Log-likelihood Ratio Based Optimum Mappings Selection for Symbol Mapping Diversity with M-QAM

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***Abstract*— In this paper, a packet combining method known as *symbol mapping diversity* (SMD) is investigated. In an SMD scheme, the bit-to-symbol mapping in M-ary modulation is varied among multiple transmissions of the same packet (e.g., ARQ). It has been shown that multiple transmissions of the same packet using SMD provide better bit error performance than a scheme where the same bit-to-symbol mapping is used for all transmissions of the same packet. A new contribution in this paper is that we propose a new method to obtain optimum bit-to-symbol mappings for SMD schemes with M-QAM. In the proposed method, the optimum mappings are so chosen to maximize a bit log-likelihood ratio (LLR) based metric. We also derive analytical expressions for the bit error rate (BER) of the SMD scheme using LLR based bit decision for 8-QAM. Analytical results are shown to agree very well with the simulation results. Our performance results show that the optimum mappings selected based on our proposed LLR method performs better (in terms of BER) than the mappings selected through other methods proposed in the literature (e.g., the method proposed by Samra *et al* which minimized an upper bound on the BER to obtain optimum mappings).**

Keywords – *M-ary modulation, symbol mapping diversity, packet combining, log-likelihood ratio, ARQ.*

I. Introduction

Achieving high data rates and low error rates is vital in wireless communications. High data rates can be achieved using spectrally efficient higher-order modulations (e.g., M-ary QAM, M-ary PSK) [1]. Low error rates can be achieved using packet retransmissions in case of uncorrectable errors in a packet (e.g., ARQ). Various packet combining methods have been proposed in the literature to achieve diversity among these multiple transmissions of the same packet. One of the well known packet combining methods is the Chase combining, which is a maximum-likelihood combining scheme that concatenates multiple copies of a codeword into a single codeword [2]. Other works on packet combining methods include [3]-[6].

A method to achieve packet combining diversity that has been recently investigated is the *symbol mapping diversity* (SMD), where the bit-to-symbol mapping in M-ary modulation is varied for each packet (re)transmission, i.e., L packet (re)transmissions of the same packet are made using L distinct bit-to-symbol mappings [7]-[9]. Such a system has been shown to enhance the diversity among the multiple (re)transmissions and provide improved error performance compared to a system where bit-to-symbol mapping is not employed, i.e., where L packet (re)transmissions of the same packet are made using the same mapping [7].

This work was supported in part by the Indo-French Centre for Promotion of Advanced Research, New Delhi, under Project 2900-IT.

A key question in the design of SMD schemes is how to choose optimum bit-to-symbol mappings for multiple packet (re)transmissions of the same packet. Note that, for an M-ary constellation, there are $M!$ possible mappings to choose from. Samra *et al*, in [7],[8], addressed this question and proposed a method to obtain the optimum mappings on AWGN and fading channels. In their method, the optimum mappings are obtained by minimizing an upper bound on the bit error rate (BER) of the SMD system. In this paper, we propose an alternate metric to optimize and obtain the optimum mappings for M-QAM. Specifically, we use the *log-likelihood ratios (LLR)* of the bits forming a M-QAM symbol in the optimum selection of the mappings.

The motivation for our investigation of alternate methods to choose the optimum mappings arises from the fact that the method proposed by Samra *et al* in [7] chooses the mappings that minimize only a bound on the BER (not the exact BER), and hence better mappings may be possible. Here, we propose to choose the mappings for multiple (re)transmissions such that the sum of the magnitudes of the LLR of the bits forming the M-QAM symbols in different (re)transmissions is maximized. We show that the mappings obtained by maximizing our proposed LLR based metric performs better than the mappings obtained by the method proposed by Samra *et al* in [7]. We also derive analytical expressions for the BER of the SMD scheme using LLR based bit decision for 8-QAM. Analytical results are shown to agree very well with the simulation results.

The rest of this paper is organized as follows. In Section II, we describe the SMD scheme and present the optimum mappings selection based on minimizing an upper bound on the BER. In Section III, we present our proposed LLR metric based mappings selection. BER analysis and the BER performance results are also presented in Section III. Conclusions are presented in Section IV.

II. Symbol Mapping Diversity

In an M-ary modulation scheme, a bit block B consisting of $m_q = \log_2 M$ bits are taken and mapped to a point in the signal constellation via a bit-to-symbol mapping function ψ, and this signal point $\psi(B)$ is transmitted on the channel. The number of possible bit-to-symbol mappings are $M!$. In order to achieve packet combining diversity, the same bit block may be transmitted more than once. Let L be the number of such transmissions. Multiple transmissions of the same bit block B can either use the same bit-to-symbol mapping in all transmissions, or vary the bit-to-symbol mapping in each transmission; we call the former scheme as the maximum-

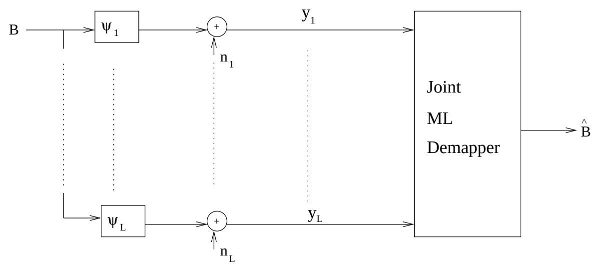

Fig. 1. Symbol mapping diversity scheme

likelihood combining diversity (MLD) scheme and the latter scheme as the symbol mapping diversity (SMD) scheme.

Figure 1 shows the SMD scheme where the same bit block B is sent L times using L different mappings $\psi_1, \psi_2, \cdots, \psi_L$. Assuming AWGN, the receiver obtains the received samples

$$y_i = \psi_i(B) + n_i, \qquad i = 1, 2, \cdots, L, \tag{1}$$

where $n_i = n_{iI} + jn_{iQ}$ is a complex Gaussian r.v of zero mean and variance $\sigma^2/2$ per dimension. The received samples y_i's are combined at the receiver to make an estimate of the transmitted bit block, $\widehat{B}$. Given the observations $y_1, \cdots, y_L$ in (1), the receiver decides that the bit block $\widehat{B}$ was transmitted according to the maximum-likelihood decision rule

$$\min_{\widehat{B} = 0, 1, \cdots, M-1} \sum_{i=1}^{L} |y_i - \psi_i(\widehat{B})|^2. \tag{2}$$

Note that for the MLD scheme, all the L mappings are the same, i.e., $\psi_1 = \psi_2 = \cdots = \psi_L = \psi$. For the SMD scheme, a key question is how to obtain optimal mapping functions $\psi_1, \psi_2, \cdots, \psi_L$. One way is to obtain expressions for the BER/SER of the SMD scheme and choose the mappings that minimize this BER/SER. In the following subsection, we present a method (proposed by Samra *et al* [7]) where the mapping functions $\psi_1, \psi_2, \cdots, \psi_L$ are obtained by minimizing an upper bound on the BER of the SMD system. An alternate way, which we propose in this paper, is to choose the mappings so as to maximize a LLR based metric.

A. Mappings based on Minimizing a BER Upper Bound

An upper bound on the BER of the SMD scheme can be derived as follows. Let B denote the decimal representation of the bit block transmitted L times, and let $\widehat{B}$ be the decimal representation of the decoded bit block at the receiver. The probability of block error is given by

$$P_B = \sum_{B=0}^{M-1} \Pr\big(\widehat{B} \neq B | B\big) \ \Pr(B), \tag{3}$$

where $\Pr(B)$ is the apriori probability that B is transmitted, which is assumed to be equally likely with probability $1/M$. The union bound on the $\Pr\big(\widehat{B} \neq B | B\big)$ can be written as

$$\Pr\big(\widehat{B} \neq B \,|\, B\big) \ \leq \ \sum_{\substack{C=0 \\ C \neq B}}^{M-1} \Pr\Big(\alpha_L(C) < \alpha_L(B) \,\big|\, B\Big), \tag{4}$$

where $\alpha_L(B)$ is the minimization metric given in (2), and $\Pr\Big(\alpha_L(C) < \alpha_L(B) \,\big|\, B\Big)$ is the pairwise error probability (PEP) of the transmitted bit block B being decoded as bit block C. Using the above, an upper bound on the BER can be obtained as

$$P_b(L) \ \leq \ \frac{1}{M} \sum_{B=0}^{M-1} \sum_{\substack{C=0 \\ C \neq B}}^{M-1} \mathcal{N}(B, C) \ \Pr\Big(\alpha_L(C) < \alpha_L(B) \,\big|\, B\Big), \tag{5}$$

where $\mathcal{N}\big(B, C\big)$ is a function to account for the number of bit errors caused by the block error, which is given by

$$\mathcal{N}(B, C) \ = \ \frac{\text{\# of differing bits between } B \text{ and } C}{M}. \tag{6}$$

Now, using the maximum-likelihood criterion in (2), $\Pr\Big(\alpha_L(C) < \alpha_L(B) \,\big|\, B\Big)$ can be written as

$$\Pr\left\{ \sum_{i=1}^{L} \big|y_i - \psi_i(C)\big|^2 < \sum_{i=1}^{L} \big|y_i - \psi_i(B)\big|^2 \ \Bigg|\ B, C \right\}. \tag{7}$$

Since $y_i = \psi_i(B) + n_i$, the above equation can reduce to

$$\Pr\left\{ \sum_{i=1}^{L} \mathcal{D}^2\Big[\psi_i(B), \psi_i(C)\Big] + 2\mathcal{D}\Big[\psi_i(B), \psi_i(C)\Big] n_{iI} \ < \ 0 \ \Bigg|\ B, C \right\}, \tag{8}$$

where $\mathcal{D}[u, v]$ is the Euclidean distance between points u and v. Assuming independence of the Gaussian noise variable n_i, the above PEP expression can further be simplified as

$$\Pr\Big(\alpha_L(C) < \alpha_L(B) \,\big|\, B\Big) = \ Q\left(\sqrt{\frac{1}{4\sigma^2} \sum_{i=1}^{L} \mathcal{D}^2\Big[\psi_i(B), \psi_i(C)\Big]}\right). \tag{9}$$

Substituting (9) in (5) gives the expression for the upper bound on the BER as

$$P_b(L) \ \leq \frac{1}{M} \sum_{B=0}^{M-1} \sum_{\substack{C=0 \\ C \neq B}}^{M-1} \mathcal{N}(B, C) \ Q\left(\sqrt{\frac{1}{4\sigma^2} \sum_{i=1}^{L} \mathcal{D}^2\Big[\psi_i(B), \psi_i(C)\Big]}\right) \tag{10}$$

The next step is to determine the L mappings $\psi_1, \psi_2, \cdots, \psi_L$, which minimize the BER upper bound in (10), i.e.,

$$\min_{\psi_1, \cdots, \psi_L \in \Psi} P_b(L), \tag{11}$$

where Ψ denotes the set of all possible mappings. Since the number of possible mappings is $M!$, the above minimization becomes a large combinatorial optimization problem whose solution space contains $(M!)^L$ solutions. To solve this problem, a simpler sub-optimal iterative solution by computing the Lth mapping from the previous $L-1$ mappings can be employed, where the optimization problem simplifies to

$$\min_{\psi_L \in \Psi} \sum_{B=0}^{M-1} \sum_{\substack{C=0 \\ C \neq B}}^{M-1} g\Big[B, \psi_L(B), C, \psi_L(C)\Big], \tag{12}$$

where $g\big[B, u, C, v\big]$ is the pairwise BER that results by mapping B to symbol u and C to symbol v in the Lth mapping, given by

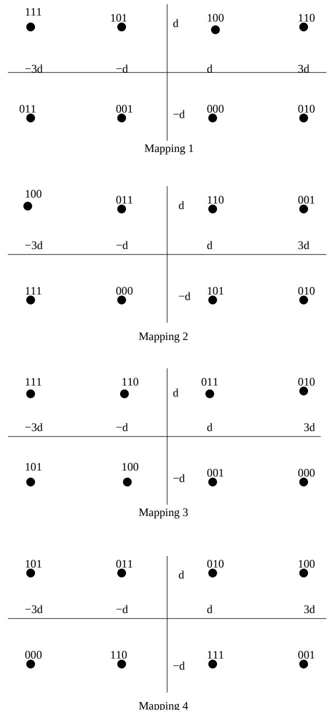

Fig. 2. Bit-to-symbol mappings obtained by minimizing the BER upper bound. Mapping 1: ψ_1, Mapping 2: ψ_2, Mapping 3: ψ_3, Mapping 4: ψ_4. 8-QAM.

$$g\left[B,u,C,v\right] = \frac{\mathcal{N}(B,C)}{M} Q\left(\sqrt{\frac{1}{4\sigma^2}\left(h\left[B,C\right] + \mathcal{D}^2\left[u,v\right]\right)}\right), \quad (13)$$

where

$$h\left[B,C\right] = \sum_{i=1}^{L-1} \mathcal{D}^2\left[\psi_i(B), \psi_i(C)\right] \quad (14)$$

denotes the sum of the squared Euclidean distances in the previously chosen $L-1$ mappings.

We obtained the optimum mappings for 8-QAM by carrying out the optimization in (12). The resulting optimum mappings $\psi_1, \psi_2, \psi_3, \psi_4$ are shown in Fig. 2. It is noted that only ψ_1 is a Gray map and others are not Gray maps.

III. Optimum Mappings Based on LLR Metric

In this section, we present our proposed method of obtaining optimum mappings for bit-to-symbol mapping diversity. Our method takes advantage of the soft information given by the LLRs of the bits forming a M-QAM symbol. Specifically, we choose the mappings that maximize a bit LLR based metric.

In M-QAM, $m_q = \log_2 M$ bits constitute a QAM symbol. Let s be the QAM symbol sent on an AWGN channel, and let $r_1, r_2, \cdots, r_{m_q}$ denote the m_q bits constituting the QAM symbol s. The received signal z corresponding to the transmitted symbol s is then given by

$$z = s + n, \quad (15)$$

where $n = n_I + jn_Q$ is a complex Gaussian r.v of zero mean and variance $\sigma^2/2$ per dimension. The LLR for bit r_i, $i = 1, 2, \cdots, m_q$, is defined as

$$LLR(r_i) = \log\left(\frac{Pr\{r_i = 1|z\}}{Pr\{r_i = 0|z\}}\right). \quad (16)$$

Define two set partitions $S_i^{(1)}$ and $S_i^{(0)}$ such that $S_i^{(1)}$ comprises symbols with $r_i = 1$, and $S_i^{(0)}$ comprises symbols with $r_i = 0$. Then we have

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha \in S_i^{(1)}} Pr\{s = \alpha|z\}}{\sum_{\beta \in S_i^{(0)}} Pr\{s = \beta|z\}}\right) \quad (17)$$

Assuming that all symbols are equally likely and using Bayes' rule, we have

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha \in S_i^{(1)}} f_{z|s}\{z|s = \alpha\}}{\sum_{\beta \in S_i^{(0)}} f_{z|s}\{z|s = \beta\}}\right). \quad (18)$$

Since $f_{z|s}\{z|s = \alpha\} = \frac{1}{\sigma\sqrt{\pi}}\exp\left(\frac{-1}{\sigma^2}\|z - \alpha\|^2\right)$, (18) can be written as

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha \in S_i^{(1)}} \exp\left(\frac{-1}{\sigma^2}\|z - \alpha\|^2\right)}{\sum_{\beta \in S_i^{(0)}} \exp\left(\frac{-1}{\sigma^2}\|z - \beta\|^2\right)}\right). \quad (19)$$

Using the approximation $\log\left(\sum_j \exp(-X_j)\right) \approx -\min_j(X_j)$, we can approximate (19) as

$$LLR(r_i) = \frac{1}{\sigma^2}\left\{\min_{\beta \in S_i^{(0)}} \|z-\beta\|^2 - \min_{\alpha \in S_i^{(1)}} \|z-\alpha\|^2\right\}. \quad (20)$$

We formulate our optimization problem as follows. We iteratively compute the Lth mapping from the $L-1$ previous mappings. Our optimization problem is choose the Lth mapping such that

$$\max_{\psi_L \in \Psi} \sum_{j=1}^{M} \sum_{i=1}^{\log_2 M} \left| h(i,j) + \overline{LLR}_{ij}^{(L)} \right|, \quad (21)$$

where

$$h(i,j) = \sum_{m=1}^{L-1} \overline{LLR}_{ij}^{(m)} \quad (22)$$

denotes the sum of LLRs of a given bit in the previous $L-1$ mappings, and $\overline{LLR}_{ij}^{(m)}$ is the average LLR computed for the ith bit of the jth symbol in the mapping of the mth transmission and the averaging is over the noise samples. The terms inside the absolute operator in (21) essentially give the sum of the LLRs of a specific bit in a specific bit block B in L different mappings; the summation with index i adds the magnitudes of all these LLR sums for all the bits in a specific bit block (there are $m_q = \log_2 M$ of them); and the summation with index j is over all the possible bit blocks (there are M of them).

We carried out the optimization in (21) and obtained the bit-to-symbol mappings $\psi_1, \psi_2, \psi_3, \psi_4$ for 8-QAM. The mappings thus obtained are shown in Fig. 3. It is interesting to note that all the four maps obtained through this LLR metric maximization procedure are Gray maps. In the following subsection, we present the BER analysis of the SMD scheme and the BER performance results.

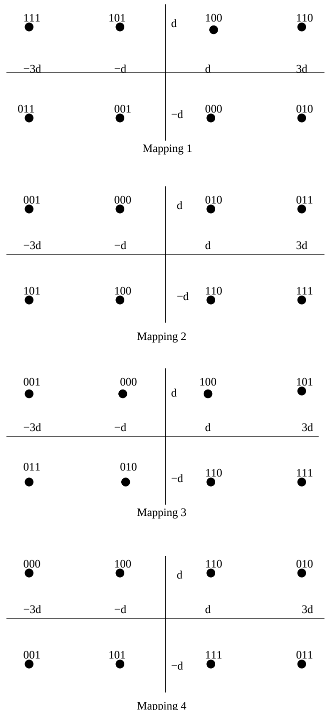

Fig. 3. Bit-to-symbol mappings obtained by maximizing the LLR based metric. Mapping 1: ψ_1, Mapping 2: ψ_2, Mapping 3: ψ_3, Mapping 4: ψ_4. 8-QAM.

A. BER Analysis and Results

We are interested in evaluating and comparing the BER performance of the SMD scheme using the LLR based mappings versus using the BER upper bound based mappings. We consider two decision methods; Euclidean distance (ED) based symbol decision and LLR based bit decision. In ED based symbol decision, the symbol with the least distance (in squared Euclidean distance sense) from the received signal point is decoded as the received symbol. For this ED based symbol decision, an upper bound on the BER can be analytically evaluated using (10) for a given choice of mappings. In LLR based bit decision, on the other hand, decisions are made bit-wise (not symbol-wise as in ED based symbol decision); ith bit of the QAM symbol is decoded as 1 if $LLR(r_i) > 0$, and as 0 otherwise. For the case of LLR based bit decision, analytical expressions for the BER can be derived. The complexity of the analysis increases with L, however. This is because, unlike MLD scheme where the decision boundaries for each data block sent remain the same in all L transmissions, in SMD scheme the decision boundaries for a given bit block keep changing from one transmission to the other. Tracking down the joint decision boundaries for the L transmissions of a given bit block analytically becomes increasingly complex as L increases. We, however, have carried out such an exercise for the case of $L = 2$ for 8-QAM, and derived the analytical BER expression. The outline of the derivation is given in the Appendix.

In Fig. 4, we illustrate the BER performance of the SMD scheme using LLR based bit decision for the case of $L = 2$ for 8-QAM. The optimum mappings obtained through maximizing the LLR metric (shown in Fig. 3) are used. Results obtained through both analysis (evaluated using the BER expression derived in the Appendix) as well as bit error simulations are shown. It is observed that both analytical and simulation results agree very well.

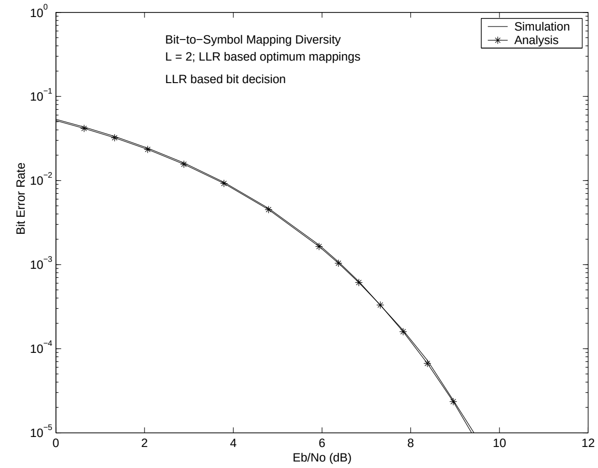

Fig. 4. BER performance of the SMD scheme for 8-QAM on AWGN channels. $L = 2$. LLR based optimum mappings. Analysis and simulations.

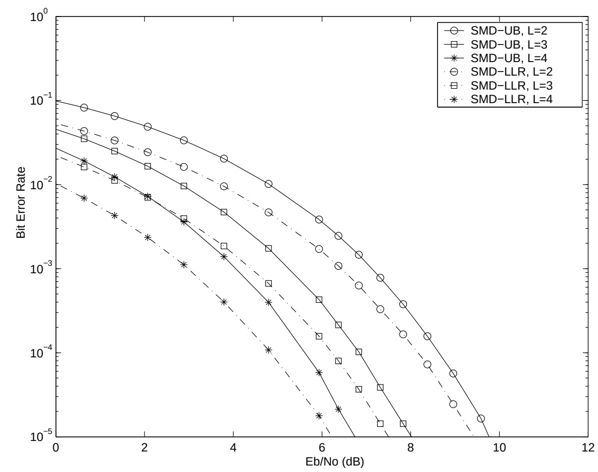

Fig. 5. BER performance comparison between LLR based mappings (SMD-LLR) and BER upper bound based mappings (SMD-UB) for 8-QAM on AWGN channels. $L = 2, 3, 4$. LLR based bit decision.

In Fig. 5, we show the BER performance comparison between using our LLR based mappings (SMD-LLR) shown in Fig. 3 versus using the BER upper bound based mappings (SMD-UB) shown in Fig. 2, for $L = 2, 3, 4$. LLR based bit decision is considered. The BER plots for $L = 2$ are obtained through analytical expressions. Since analysis for $L = 3, 4$ is difficult, we obtained the performance plots for $L = 3, 4$ through simulations. From Fig. 5, it can be observed that the LLR based mappings result in better BER performance than the BER upper bound mappings. For example, at a BER of 10^{-2} the LLR based mappings result in about 1 dB of E_b/N_0 advantage compared to the BER upper bound based mappings. A similar comparison is made in Fig. 6 for the case of Euclidean distance based symbol decision. Here again, we see that the optimum mappings obtained by the proposed LLR based method result in better performance compared to those obtained by the BER upper bound based method proposed by Samra *et al* in [7].

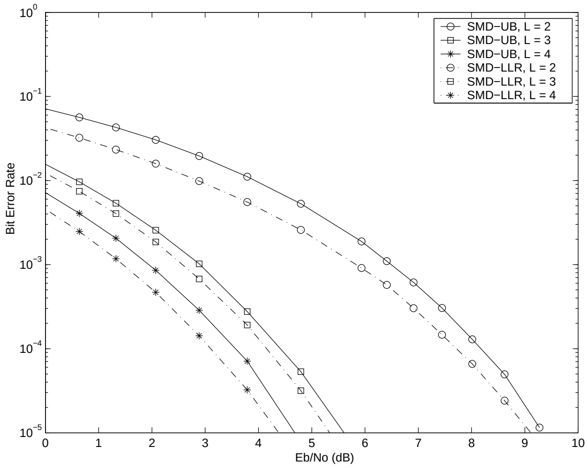

Fig. 6. BER performance comparison between LLR based mappings (SMD-LLR) and BER upper bound based mappings (SMD-UB) for 8-QAM on AWGN channels. $L = 2, 3, 4$. ED based symbol decision.

IV. Conclusions

We proposed a new method to obtain optimum bit-to-symbol mappings for SMD schemes with M-QAM. In the proposed method, the optimum mappings were so chosen to maximize a bit log-likelihood ratio (LLR) based metric. We also derived analytical expressions for the BER of the SMD scheme using LLR based bit decision for 8-QAM. Analytical results were shown to agree well with the simulation results. Our performance results showed that the optimum mappings selected based on our proposed LLR method results in better BER performance than the mappings selected through minimizing a BER upper bound proposed earlier in the literature. Several extensions are possible to this work. For example, LLR based optimum mappings for fading channels can be obtained. SMD schemes using other M-ary modulations like M-PSK can be also investigated.

Appendix - BER Analysis for $L = 2$

Consider the 8-QAM symbol mappings ψ_1 and ψ_2 in Fig. 3. Consider LLR based bit decision. Taking $2d$ as the spacing between adjacent symbols as shown in Fig. 3, and based on the definition of bit LLRs in Sec. III, the LLRs for the bits r_1, r_2, r_3 for the 1st mapping ψ_1 can be obtained as

$$LLR1(r_1) = \begin{cases} -dz_I & |z_I| \leq 2d \\ 2d(d - z_I) & z_I > 2d \\ -2d(d + z_I) & z_I < -2d, \end{cases} \tag{23}$$

$$LLR1(r_2) = \quad d(|z_I| - 2d) \qquad \forall \quad z_I, \tag{24}$$

$$LLR1(r_3) = \quad dz_Q \qquad \forall \quad z_Q. \tag{25}$$

Likewise, the LLRs for the bits r_1, r_2, r_3 for the 2nd mapping ψ_2 can be obtained as

$$LLR2(r_1) \quad = \quad d(|z_I| - 2d) \qquad \forall \quad z_I, \tag{26}$$

$$LLR2(r_2) \quad = \quad \begin{cases} dz_I & |z_I| \leq 2d \\ 2d(z_I - d) & z_I > 2d \\ 2d(z_I + d) & z_I < -2d, \end{cases} \tag{27}$$

$$LLR2(r_3) \quad = \quad -dz_Q \qquad \forall \quad z_Q. \tag{28}$$

The expression for the average BER, P_b is given by

$$P_b \quad = \quad \frac{1}{3}\Big(P_{b_1} + P_{b_2} + P_{b_3}\Big). \tag{29}$$

where $$P_{b_i} = \frac{1}{2}\left(P_{b_i|r_i=0}\right) + \frac{1}{2}\left(P_{b_i|r_i=1}\right), \quad i = 1, 2, 3. \tag{30}$$

Taking into account the different decision boundaries for a symbol in the ψ_1 and ψ_2, $P_{b_1|r_1=0}$ can be written as

$$\begin{aligned} p_{b_1|r_1=0} &= \Pr\left\{LLR1(r_1) + LLR2(r_1) \geq 0 \,|\, a_{I_1}, a_{I_2}\right\} \\ &= \Pr\left\{-z_{I_1}d + d(|z_{I_2}| - 2d) \geq 0,\ |z_{I_1}| \leq 2d\right\} \\ &\quad + \Pr\left\{2d(d - z_{I_1}) + d(|z_{I_2}| - 2d) \geq 0,\ z_{I_1} > 2d\right\} \\ &\quad + \Pr\left\{-2d(d + z_{I_1}) + d(|z_{I_2}| - 2d) \geq 0,\ z_{I_1} < -2d\right\}. \end{aligned} \tag{31}$$

When $r_1 = 0$, the possible values of z_{I_1} are $d + n_1$, $3d + n_1$, and the possible values of z_{I_2} are $-d + n_2$, $d + n_2$, where $n_1, n_2 \sim \mathcal{N}(0, \sigma_I^2)$ and $\sigma_I^2 = \sigma_Q^2 = \sigma^2/2$. For the case when $z_{I_1} = d + n_1$ and $z_{I_2} = -d + n_2$, $P_{b_1|r_1=0,a_{I_1}=d,a_{I_2}=-d}$ can be written as

$$\begin{aligned} P_{b_1|r_1=0,a_{I_1}=d,a_{I_2}=-d} &= \Pr\left\{LLR1(r_1) + LLR2(r_1) \geq 0 \,|\, a_{I_1} = d, a_{I_2} = -d\right\} \\ &= \int_{x=-3d}^{d}\int_{y=x+4d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=-3d}^{d}\int_{y=-\infty}^{-x-2} f_X(x) f_Y(y)\,dy\,dx \\ &\quad + \int_{x=d}^{\infty}\int_{y=2x+3d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=d}^{\infty}\int_{y=-\infty}^{-2x-d} f_X(x) f_Y(y)\,dy\,dx \\ &\quad + \int_{x=-\infty}^{-3d}\int_{y=d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=-\infty}^{-3d}\int_{y=-\infty}^{d} f_X(x) f_Y(y)\,dy\,dx, \end{aligned} \tag{32}$$

and for the case when $z_{I_1} = 3d + n_1$ and $z_{I_2} = d + n_2$, $P_{b_1|r_1=0,a_{I_1}=3d,a_{I_2}=d}$ can be written as

$$\begin{aligned} P_{b_1|r_1=0,a_{I_1}=3d,a_{I_2}=d} &= \Pr\left\{LLR1(r_1) + LLR2(r_1) \geq 0 \,|\, a_{I_1} = 3d, a_{I_2} = d\right\} \\ &= \int_{x=-5d}^{-d}\int_{y=x+4d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=-5d}^{-d}\int_{y=-\infty}^{-x-6d} f_X(x) f_Y(y)\,dy\,dx \\ &\quad + \int_{x=-d}^{\infty}\int_{y=2x+5d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=-d}^{\infty}\int_{y=-\infty}^{-2x-7d} f_X(x) f_Y(y)\,dy\,dx \\ &\quad + \int_{x=-\infty}^{-5d}\int_{y=-d}^{\infty} f_X(x) f_Y(y)\,dy\,dx + \int_{x=-\infty}^{-5d}\int_{y=-\infty}^{-d} f_X(x) f_Y(y)\,dy\,dx, \end{aligned} \tag{33}$$

where $f_X(x)$, $f_Y(y)$ are pdfs of n_1, n_2, respectively, given by $f_X(x) = \frac{1}{\sqrt{2\pi\sigma_I^2}} e^{\frac{-x^2}{2\sigma_I^2}}$ and $f_y(y) = \frac{1}{\sqrt{2\pi\sigma_I^2}} e^{\frac{-y^2}{2\sigma_I^2}}$. From (32) and (33), we have

$$p_{b_1|r_1=0} = \frac{1}{2}\left(P_{b_1|r_1=0,a_{I_1}=d,a_{I_2}=-d}\right) + \frac{1}{2}\left(P_{b_1|r_1=0,a_{I_1}=3d,a_{I_2}=d}\right) \tag{34}$$

Other conditional probabilities in (30), can be derived likewise. Eqn. (34) and expressions for other conditional probabilities in (30) when substituted in (29) gives the average BER expression for the SMD scheme with LLR based bit decision (Note: Expressions for the other conditional probabilities and the final expression for the average BER are given in [9]).

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