(f_1, f_2, \cdots, f_K\Big), \quad (20)$$

where f_k is given by

$$\begin{aligned} f_k = \; & 1 - \sum_{k_1 \neq k}^{K} \rho_{kk_1}\rho_{k_1k} + \sum_{k_1 \neq k}^{K} \sum_{k_2 \neq k, k_1}^{K} \rho_{kk_2}\rho_{k_2k_1}\rho_{k_1k} \\ & - \sum_{k_1 \neq k}^{K} \sum_{k_2 \neq k, k_1}^{K} \sum_{k_3 \neq k, k_2}^{K} \rho_{kk_3}\rho_{k_3k_2}\rho_{k_2k_1}\rho_{k_1k} \cdots \end{aligned} \quad (21)$$

For the case of equi-correlated users, f_k in (21) can be shown to converge to $1 - \big((K-1)\rho^2/(1+(K-2)\rho\big)$, and there are no interference terms in (19). Also, note that the outputs of the $\mathbf{G}$ filter in (16) and the $\mathbf{G_p}$ filter in (19) have the same SNR for $m \to \infty$.

B. A Modified MMSE Converging Filter, ${\mathbf{G_{p\mu}}}^{(m)}$

As pointed out in Sec. I, Guo *et al*, in [3], have proposed modifications to the $\mathbf{G}^{(m)}$ filter so that the resulting modified matrix filter converges to the MMSE detector instead of the decorrelating detector, by exploiting the equivalence of the LPIC to the steepest descent method (SDM) of optimization for minimizing the MSE. They also derived optimum step sizes for various stages, which ensured convergence to the MMSE detector in K stages, where K is the number of users. We refer to this MMSE converging matrix filter proposed by Guo *et al* in [3] as the $\mathbf{G}_{\mu}{}^{(m)}$ filter, which is given by [3]

$$\mathbf{y}^{(m)} = \underbrace{\left(\mu_m \mathbf{I} + \sum_{i=1}^{m-1} \mu_{m-i} \prod_{j=1}^{i} (\mathbf{I} - \mu_{m-i+j}(\mathbf{R} + \sigma^2 \mathbf{I}))\right)}_{\mathbf{G}_{\mu}{}^{(m)}} \mathbf{y}^{(1)}, \quad (22)$$

where μ_i is the step size at stage i, the optimum values of which were obtained to be

$$\mu_i = \frac{1}{\lambda_i + \sigma^2}, \quad i = 1, 2, \cdots, K, \quad (23)$$

where $\lambda_i,\ i = 1, 2, \cdots, K$ are eigenvalues of matrix $\mathbf{R}$.

We note that a similar SDM view can be taken to modify our proposed matrix filter $\mathbf{G}_{\mathbf{p}}{}^{(m)}$ so that it can converge to the MMSE detector. We refer to such a modified version of our proposed filter as $\mathbf{G}_{\mathbf{p}\mu}{}^{(m)}$ filter, where we avoid the generation of new interference and noise terms as in $\mathbf{G}_{\mathbf{p}}{}^{(m)}$, while using the step sizes obtained for $\mathbf{G}_{\mu}{}^{(m)}$ in [3]. Accordingly, the modified version of our proposed filter can be written as

$$\mathbf{y}^{(m)} = \underbrace{\left(\mu_m \mathbf{I} + \sum_{i=1}^{m-1} \mu_{m-i} \mathbf{J}_i\right)}_{\mathbf{G}_{\mathbf{p}\mu}{}^{(m)}} \mathbf{y}^{(1)}, \quad (24)$$

where $\mathbf{J}_i$ is given by

$$\mathbf{J}_i = \left[\mathbf{J}_{i-1}\left(\mathbf{I} - \mu_{K-i+1}(\mathbf{R} + \sigma^2 \mathbf{I})\right)\right]^{\odot}, \quad \mathbf{J}_0 = \mathbf{I}. \quad (25)$$

C. A Weighted Matrix Filter, $\mathbf{G}_{\mathbf{pw}}{}^{(m)}$

In the $\mathbf{G}_{\mu}{}^{(m)}$ and $\mathbf{G}_{\mathbf{p}\mu}{}^{(m)}$ filters above, different step sizes are used in different stages (but the same step size for all users in a stage). Improved performance can be achieved if different scaling factors (weights) are used for different users in different stages. Accordingly, we propose a weighted version of our proposed filter $\mathbf{G}_{\mathbf{p}}{}^{(m)}$. We refer to this weighted version as $\mathbf{G}_{\mathbf{pw}}{}^{(m)}$, which is derived as follows.

In a weighted LPIC (WLPIC), the MAI estimate in a given stage is scaled by a weight before cancellation (unit weight corresponds to CLPIC and zero weight corresponds to MF). For example, the mth stage output of the desired user k, $y_{k,\mathbf{w}}^{(m)}$, in a WLPIC is given by

$$y_{k,\mathbf{w}}^{(m)} = y_k^{(1)} - w_k^{(m)} \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_{j,\mathbf{w}}^{(m-1)}, \quad (26)$$

where $w_k^{(m)}$ is the weight with which the MAI estimate for the kth user in the mth stage is scaled. The above weighted cancellation operation in (26) can be written in matrix form, for $m \geq 2$, as

$$\mathbf{y}^{(m)} = \Big(\mathbf{I} + \mathbf{W}^{(m)}(\mathbf{I}-\mathbf{R}) + \mathbf{W}^{(m)}(\mathbf{I}-\mathbf{R})\mathbf{W}^{(m-1)}(\mathbf{I}-\mathbf{R}) + \cdots + \mathbf{W}^{(m)}(\mathbf{I}-\mathbf{R})\mathbf{W}^{(m-1)}(\mathbf{I}-\mathbf{R})\cdots\mathbf{W}^{(2)}(\mathbf{I}-\mathbf{R})\Big)\mathbf{y}^{(1)}, \quad (27)$$

where $\mathbf{W}^{(m)}$ is the weight matrix at the mth stage, given by $\mathbf{W}^{(m)} = \text{diag}\left(w_1^{(m)}, w_2^{(m)}, \cdots, w_K^{(m)}\right)$, and $\mathbf{W}^{(1)} = \mathbf{0}$.

Now, as in $\mathbf{G}^{(m)}$, in order to avoid the generation of new interference and noise terms, we modify (27) as follows:

$$\mathbf{y}^{(m)} = \underbrace{\left(\sum_{j=0}^{m-1} \tilde{\mathbf{B}}_j\right)}_{\mathbf{G}_{\mathbf{pw}}{}^{(m)}} \mathbf{y}^{(1)}, \quad (28)$$

where

$$\tilde{\mathbf{B}}_n = \left[(\tilde{\mathbf{B}}_{n-1})(\mathbf{W}^{(m-1+n)})(\mathbf{I}-\mathbf{R})\right]^{\odot}, \quad \tilde{\mathbf{B}}_0 = \mathbf{I}. \quad (29)$$

Note that $\mathbf{G}_{\mathbf{pw}}{}^{(m)}$ becomes $\mathbf{G}_{\mathbf{p}}{}^{(m)}$ when $\mathbf{W}^{(m)} = \mathbf{I}, \forall\, m > 1$.

D. Optimum Weight Matrix $\mathbf{W}_{opt}^{(m)}$

The mth stage output of the kth user when $\mathbf{G}_{\mathbf{pw}}{}^{(m)}$ filter is used can be written as

$$\begin{aligned}\left(y_k^{(m)}\right)_{\mathbf{G}_{\mathbf{pw}}} &= y_k^{(1)} - w_k^{(m)} \sum_{i\neq k}^{K} r_{k,i}^{(m)} y_i^{(1)} \\ &= \underbrace{x_k\left(1 - w_k^{(m)} \sum_{i\neq k}^{K} r_{k,i}^{(m)} \rho_{ki}\right)}_{\text{desired signal}} + \underbrace{n_k - w_k^{(m)} \sum_{i\neq k}^{K} r_{k,i}^{(m)} n_i}_{\text{noise}} \\ &\quad + \underbrace{\sum_{i\neq k}^{K} \left(\rho_{ki} - w_k^{(m)}\left(r_{k,i}^{(m)} + \sum_{k_1\neq i,k}^{K} r_{k,k_1}^{(m)} \rho_{k_1 i}\right)\right) x_i}_{\text{interference}}, \quad (30)\end{aligned}$$

where

$$\begin{aligned} r_{k,i}^{(m)} = \Bigg(&\rho_{ki} - \sum_{k_1\neq k,i}^{K} w_{k_1}^{(m-1)} \rho_{kk_1}\rho_{k_1 i} + \sum_{k_1\neq k,i}^{K} w_{k_1}^{(m-2)} \sum_{k_2\neq k,k_1}^{K} \\ & w_{k_2}^{(m-1)} \rho_{kk_2}\rho_{k_2k_1}\rho_{k_1 i} - \cdots + (-1)^m \sum_{k_1\neq k,i}^{K} w_{k_1}^{(2)} \sum_{k_2\neq k,k_1}^{K} w_{k_2}^{(3)} \\ & \cdots \sum_{k_{m-3}\neq k,k_{m-4}}^{K} w_{k_{m-3}}^{(m-2)} \sum_{k_{m-2}\neq k,k_{m-3}}^{K} w_{k_{m-2}}^{(m-1)} \\ & \rho_{kk_{m-2}}\rho_{k_{m-2}k_{m-3}} \cdots \rho_{k_3k_2}\rho_{k_2k_1}\rho_{k_1 i}\Bigg). \quad (31)\end{aligned}$$

Since the interference and noise terms in on the RHS of (30) are the sum of linear combinations of complex Gaussian r.v's (since the channel fading coefficients h_k are assumed to be complex Gaussian), the average SINR for the kth user at the mth stage output can be obtained, in closed-form, as

$$\overline{\text{SINR}}_k^{(m)} = \frac{A_k^2\left(1 - a w_k^{(m)}\right)^2}{\sigma_I^2 + \sigma_N^2}, \quad (32)$$

where

$$a = \sum_{i\neq k}^{K} r_{k,i}^{(m)} \rho_{ki}, \quad \sigma_I^2 = b + \left(w_k^{(m)}\right)^2 c - 2\, w_k^{(m)} d,$$

$$\sigma_N^2 = \sigma^2\left(1 + \left(w_k^{(m)}\right)^2 e - 2\, w_k^{(m)} a\right), \quad b = \sum_{i\neq k}^{K} \rho_{ki}^2 A_i^2,$$

$$c = \sum_{i\neq k}^{K}\left(r_{k,i}^{(m)} + \sum_{k_1\neq i,k}^{K} r_{k,k_1}^{(m)}\rho_{k_1 i}\right)^2 A_i^2, \quad e = \sum_{i\neq k}^{K}\sum_{j\neq k}^{K} r_{k,i}^{(m)} r_{k,j}^{(m)} \rho_{ij},$$

$$d = \sum_{i\neq k}^{K} \rho_{ki}\left(r_{k,i}^{(m)} + \sum_{k_1\neq i,k}^{K} r_{k,k_1}^{(m)}\rho_{k_1 i}\right) A_i^2.$$

By differentiating the average SINR expression in (32) w.r.t $w_k^{(m)}$, the optimum weights $w_{k,opt}^{(m)}$ can be obtained, in closed-form, as

$$w_{k,opt}^{(m)} = \frac{d - ab}{c - ad + \sigma^2(e - a^2)}. \quad (33)$$

IV. Results and Discussion

In this section, we present a comparison of the bit error rate (BER) performance of different matrix filters. The various matrix filters considered include:

1) the conventional filter, $\mathbf{G}^{(m)}$, given by (10),
2) the proposed filter, ${\mathbf{G}_\mathbf{p}}^{(m)}$, given by (14),
3) the MMSE converging filter in [3], ${\mathbf{G}_\mu}^{(m)}$, given by (22),
4) the modified MMSE converging filter, ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$, given by (24), and
5) the proposed weighted filter, ${\mathbf{G}_\mathbf{pw}}^{(m)}$, given by (28).

In Fig. 1, we plot the BER performance of the conventional filter, $\mathbf{G}^{(m)}$, and the proposed filter, ${\mathbf{G}_\mathbf{p}}^{(m)}$, as a function of the stage index, m, for $K = 10$, processing gain $P = 64$, and SNR = 15 dB, for both no near-far (i.e., $A_1 = A_2 = \cdots = A_K$) as well as near-far conditions. In all the simulations, user 1 is taken to be the desired user. Random binary sequences are used as spreading sequences. For the near-far condition, odd-indexed users (users $3, 5, 7, \cdots$) transmit with the same amplitude as the desired user 1, whereas the even-indexed users (users $2, 4, 6, \cdots$) transmit at 10 times larger amplitude than the desired user. The performance of the MF detector and the DC detector are also plotted for comparison. From Fig. 1, it can be seen that the conventional $\mathbf{G}^{(m)}$ filter approaches the DC detector performance rather slowly for increasing m. Observe that the performance of the proposed ${\mathbf{G}_\mathbf{p}}^{(m)}$ filter and the conventional $\mathbf{G}^{(m)}$ filter are the same for $m = 2$ because of no desired signal loss recovery at the 2nd stage of both $\mathbf{G}^{(m)}$ and ${\mathbf{G}_\mathbf{p}}^{(m)}$. However, for $m \geq 3$, the ${\mathbf{G}_\mathbf{p}}^{(m)}$ filter performs better than the $\mathbf{G}^{(m)}$ filter. This is because the ${\mathbf{G}_\mathbf{p}}^{(m)}$ filter, as intended, avoids the generation of new interference and noise terms (e.g., $\boxed{B_I}$ and $\boxed{B_N}$ terms for $m = 3$) compared to the $\mathbf{G}^{(m)}$ filter. The ${\mathbf{G}_\mathbf{p}}^{(m)}$ filter is found to offer greater advantage in near-far conditions, since strong other-user interference terms in $\boxed{B_I}$ are avoided in the ${\mathbf{G}_\mathbf{p}}^{(m)}$ filter.

Next, in Fig. 2, we present a comparison of the performance of the MMSE converging ${\mathbf{G}_\mu}^{(m)}$ filter in [3], and the modified MMSE converging ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ filter, for the same system conditions in Fig. 1. The performance of the MF detector and the MMSE detector are also plotted for comparison. Here again, the ${\mathbf{G}_\mu}^{(m)}$ and ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ filters perform the same for $m = 2$. Also, both ${\mathbf{G}_\mu}^{(m)}$ and ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ are seen to approach the MMSE performance as m is increased. For $m \geq 3$, ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ performs better than ${\mathbf{G}_\mu}^{(m)}$ because of the avoidance of new interference and noise terms. In generating the plot for ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$, we have used the step sizes in (23), which are actually optimum for ${\mathbf{G}_\mu}^{(m)}$. Even with these step sizes (which can be suboptimum for ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$), the proposed ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ filter approaches MMSE performance faster than the ${\mathbf{G}_\mu}^{(m)}$ filter.

Finally, in Fig. 3, we illustrate the performance of all the matrix filters considered in this paper, including the proposed weighted filter, ${\mathbf{G}_\mathbf{pw}}^{(m)}$, under the no near-far condition. The performance of the MF, DC and MMSE detectors are also

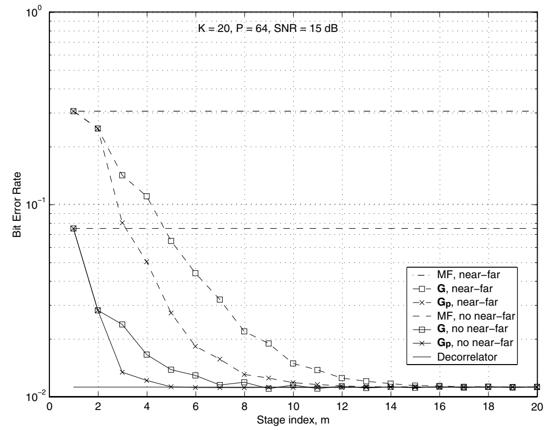

Fig. 1. BER performance of various linear matrix filters – i) conventional filter $\mathbf{G}^{(m)}$, and ii) proposed filter ${\mathbf{G}_\mathbf{p}}^{(m)}$. $K = 20$, $P = 64$, SNR = 15 dB. Near-far as well as no near-far conditions.

plotted. It can be observed that among all the filters considered, the proposed weighted filter ${\mathbf{G}_\mathbf{pw}}^{(m)}$ performs the best for small values of m ($m < 6$, for example). In other words, ${\mathbf{G}_\mathbf{pw}}^{(m)}$ performs best in terms of convergence, i.e., fewer stages are sufficient to yield close to DC detector performance. This may be expected, because in the ${\mathbf{G}_\mu}^{(m)}$ and ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ filters the optimum step sizes are obtained only on a per-stage basis, whereas in the ${\mathbf{G}_\mathbf{pw}}^{(m)}$ filter the optimum weights are obtained on a per-stage as well as a per-user basis. Th computation of the optimum weights, $w_{k,opt}^{(m)}$, for the ${\mathbf{G}_\mathbf{pw}}^{(m)}$ filter, using the closed-form expression in (33), adds to the receiver complexity. However, since these optimum weights are computed by using the *average* SINR expression, the weights computation can be carried out offline once (or whenever users exit from or enter into the system, which changes the correlation matrix), and this need not add to the per-bit complexity of the canceller. Also, in terms of convergence as well as complexity, the proposed filter ${\mathbf{G}_{\mathbf{p}\mu}}^{(m)}$ is also quite attractive.

V. Conclusions

We proposed improved LPIC schemes by viewing an LPIC as a linear matrix filter. Specifically, we proposed new linear matrix filters which achieved better performance than other linear matrix filters in the literature. This was made possible by avoiding the generation of certain new interference and noise terms by making the diagonal elements of a certain matrix equal to zero in each stage, without adding complexity. We point out that, although we have illustrated the proposed matrix filters for a single carrier CDMA system, they are also applicable to other multiuser systems such as multicarrier CDMA, OFDMA, and multiuser MIMO, which can be characterized by a linear vector channel model.

Appendix A
3rd Stage Output Expression for $\mathbf{G}^{(m)}$ Filter

In this appendix, we write the expression for the 3rd stage output of the $\mathbf{G}^{(m)}$ filter (i.e., CLPIC) in an expanded form. From (10), $\mathbf{y}^{(3)}$ can be written as

$$\begin{aligned}\mathbf{y}^{(3)} &= \left[\mathbf{I}+(\mathbf{I}-\mathbf{R})+(\mathbf{I}-\mathbf{R})^2\right]\mathbf{y}^{(1)}\\ &= \mathbf{y}^{(2)}+(\mathbf{I}-\mathbf{R})^2\,\mathbf{y}^{(1)}\\ &= y_k^{(2)}+\sum_{j=1}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}y_j^{(1)}\\ &= y_k^{(2)}+\underbrace{\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}y_k^{(1)}}_{T_1:\text{ case of } j=k}+\underbrace{\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}y_j^{(1)}}_{T_2:\text{ case of } j\neq k}. \qquad (34)\end{aligned}$$

We point out that the term T_1 in the above equation recovers the desired signal lost in the 2nd stage, and the term T_2 removes the interference terms generated in the 2nd stage. Substituting (11) and (3) in (34), we can write

$$\begin{aligned}y_k^{(3)} &= x_k\left(1-\sum_{j\neq k}^{K}\rho_{kj}\rho_{jk}\right)-\sum_{j\neq k}^{K}\sum_{l\neq j,k}^{K}\rho_{kj}\rho_{jl}x_l+n_k-\sum_{j\neq k}^{K}\rho_{kj}n_j\\ &\quad+\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}\left[x_k+\sum_{j\neq k}^{K}\rho_{kj}x_j+n_k\right]\\ &\quad+\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}\left[x_j+\sum_{l\neq j}^{K}\rho_{jl}x_l+n_j\right]\\ &= x_k-\sum_{j\neq k}^{K}\rho_{kj}\rho_{jk}x_k-\sum_{j\neq k}^{K}\sum_{l\neq j,k}^{K}\rho_{kj}\rho_{jl}x_l+n_k-\sum_{j\neq k}^{K}\rho_{kj}n_j\\ &\quad+\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}x_k+\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}\sum_{j\neq k}^{K}\rho_{kj}x_j+\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}n_k\\ &\quad+\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}x_j+\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}\sum_{l\neq j}^{K}\rho_{jl}x_l\\ &\quad+\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}n_j\\ &= x_k-\underbrace{\sum_{j\neq k}^{K}\rho_{kj}\rho_{jk}x_k}_{\boxed{\text{A}}}-\underbrace{\sum_{j\neq k}^{K}\sum_{l\neq j,k}^{K}\rho_{kj}\rho_{jl}x_l}_{\boxed{\text{C}}}+n_k-\sum_{j\neq k}^{K}\rho_{kj}n_j\\ &\quad+\underbrace{\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}x_k}_{\boxed{\text{A}}}+\underbrace{\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}\sum_{j\neq k}^{K}\rho_{kj}x_j}_{\boxed{B_I}}+\underbrace{\sum_{i\neq k}^{K}\rho_{ki}\rho_{ik}n_k}_{\boxed{B_N}}\\ &\quad+\underbrace{\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}x_j}_{\boxed{\text{C}}}+\underbrace{\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}\rho_{jk}x_k}_{\boxed{\text{D}}}\\ &\quad+\underbrace{\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}\sum_{l\neq k,j}^{K}\rho_{jl}x_l}_{\boxed{E_I}}+\underbrace{\sum_{j\neq k}^{K}\sum_{i\neq k,j}^{K}\rho_{ki}\rho_{ij}n_j}_{\boxed{E_N}}. \qquad (35)\end{aligned}$$

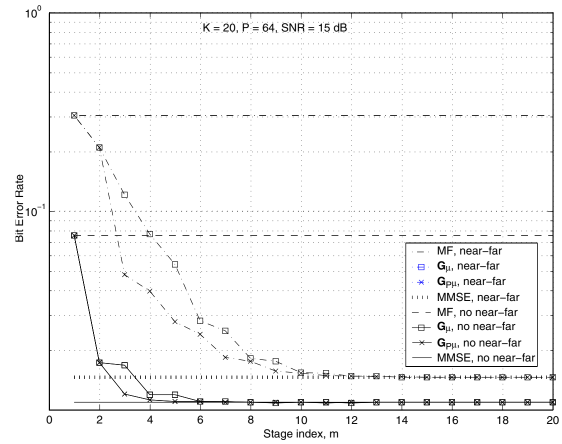

Fig. 2. BER performance of various linear matrix filters – $i)$ MMSE converging filter $\mathbf{G}_{\mu}{}^{(m)}$, and $ii)$ modified MMSE converging filter $\mathbf{G}_{\mathbf{p}\mu}{}^{(m)}$. $K = 20$, $P = 64$, SNR = 15 dB. Near-far as well as no near-far conditions.

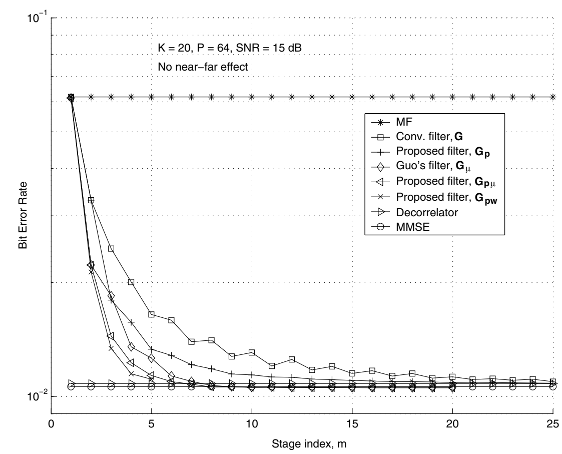

Fig. 3. BER performance of various linear matrix filters – $i)$ $\mathbf{G}^{(m)}$ filter, $ii)$ proposed $\mathbf{G}_{\mathbf{p}}{}^{(m)}$ filter, $iii)$ MMSE converging $\mathbf{G}_{\mu}{}^{(m)}$ filter, $iv)$ modified MMSE converging filter $\mathbf{G}_{\mathbf{p}\mu}{}^{(m)}$, and $v)$ proposed weighted filter, $\mathbf{G}_{\mathbf{pw}}{}^{(m)}$. $K = 20$, $P = 64$, SNR = 15 dB. No near-far effect.

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