Performance Analysis of a Dual-Battery Scheme for Energy Efficiency in Wireless Mobile Terminals

P. T. Dahake and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012

***Abstract*— In this paper, we propose and analyze the performance of a *dual-battery scheme* which exploits the relaxation phenomenon in batteries (under pulsed discharge conditions) to increase the number of packets served during the life time of a battery in wireless mobile terminals. We first adopt a queueing theory based approach to model and analyze a dual-battery scheme. The batteries serve the packets based on random scheduling, i.e., a packet gets served by either the first or the second battery with probabilities p and $(1-p)$, respectively. We present an approximate analysis to derive the expression for the expected number of packets served in the dual-battery scheme. We then study the performance of the proposed dual-battery scheme using the Lithium-ion battery simulation program from UC, Berkeley. We show that the dual-battery scheme achieves increased number of packets served compared to a single battery scheme with intentional vacations, without compromising on the packet delay performance.**

Keywords – *Energy efficiency, relaxation phenomenon, dual-battery scheme, wireless mobile terminals*

I. Introduction

A challenging aspect in wireless mobile communications is exploring different ways by which the 'talk-time' in wireless mobile terminals can be maximized. In addition to developing low-power circuits/devices, efficient batteries and fuel cells, other means of energy savings (for example, through design of energy conscious protocols) in wireless networks have drawn significant research attention. Another promising approach to improving energy efficiency in mobile terminals is to exploit the *relaxation phenomenon* in batteries [1]. Several studies characterizing battery discharge behavior have shown that a battery can deliver more if it is discharged in a pulsed mode rather than in a continuous mode [2]. This is because of the relaxation phenomenon in batteries by which a battery can recover its potential if left idle after a discharge [1],[2].

The ability of a battery to recover its potential when left idle following a discharge has motivated several studies to exploit this phenomenon to increase battery life in wireless mobile terminals [3]-[6]. The bursty nature of many data traffic sources suggests that data transmission in communication devices may provide natural opportunities (idle periods) for such recovery. Single battery schemes which allow *intentional vacations* to the battery in order to exploit the relaxation phenomenon have been proposed and analyzed in [6], where it has been shown that allowing intentional vacations can increase the number of packets served. However, such battery life gains came at the cost increased delay performance of the packets because packets are buffered and not sent during the intentional vacations.

Our focus in this paper is to investigate multiple battery architectures with a motivation to exploit the relaxation phenomenon, but without compromising on the packet delay performance. In multiple battery schemes, when one battery serves a packet other batteries remain idle (during which the idle batteries can recover). Therefore, multiple battery schemes naturally allow recovery periods even without intentional vacations, which can avoid the penalty of increased packet delay due to intentional vacations. The key contribution in this paper is the proposal and analysis of a *dual-battery scheme*, in which the batteries serve the packets based on random scheduling, i.e., a packet gets served by either the first or the second battery with probabilities p and $(1-p)$, respectively. We evaluate the performance of the proposed dual-battery scheme through $a)$ a queueing theory based analysis, and $b)$ simulations using the Lithium-ion battery simulation program from UC, Berkeley [7]. We show that the dual-battery scheme achieves increased number of packets served compared to a single battery scheme with intentional vacations, without loosing on the packet delay performance.

II. Battery Discharge/Recharge Model

As in [6], we model the discharge and recharge behavior of the battery as shown in Fig. 1. While serving packets in busy periods, the battery is assumed to loose charge linearly at a constant slope of unity. During idle periods, the battery is assumed to recharge linearly with varying slopes depending on the battery level at the beginning of the idle period. Although the discharge/recharge behavior in batteries are nonlinear, the linear model considered here is an approximate, yet useful, model which allows mathematical analysis of the performance of various schemes that can exploit the relaxation phenomenon to achieve increased battery life. The recharge model is more clearly explained as follows. We divide the range of charge from 0 to N using P threshold values, $\theta_1, \theta_2, \ldots, \theta_p$. The recharge slope is taken to be $r_1, r_2, \ldots, r_{p+1}$, respectively, when the battery level at the beginning of the idle period is in the range θ_1 to N, θ_2 to θ_1,…,0 to θ_p. By choosing $r_1 > r_2 > \ldots > r_{p+1}$, the model ensures that the ability to recharge reduces with decreasing battery level, as in real batteries. Thus, the parameters P, θ_i's, and r_i's characterize the recharge behavior.

This work was supported in part by the Swarnajayanti Fellowship, Department of Science and Technology, New Delhi, Government of India, under Project Ref: No.6/3/2002-S.F.

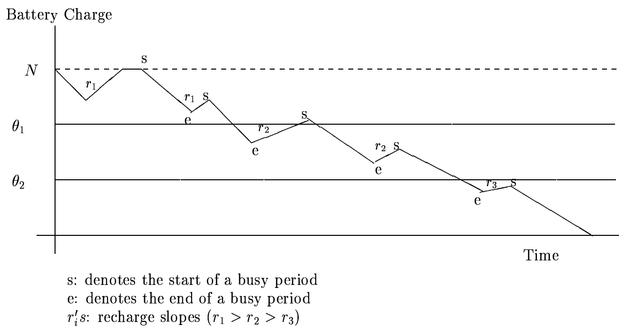

Fig. 1. Battery discharge/recharge model.

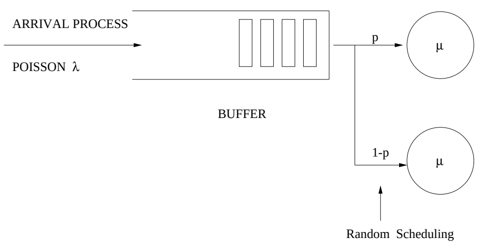

Fig. 2. Dual-battery scheme with random scheduling.

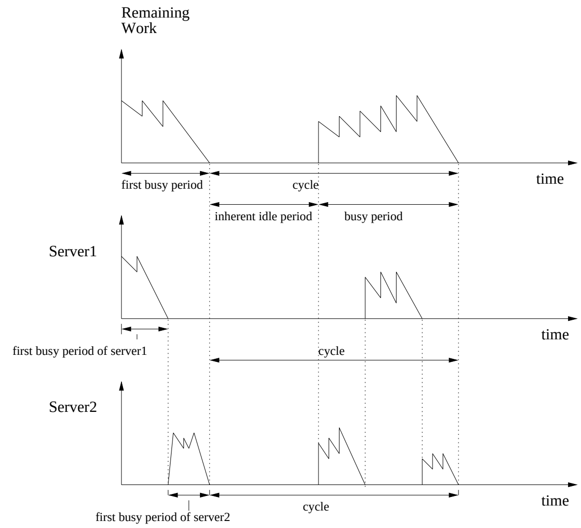

Fig. 3. Definition of a cycle in the dual-battery scheme.

III. Dual-Battery Scheme

The proposed dual-battery scheme is shown in Fig. 2. Two batteries each with N charge units are considered. We model the batteries as servers with finite capacities and packets as customers to serve. We assume that the packet arrival process at the mobile terminal is Poisson with rate λ, and the service time distribution is exponential with parameter μ. A packet can be served either by the first or by the second battery with probabilities p and $(1-p)$, respectively. When one battery serves a packet, the other battery remains idle (i.e., takes a natural vacation). The batteries are assumed to discharge (during packet transmissions) and recharge (during idle periods) as per the discharge/recharge model described in Sec. II.

A. Performance Analysis

We are interested in analyzing the performance of the dual-battery scheme, in terms of mean number of packets served and mean packet delay. We define a cycle as shown in Fig. 3. In order to find the expected number of packets served by the system, we find $a)$ the expected number of packets served by both the batteries till one of the battery expires (i.e., battery charge goes to zero), and $b)$ the expected number of packets served by the remaining live battery till it expires. To do that, we carry out the following analytical steps:

1) obtain the distribution of the amount of charge left in each battery at the end of its first busy period
2) obtain the expected number of cycles after the first busy period of each battery till its charge goes to zero
3) obtain the expected number of packets served in each cycle in each battery till any one battery expires
4) after any one battery expires, using the charge left in the remaining live battery, evaluate the expected number of packets served by it till it expires, by considering the system as a single battery system without vacations.

Consider the busy period of a $M/M/1$ queue. Let X be a r.v denoting the number of packets served in a $M/M/1$ busy period. Let $X=n$ packets are served in this busy period. Let k, $k=1,2,\ldots,n$, be the number of packets served by the first battery during a $M/M/1$ busy period. Then $(n-k)$ packets will be served by the second battery in the same busy period. We use the following notations:
T_b, T_i: r.v's denoting busy & idle periods of $M/M/1$ queue
T_{1b}, T_{1i}: r.v's denoting busy & idle periods of 1st battery
T_{2b}, T_{2i}: r.v's denoting busy & idle periods of 2nd battery
S_i : exponentially distributed r.v with parameter μ.

We are interested in obtaining the distribution of the busy periods of the two batteries. Exact pdf expressions for these busy periods are difficult to obtain. Hence, in order to facilitate the analysis, we assume that the first battery serves the k out of n packets in a $M/M/1$ busy period continuously, and likewise the second battery serves the remaining $n-k$ packets continuously. Note that in the actual system, the service of k packets in a $M/M/1$ busy period by the first battery can be discontinuous (i.e., service of packets can alternate randomly between the two batteries depending on the scheduling probability, p). Hence this assumption is expected to give approximate results. Later, we will compare the results obtained through this approximate analysis with exact simulation results.

With the above assumption, the cdf of the busy period of the first battery can be written as

$$P(T_{1b} \le t) = \Pr\Big\{ n \text{ packets are served in } M/M/1 \text{ busy period, and } k \text{ out of } n \text{ packets are served by 1st battery, and } \sum_{i=1}^{k} S_i \le t \Big\}. \quad (1)$$

The probability of having n packets served in the $M/M/1$ busy period is given by [8]

$$P(X=n)=\frac{1}{n}\binom{2n-2}{n-1}\rho^{n-1}(1+\rho)^{1-2n}, \quad (2)$$

where $\rho=\lambda/\mu$. Eqn. (1) can then be written as

$$\begin{aligned} P(T_{1b}\le t) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\Pr\left(\sum_{i=1}^{k}S_i\le t\,|\,X=n,\,k\text{ packets are served by 1st battery}\right)\\ &\cdot P(X=n)\cdot\Pr(k\text{ out of }n\text{ packets are served by the first battery})\\ &= \sum_{n=1}^{N}\sum_{k=1}^{n}\Pr\left(\sum_{i=1}^{k}S_i\le t\right)\frac{1}{n}\binom{2n-2}{n-1}\rho^{n-1}(1+\rho)^{1-2n}\\ &\cdot\binom{n}{k}p^k(1-p)^{n-k}. \end{aligned} \quad (3)$$

Since S_i's are $\sim\exp(\mu)$, $S=\sum_{i=1}^{k}S_i$ has Erlang distribution. The pdf of the busy period of the first battery can then be obtained from (3), as

$$\begin{aligned} f_{T_{1b}}(t) = \sum_{n=1}^{N}\sum_{k=1}^{n}\frac{1}{n}\binom{2n-2}{n-1}\rho^{n-1}(1+\rho)^{1-2n}\\ \binom{n}{k}p^k(1-p)^{n-k}f_S(t) \end{aligned} \quad (4)$$

where f_S is given by the Erlang distribution, $f_S(t)=\frac{\mu(\mu t)^{k-1}\exp(-\mu t)}{(k-1)!}$. Similarly, the cdf and pdf of the busy period of the second battery can be obtained as

$$\begin{aligned} P(T_{2b}\le t) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\Pr\left(\sum_{i=1}^{n-k}S_i\le t\right)\frac{1}{n}\binom{2n-2}{n-1}\rho^{n-1}(1+\rho)^{1-2n}\\ &\cdot\binom{n}{n-k}p^k(1-p)^{n-k}, \end{aligned} \quad (5)$$

$$\begin{aligned} f_{T_{2b}}(t) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\frac{1}{n}\binom{2n-2}{n-1}\binom{n}{n-k}\left(\frac{\lambda+\mu}{\lambda}\right)\left(\frac{\rho(1-p)}{(1+\rho)^2}\right)^n\\ &\left(\frac{p}{1-p}\right)^k\cdot\left(\frac{\mu(\mu t)^{n-k-1}\exp(-\mu t)}{(n-k-1)!}\right). \end{aligned} \quad (6)$$

Next, we are interested in the distribution of the idle periods of the two batteries. The idle period of the first battery in a cycle consists of two components, namely, the inherent idle period in a $M/M/1$ queue and the busy period of the second battery. Hence, the pdf of the idle period of the first battery can be written as

$$f_{T_{1i}}(t) = f_{T_{2b}}(t)\star f_{T_i}(t), \quad (7)$$

where $\star$ denotes convolution operation, and $f_{T_i}(t)=\lambda e^{-\lambda t}$. Eqn. (7), in transform domain, can be written as

$$f_{T_{1i}}(s) = f_{T_{2b}}(s).f_{T_i}(s), \quad (8)$$

where $f_{T_i}(s)$ and $f_{T_{2b}}(s)$ are given by [9]

$$f_{T_i}(s) = \frac{\lambda}{(s+\lambda)}, \quad (9)$$

$$\begin{aligned} f_{T_{2b}}(s) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\frac{1}{n}\binom{2n-2}{n-1}\binom{n}{n-k}\left(\frac{\lambda+\mu}{\lambda}\right)\left(\frac{\rho(1-p)}{(1+\rho)^2}\right)^n\\ &\cdot\left(\frac{p}{1-p}\right)^k\left(\frac{\mu}{s+\mu}\right)^{n-k}. \end{aligned} \quad (10)$$

Also, we can obtain the following Laplace Transform relation

$$\begin{aligned} \frac{\lambda}{s+\lambda}\left(\frac{\mu}{s+\mu}\right)^{n-k} \rightleftharpoons \frac{e^{-\lambda t}\mu^{n-k}(\mu-\lambda)^{-(n-k)}}{\Gamma(n-k)}\\ \cdot\frac{\lambda\left(\Gamma(n-k)-\Gamma(k,(\mu-\lambda)t)\right)}{\Gamma(n-k)}, \end{aligned} \quad (11)$$

where $\Gamma(k)$ and $\Gamma(a,z)$ are, respectively, the Euler Gamma function and the incomplete Gamma function, given by $\Gamma(k)=(k-1)!$, k: integer, and $\Gamma(a,z)=\int_{t=z}^{\infty}t^{a-1}e^{-t}dt$. From the above, $f_{T_{1i}}(t)$ can be written as

$$\begin{aligned} f_{T_{1i}}(t) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\frac{1}{n}\binom{2n-2}{n-1}\binom{n}{n-k}\left(\frac{\lambda+\mu}{\lambda}\right)\left(\frac{\rho(1-p)}{(1+\rho)^2}\right)^n\left(\frac{p}{1-p}\right)^k\\ &\cdot\left(\frac{e^{-\lambda t}\mu^{n-k}(\mu-\lambda)^{-(n-k)}\lambda\left(\Gamma(n-k)-\Gamma(n-k,(\mu-\lambda)t)\right)}{\Gamma(n-k)}\right). \end{aligned} \quad (12)$$

Similarly, since the busy period of the first battery contributes to the idle period of the second battery, the pdf of the idle period of the second battery, $f_{T_{2i}}(t)$, can be written as

$$f_{T_{2i}}(t) = f_{T_{1b}}(t)\star f_{T_i}(t), \quad (13)$$

which can be obtained as

$$\begin{aligned} f_{T_{2i}}(t) &= \sum_{n=1}^{N}\sum_{k=1}^{n}\frac{1}{n}\binom{2n-2}{n-1}\binom{n}{k}\left(\frac{\lambda+\mu}{\lambda}\right)\left(\frac{\rho(1-p)}{(1+\rho)^2}\right)^n\left(\frac{p}{1-p}\right)^k\\ &\cdot\left(\frac{e^{-\lambda t}\mu^{k}(\mu-\lambda)^{-k}\lambda\left(\Gamma(k)-\Gamma(k,(\mu-\lambda)t)\right)}{\Gamma(k)}\right). \end{aligned} \quad (14)$$

We are now interested in obtaining the expected number of cycles after the first busy period of each battery till its charge goes to zero. Let τ_1 be a r.v denoting the number of cycles till the charge of the first battery goes to zero, and let τ_2 be a r.v denoting the number of cycles till the charge of the second battery goes to zero. We first find $E(\tau_1)$. Let

$$Z_{1i}=Y_{1i}-B_{1i}, \quad (15)$$

where B_{1i} is the charge consumed during the busy period of the first battery in the i^{th}cycle, Y_{1i} is the amount of recharge gained during the idle period in the i^{th} cycle, and Z_{1i} is net charge gained (or lost) in the i^{th} cycle. Let z_1 be the r.v which denotes the charge at the end of the first busy period of the first battery. We need to find

$$\tau_1 = inf\left\{n:\sum_{i=1}^{n}Z_{1i}\le -z_1\right\}. \quad (16)$$

Let $E_{z_1}(\tau_1)$ denote expected number of cycles given that the charge after the first busy period is z_1. Then we have

$$E_{z_1}(\tau_1) = 1+\int_{-z_1}^{N-z_1}E_{z_{11}}[\tau_1].dF_{Z_{11}}(z_{11}), \quad (17)$$

where $F_{Z_{11}}(z_{11})$ is the cdf of Z_{11}. To obtain the distribution of Z_{1i}, we need to obtain the distribution of Y_{1i} and B_{1i}. The distribution of Y_{1i} is obtained as follows. Let z' denote the charge at the beginning of i^{th} cycle and let T_{1i} denote the duration of the idle period of the i^{th} cycle of the first battery. Then $Y_{1i}=\min(r_kT_{1i},N-z')$, and

$$F_{Y_{1i}}(y) = \begin{cases} F_{T_{1i}}(y/r_k) & y\le N-z'\\ 1 & y>N-z', \end{cases} \quad (18)$$

where r_k, $k \in \{1, 2, \ldots\ldots p+1\}$ is the rate of recharge (as described in Sec. II), which depends on available battery charge at the start of the i^{th} cycle. The value of k, $k \in \{1, 2, \ldots\ldots p+1\}$ is determined by the charge threshold values between which z' lies. Since the battery discharge slope is unity, the distribution of B_{1i} is the same as the distribution of T_{1b}. Hence, the cdf of z_{11} in (17) can be written as

$$F_{Z_{11}}(z_{11}) = \int_{max(0,-z_{11})}^{\infty} F_{Y_{11}}(z_{11}+t) f_{T_{1b}}(t) dt. \quad (19)$$

To obtain $E[\tau_1]$, we average $E_{z_1}(\tau_1)$ as

$$E[\tau_1] = \int_0^N E_{N-z_1}[\tau_1] F_{T_{1b}}(z_1). \quad (20)$$

We can obtain $E[\tau_2]$ by following similar steps used to obtain $E[\tau_1]$ in the above.

Let $x = min\{E[\tau_i], i = 1, 2\}$. Then the average number of packets served by both the batteries till one of the battery expires is given by

$$C_1 = (1+x)\frac{p}{(1-\rho)} + (1+x)\frac{1-p}{(1-\rho)} = (1+x)\frac{1}{(1-\rho)}. \quad (21)$$

Now, we need to determine the remaining charge available in the battery which is alive (not expired yet) at the expiry time of the other battery. Let $min\{E[\tau_i], i = 1, 2\}$ correspond to $i = j$ and $\bar{j}$ denote the complement of j, i.e., when $j = 1$, $\bar{j} = 2$ and vice versa. Then we have

$$x = \int_0^{N_1} E_{N_1 - z_{\bar{j}}}[\tau] dF_{T_{\bar{j}b}}(z_{\bar{j}}), \quad (22)$$

where $N - N_1$ is the charge left in the live battery at the expiry time of the other battery. Note that, after one battery expires, the live battery serves packets like as a single battery system without vacations. Hence, the expected number of cycles elapsed till $N - N_1$ charge units available in the live battery are exhausted can be obtained as

$$E[\tau] = \int_0^{N-N_1} E_{N-N_1-z}[\tau] F_{T_b}(z), \quad (23)$$

where $f_{T_b}(z)$ is the pdf of the busy period in a single battery scheme. Therefore, the expected number of packets served, C_2, by the live battery can be evaluated as described in [6]. The expected number of packets served by the *system* till both the batteries expire is then given by $C = C_1 + C_2$.

IV. Results and Discussion

We computed the expected number of packets served by the proposed dual-battery scheme. The following system parameters are considered: $N = 100$ charge units for each battery, number of thresholds $P = 3$, threshold values $\theta_1 = 75, \theta_2 = 50, \theta_3 = 25$, recharge slope values $r_1 = 0.4, r_2 = 0.3, r_3 = 0.2, r_4 = 0.1$, and service time parameter $\mu = 1$. Fig. 4 shows the expected number of packets served, C, as a function ρ when the battery scheduling probability $p = 0.5$, obtained through the analysis. The results obtained through simulations (simulating the same analytical system model but without the approximations made in the analysis) are also plotted. It is observed that the dual-battery scheme offers increased number of packets served (more than $2N$ which will be the number of packets served if the batteries continuously serve packets without any idle/vacation periods), particularly at low arrival rates where the recharge due to idle periods can be more. It is further observed that the analytical results reasonably match with the simulation results. The difference between the performances predicted by analysis and simulations is mainly due to the approximation we made in deriving the busy period distribution of the batteries. We essentially approximated the possible pulsed discharge during a busy period as continuous discharge. Because of this, the charge recovery is not fully accounted for, and this is the reason why the analysis always underestimates the performance compared to the simulations, as observed in Fig. 4. Also, the delay performance of the dual-battery scheme will be the same as that of the single battery scheme without vacation (i.e., same as the delay in a simple $M/M/1$ queue [8]).

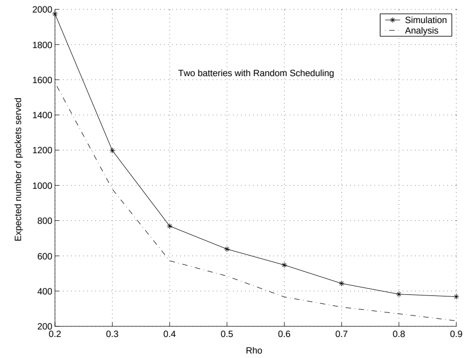

Fig. 4. Expected number of packets served vs ρ for $p = 0.5$. $\mu = 1$. $N = 100$. $\theta_1 = 75, \theta_2 = 50, \theta_3 = 25$. $r_1 = 0.4, r_2 = 0.3, r_3 = 0.2, r_4 = 0.1$.

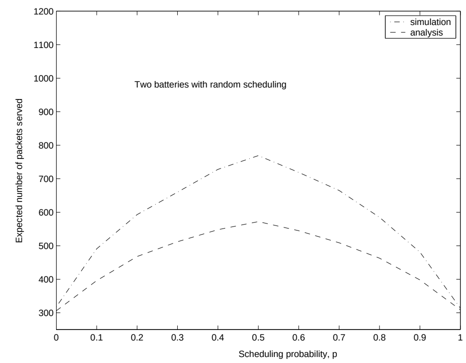

Fig. 5. Expected number of packets served vs p for $\rho = 0.4$. $N = 100$. $\theta_1 = 75, \theta_2 = 50, \theta_3 = 25$. $r_1 = 0.4, r_2 = 0.3, r_3 = 0.2, r_4 = 0.1$.

Fig. 5 shows the expected number of packets as a function of the battery scheduling probability, p, for a given ρ value of 0.4. It is noted that (as expected in random scheduling) the value of p that maximizes the expected number of packets is 0.5. The match between analysis and simulation is very close for $p = 0$ and 1. This is because when $p = 0$ or 1, the system essentially behaves like a single battery scheme (i.e., one battery will continue to serve till it expires, and only then the second battery starts serving). Because of this, only one battery serves during a given busy period. Hence, the approximation made to obtain the busy period distribution is not required when $p = 0$ or 1. Fig. 5 also illustrate that the dual-battery scheme ($0 < p < 1$) serves increased number of packets compared to an equivalent single battery scheme ($p = 0$ or 1).

Lithium-ion Battery Simulation Results:

We also studied the performance of the proposed dual-battery scheme using the battery simulation program developed by the Chemical Engineering Department, UC, Berkeley [7]. We evaluated the performance for the dual-battery scheme with random scheduling (DBS-RD scheme) as follows. First, we implement the packet arrival process, queueing and battery scheduling algorithm in a separate program. We run this program to obtain traces of the busy and idle periods of the two batteries. These busy and idle period traces are then given as inputs to the Berkeley Lithium-ion battery simulation program which incorporates the actual (non-linear) discharge/recharge characteristics of the battery. We run this battery simulation program till both batteries fall below their cut-off voltages. Statistics are collected during these simulation runs to obtain the expected number of packets served and the mean packet delay.

Following a similar procedure, for comparison purposes, we evaluated the performance of two 'single battery like' (SBL) schemes, in which the first battery will continue to serve packets till it drops below the cutoff voltage and only then the second battery starts serving. We consider a SBL scheme with *exhaustive service)* (i.e., no intentional vacation), and another SBL scheme with *non-exhaustive service* (i.e., allow intentional vacations). In non-exhaustive service scheme, the battery takes a exponentially distributed vacation time (with parameter δ) after continuously serving L packets. We compare the performance of a) DBS-RD scheme, b) SBL Exhaustive Service (SBL-ES) scheme, and c) SBL Non-exhaustive Service (SBL-NS) scheme. Since the total theoretical capacity is taken to be same for the SBL and the DBS-RD schemes, the performance difference between these schemes arise mainly due to the way in which the batteries are discharged in each scheme.

Figs. 6 and 7 show the performance comparison between the DBS-RD, SBL-ES and SBL-NS schemes. The following observations can be made from Figs. 6 and 7. The DBS-RD scheme performs better than SBL-ES scheme (no intentional vacation) in terms of expected number of packets served. Their delay performances, however, are almost the same. The number of packets served is increased in SBL-NS scheme (compared to SBL-ES scheme) by allowing intentional vacations once every L packets served. Even with intentional vacations with $L = 4$, the SBL-NS scheme performs poorer than the DBS-RD scheme in terms of number of packets served. Further, because of intentional vacations, the delay performance of the SBL-NS scheme is much poorer than the DBS-RD scheme. Thus, the proposed dual-battery architecture can provide both increased number of packets served as well as lesser mean delay compared to single battery schemes with intentional vacations.

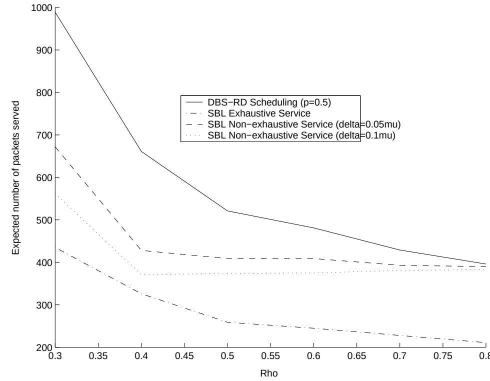

Fig. 6. Expected number of packets vs ρ for a) DBS-RD scheme, $p = 0.5$, b) SBL-ES scheme, and c) SBL-NS scheme. $\mu^{-1} = 0.02$ sec. $\delta = 0.05\mu$, 0.1μ.

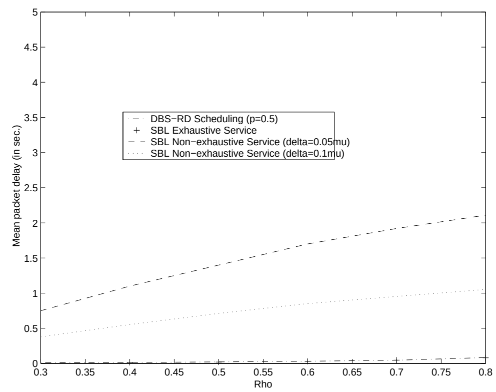

Fig. 7. Mean delay vs ρ for a) DBS-RD scheme, $p = 0.5$, b) SBL-ES scheme, and c) SBL-NS scheme. $\mu^{-1} = 0.02$ sec. $\delta = 0.05\mu$, 0.1μ.

V. Conclusions

We proposed and analyzed the performance of a dual-battery scheme which exploits the relaxation phenomenon in batteries. Using a queueing theory based approach, we analyzed the performance of the dual-battery scheme with random scheduling. We also carried out a detailed simulation study of the proposed dual-battery scheme using the battery simulation program from UC, Berkeley. It was shown that the dual-battery scheme achieves increased number of packets served compared to a single battery scheme with intentional vacations, without loosing on the packet delay performance. Analysis of generalized multiple battery schemes (more than two batteries) and practical realization of multiple battery architectures are possible future extensions to this work.

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