A Low-Complexity Precoder for Large Multiuser MISO Systems

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***Abstract*— In this paper, we consider the problem of precoding in *large* multiuser MISO systems, where by 'large' we mean i) large number of transmit antennas (N_t) at the base station of the order of tens to hundreds of transmit antennas, and ii) large number of downlink users (N_u) of the order of tens to hundreds of users where each user has one receive antenna. Such large MISO systems will be of immense interest because of the high capacities (sum-rates) of the order of hundreds of bits/channel use possible in such systems. We propose a vector perturbation based *low-complexity precoder*, termed as *norm descent search (NDS)* precoder, which has a complexity of just $O(N_uN_t)$ per information symbol. This low complexity attribute of the precoder is achieved by searching for the perturbation vector over a reduced search space. Interestingly, in terms of BER performance, the proposed precoder achieves increasingly better BER for increasing N_t, N_u, such that for large N_t, N_u it achieves near-exponential diversity with some SNR loss, thus making it suited for large MISO systems both in terms of complexity as well as performance. The results of uncoded/turbo-coded simulations without and with channel estimation errors are presented.**

Keywords – *Large multiuser MISO systems, low-complexity precoding, dirty paper coding, ZF/MMSE precoders, vector perturbation.*

I. INTRODUCTION

There is growing interest in MIMO techniques applied to multiuser communications [1],[2]. Of particular interest is downlink communications where a base station (BS) equipped with multiple transmit antennas sends data to multiple downlink users, each having one receive antenna [3]. Availability of channel status information (CSI) at the transmitter allows interesting signal processing to be carried out at the transmitter in multiuser MIMO systems [1],[3]. Such pre-processing of signals before transmission, for instance, can achieve other-user interference suppression at the transmitter itself. For example, in a downlink multiuser MISO system, with the knowledge of both the channel matrix (obtained through feedback from receivers) as well as the information symbols of all the users, the BS can perform precoding on the information symbol vector so that the transmitted signals when they arrive at a desired user terminal will have no or less other-user interference, rendering the user terminal receiver simple. This is the idea of dirty paper coding (DPC) [4],[5], which, in Gaussian broadcast multiuser MIMO channels, has been shown to achieve sum capacity (i.e., maximum aggregation of all users' data rates) that grows linearly with the minimum of the number of antennas (N_t) and the number of users (N_u), provided the transmitter and receivers all know the channel [6],[7].

Practical MIMO transmit pre-processing techniques that aim to achieve the sum capacity promised by DPC have been a key topic of recent research [8]-[18]. Linear precoders including normalized zero-forcing (ZF) and minimum mean square error (MMSE) precoders [8]-[10], and non-linear precoders including Tomlinson-Harashima precoder (THP) [11]-[13] have relatively less complexity, but do not achieve full diversity in the system. Precoders based on vector perturbation [8],[9] and several of their variants [15]-[17] have been shown to achieve good performance at the expense of increased complexity.

It is desired that large number of users be supported in practical systems. However, in much of the multiuser MISO precoder literature, the number of downlink users reported is typically less than ten. Most precoders in the literature either do not scale well (in terms of complexity) or show poor performance for large number of users. Large multiuser MISO systems[1] will be of immense interest because of the practical importance of supporting large number of users and high capacities (sum-rates) of the order of hundreds of bits/channel use possible in such systems.

Our new contribution in this paper is that we propose a low-complexity precoder for large MISO systems. The proposed precoder, termed as norm descent search (NDS) precoder, is a vector perturbation based precoder which achieves its low-complexity attribute by searching for the perturbation vector over a reduced search space. Interestingly, in terms of BER performance, the proposed precoder achieves increasingly better BER for increasing N_t, N_u, such that for large N_t, N_u it achieves near-exponential diversity with some SNR loss, thus making it suited for large MISO systems in terms of both complexity as well as performance. We present the results of both uncoded and turbo-coded simulations without and with channel estimation errors.

The rest of the paper is organized as follows. In Sec. II, we present the system model. The proposed precoder algorithm is presented in Sec. III. Results and discussions are presented in Sec. IV. Conclusions are presented in Sec. V.

II. SYSTEM MODEL

We consider a multiuser MISO system, where a base station (BS) communicates with N_u users on the downlink. A block diagram of the system considered is shown in Fig. 1. The BS employs N_t transmit antennas and each downlink user is equipped with one receive antenna. Let $\mathbf{u}_c \in \mathbb{C}^{N_u \times 1}$ be the complex information symbol vector[2]. Precoding on the symbol vector $\mathbf{u}_c$ is carried out to obtain the precoded symbol vector $\mathbf{x}_c \in \mathbb{C}^{N_t \times 1}$, which is transmitted using N_t transmit

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[1] By 'large' multiuser MISO systems we mean systems having hundreds of BS transmit antennas and hundreds of single-antenna downlink users.

[2] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$ and $[.]^H$ denote the transpose and Hermitian operation, respectively.

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antennas such that ith symbol of $\mathbf{x}_c$ is transmitted on the ith transmit antenna, $i = 1, 2, \cdots, N_t$.

Let y_i denote the received complex signal at user i, and $\mathbf{y}_c = [y_1 y_2 \cdots y_{N_u}]^T$. Let $\mathbf{H}_c \in \mathbb{C}^{N_u \times N_t}$ denote the channel matrix such that its (i,j)th entry $h_{i,j}$ is the complex channel gain from the jth transmit antenna to the ith user's receive antenna. Assuming rich scattering, we model the entries of $\mathbf{H}_c$ as i.i.d and $\mathcal{CN}(0,1)$. Let $\mathbf{n}_c$ denote the vector of noise samples at the N_u users. The elements of $\mathbf{n}_c$ are modelled as i.i.d and $\mathcal{CN}(0, \sigma^2)$. Therefore, $\mathbf{y}_c$ can be expressed in terms of $\mathbf{H}_c$, $\mathbf{x}_c$, and $\mathbf{n}_c$ as

$$\mathbf{y}_c \quad = \quad \mathbf{H}_c \mathbf{x}_c + \mathbf{n}_c. \tag{1}$$

Let $\mathbf{u}_c, \mathbf{x}_c, \mathbf{y}_c, \mathbf{H}_c$, and $\mathbf{n}_c$ be decomposed into real and imaginary parts as follows:

$$\mathbf{u}_c = \mathbf{u}_I + j\mathbf{u}_Q, \quad \mathbf{x}_c = \mathbf{x}_I + j\mathbf{x}_Q, \quad \mathbf{y}_c = \mathbf{y}_I + j\mathbf{y}_Q,$$
$$\mathbf{H}_c = \mathbf{H}_I + j\mathbf{H}_Q, \quad \mathbf{n}_c = \mathbf{n}_I + j\mathbf{n}_Q. \tag{2}$$

Further, we define $\mathbf{u}_r \in \mathbb{R}^{2N_u \times 1}$, $\mathbf{x}_r \in \mathbb{R}^{2N_t \times 1}$, $\mathbf{H}_r \in \mathbb{R}^{2N_u \times 2N_t}$, $\mathbf{y}_r \in \mathbb{R}^{2N_u \times 1}$, and $\mathbf{n}_r \in \mathbb{R}^{2N_u \times 1}$ as

$$\mathbf{u}_r = [\mathbf{u}_I^T \ \ \mathbf{u}_Q^T]^T, \quad \mathbf{H}_r = \begin{pmatrix} \mathbf{H}_I & -\mathbf{H}_Q \\ \mathbf{H}_Q & \mathbf{H}_I \end{pmatrix},$$
$$\mathbf{x}_r = [\mathbf{x}_I^T \ \ \mathbf{x}_Q^T]^T, \quad \mathbf{y}_r = [\mathbf{y}_I^T \ \ \mathbf{y}_Q^T]^T, \quad \mathbf{n}_r = [\mathbf{n}_I^T \ \ \mathbf{n}_Q^T]^T. \tag{3}$$

Now, (1) can also be written as

$$\mathbf{y}_r \quad = \quad \mathbf{H}_r \mathbf{x}_r + \mathbf{n}_r. \tag{4}$$

In the discussions to follow, we shall work with the real-valued system in (4). For notational convenience, we drop the subscripts r in (4) and write

$$\mathbf{y} \quad = \quad \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{H} = \mathbf{H}_r \in \mathbb{R}^{2N_u \times 2N_t}$, $\mathbf{y} = \mathbf{y}_r \in \mathbb{R}^{2N_u \times 1}$, $\mathbf{x} = \mathbf{x}_r \in \mathbb{R}^{2N_t \times 1}$, $\mathbf{u} = \mathbf{u}_r \in \mathbb{R}^{2N_u \times 1}$, and $\mathbf{n} = \mathbf{n}_r \in \mathbb{R}^{2N_u \times 1}$. With the above real-valued system model, the real part of the original complex information symbols (i.e., $\mathbf{u}_c$) will be mapped to $[u_1, \cdots, u_{N_u}]$ and the imaginary part of these symbols will be mapped to $[u_{N_u+1}, \cdots, u_{2N_u}]$. For M-PAM modulation, $[u_{N_u+1}, \cdots, u_{2N_u}]$ will be zeros since M-PAM symbols take only real values. In the case of M-QAM, $[u_1, \cdots, u_{N_u}]$ can be viewed to be from an underlying M-PAM signal set and so is $[u_{N_u+1}, \cdots, u_{2N_u}]$.

A. Vector Perturbation

With the real-valued system model defined in the above, let $\mathbf{G} \in \mathbb{R}^{2N_t \times 2N_u}$ denote the precoding matrix. Therefore, the unit-norm transmitted symbol vector $\mathbf{x}$ can be written as

$$\mathbf{x} \quad = \quad \frac{\mathbf{G}\mathbf{u}}{\|\mathbf{G}\mathbf{u}\|}, \tag{6}$$

where $\|\mathbf{v}\| = \sqrt{\mathbf{v}^T\mathbf{v}}$. For example, for the well known zero-forcing (ZF) linear precoder with $N_t \geq N_u$, the precoding matrix is given by

$$\mathbf{G}_{ZF} \quad = \quad \mathbf{H}^T(\mathbf{H}\mathbf{H}^T)^{-1}, \tag{7}$$

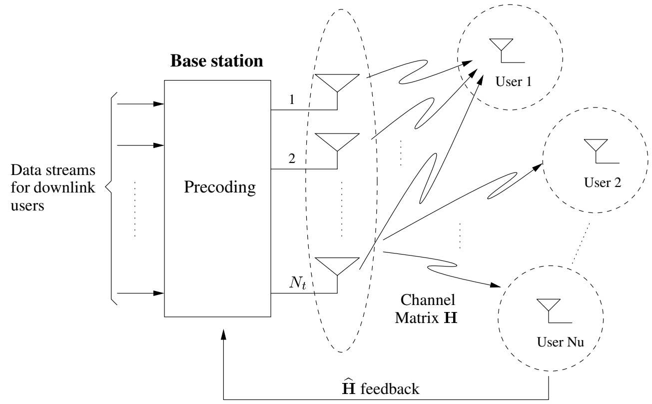

Fig. 1. Multiuser MIMO system on the downlink.

and the corresponding received signal vector $\mathbf{y}$ is given by

$$\mathbf{y} \quad = \quad \frac{\mathbf{u}}{\|\mathbf{G}\mathbf{u}\|} + \mathbf{n}. \tag{8}$$

From (8), we see that $\|\mathbf{G}\mathbf{u}\|$ has a scaling effect on the instantaneous received SNR at the users, and for poorly conditioned channels this results in a loss in SNR. It is assumed that $\|\mathbf{G}\mathbf{u}\|$ is known at the receiver so that the received signal is scaled by $\|\mathbf{G}\mathbf{u}\|$ prior to detection[3]. Hence, in order to improve performance $\|\mathbf{G}\mathbf{u}\|$ needs to be minimized. One technique suggested in the literature is to perturb the information symbol vector $\mathbf{u}$ in such a way that the transformed vector $\tilde{\mathbf{u}}$ is another point in the lattice but $\|\mathbf{G}\tilde{\mathbf{u}}\|$ is much less than $\|\mathbf{G}\mathbf{u}\|$ [9]. Specifically, we can define $\tilde{\mathbf{u}}$ as

$$\tilde{\mathbf{u}} \quad = \quad \mathbf{u} + \tau\,\mathbf{p}, \tag{9}$$

where $\mathbf{p} \in \mathbb{Z}^{2N_u \times 1}$ is the perturbation vector and τ is a positive real number. The optimal value of $\tilde{\mathbf{u}}$, denoted by $\tilde{\mathbf{u}}_{opt}$, is given by

$$\begin{aligned} \tilde{\mathbf{u}}_{opt} &= \mathbf{u} + \tau\,\mathbf{p}_{opt}, \quad \text{where} \\ \mathbf{p}_{opt} &= \underset{\mathbf{p} \in \mathbb{Z}^{2N_u \times 1}}{\arg\min} \ \|\mathbf{G}(\mathbf{u} + \tau\mathbf{p})\|^2. \end{aligned} \tag{10}$$

Exact solution of the above problem requires exponential complexity in N_u. Approximate methods (with polynomial complexity) have been proposed in the literature to solve the above problem [17]. Even these polynomial complexity precoders are prohibitively complex for large MISO systems with hundreds of transmit antennas/users. Our contribution here is to propose an approximate low-complexity solution to (10); the proposed solution is given in Sec. III.

In terms of detection at the receiver, let $\tilde{\mathbf{p}}$ be an approximate (or optimal) solution to (10). Then, the received signal vector (after scaling by $\|\mathbf{G}\mathbf{u}\|$) is given by

$$\begin{aligned} \mathbf{y} &= (\mathbf{u} + \tau\,\tilde{\mathbf{p}}) + \tilde{\mathbf{n}}, \quad \text{where} \\ \tilde{\mathbf{n}} &= \|\mathbf{G}(\mathbf{u} + \tau\,\tilde{\mathbf{p}})\|\mathbf{n}. \end{aligned} \tag{11}$$

The detected symbol vector at the receiver is given by

$$\widehat{\mathbf{u}} \quad = \quad \mathbf{y} - \tau \left\lfloor \frac{\mathbf{y} + \frac{\tau}{2}}{\tau} \right\rfloor. \tag{12}$$

[3]It has been shown via simulations that using $E(\|\mathbf{G}\mathbf{u}\|)$ instead of the instantaneous value of $\|\mathbf{G}\mathbf{u}\|$ results in almost the same performance [9].

In (12), the operation is defined on each entry of the vector since each user gets only one entry of the vector $\mathbf{y}$. τ is a positive real scalar whose value is fixed. Choice of the value of τ affects the overall performance. Too high a value is good as far as mitigating the effect of receiver noise is concerned (since the constellation replicas are placed far apart, and there is little probability that noise may push a point from one replica to another), but on the other hand a high value of τ results in a high value of $\|\mathbf{G}(\mathbf{u}+\tau\,\tilde{\mathbf{p}})\|$. It has been empirically observed that a good choice of τ is given by [9]

$$\tau = 2|c_{max}| + \delta, \tag{13}$$

where $|c_{max}|$ is the maximum value of either the real or imaginary component of the constellation symbols, and δ is the spacing between the constellation symbols. For example, 16-QAM is effectively two 4-PAM constellations in quadrature (taking values of -3, -1, 1, 3 on the real and imaginary axis). Therefore, for 16-QAM, $|c_{max}|$ is 3, δ is 2, and so τ is 8. Similarly, for 4-QAM, τ is 4.

III. Proposed NDS Precoder

In this section, we present the proposed NDS precoder (see Fig. 2 for a block diagram), which is iterative in nature and achieves a suboptimal solution to the problem in (10). Let $\tilde{\mathbf{u}}^{(k)}$ be the perturbed information symbol vector after the kth iteration. We initially start with $\tilde{\mathbf{u}}^{(0)} = \mathbf{u}$, where $\mathbf{u}$ is the unperturbed information symbol vector. We perturb $\tilde{\mathbf{u}}^{(k)}$ to get $\tilde{\mathbf{u}}^{(k+1)}$ as

$$\tilde{\mathbf{u}}^{(k+1)} \;=\; \tilde{\mathbf{u}}^{(k)} + \tau\,\mathbf{p}^{(k)}, \tag{14}$$

where $\mathbf{p}^{(k)} \in \mathbb{Z}^{2N_u\times 1}$ for M-QAM, and $\mathbf{p}^{(k)} \in \mathbb{Z}^{N_u\times 1}$ for M-PAM. To reduce the overall computational complexity of the proposed algorithm, we constrain $\mathbf{p}^{(k)}$ to have only one non-zero entry. Let $\mathbf{F} \stackrel{\triangle}{=} \mathbf{G}^T\mathbf{G}$, where $\mathbf{G} \in \mathbb{R}^{2N_t\times 2N_u}$ is the precoding matrix. Further, let $q^{(k)}$ be the power (squared-norm) of the precoded symbol vector after the kth iteration. Therefore, $q^{(k)}$ is given by

$$q^{(k)} \;=\; \|\mathbf{G}\tilde{\mathbf{u}}^{(k)}\|^2 \;=\; \tilde{\mathbf{u}}^{(k)^T}\mathbf{F}\,\tilde{\mathbf{u}}^{(k)}. \tag{15}$$

In the $(k+1)$th iteration, the algorithm finds a constrained integer vector $\mathbf{p}^{(k)}$ such that $q^{(k+1)} \le q^{(k)}$. Let

$$\Delta q^{(k+1)} \;\stackrel{\triangle}{=}\; q^{(k+1)} - q^{(k)}. \tag{16}$$

Let $\mathbf{e}_i$ denote a $2N_u$-dimensional unit vector with its ith entry only to be one, and all the other entries to be zero. Since we allow only one non-zero entry in $\mathbf{p}^{(k)}$, we can express $\mathbf{p}^{(k)}$ as a scaled integer multiple of any $\mathbf{e}_i$, $i = 1, \cdots, 2N_u$. $\Delta q^{(k+1)}$ can be negative for more than one choice of i. The natural question is therefore to select the appropriate i. Let us denote by $\Delta q_i^{(k+1)}$, the value of $\Delta q^{(k+1)}$ when $\mathbf{p}^{(k)}$ is a scaled integer multiple of $\mathbf{e}_i$. For each i, there exists an integer $\lambda_i^{(k)}$ which minimizes $\Delta q_i^{(k+1)}$. Let this minimum value of $\Delta q_i^{(k+1)}$ be denoted by $\Delta q_{i,opt}^{(k+1)}$. We can therefore express $\Delta q_{i,opt}^{(k+1)}$ as

$$\Delta q_{i,opt}^{(k+1)} = {\lambda_i^{(k)}}^2\tau^2\,\mathbf{F}_{i,i} + 2\lambda_i^{(k)}\tau\,{z_i}^{(k)}, \tag{17}$$

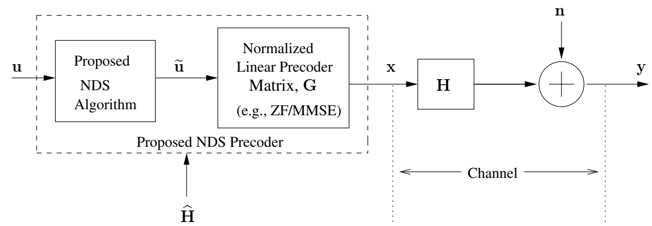

Fig. 2. Proposed norm-descent search (NDS) precoder

where $\mathbf{F}_{i,i}$ is the ith diagonal entry of $\mathbf{F}$, ${z_i}^{(k)}$ is the ith entry of the vector

$$\mathbf{z}^{(k)} \;\stackrel{\triangle}{=}\; \mathbf{F}\tilde{\mathbf{u}}^{(k)}, \tag{18}$$

and

$$\begin{aligned}
\lambda_i^{(k)} &= \underset{\lambda\in\mathbb{Z}}{\text{arg min}}\; \Delta q_i^{(k+1)},\\
&= \underset{\lambda\in\mathbb{Z}}{\text{arg min}}\; \|\mathbf{G}(\tilde{\mathbf{u}}^{(k)}+\lambda\tau\mathbf{e}_i)\|^2 - \|\mathbf{G}\tilde{\mathbf{u}}^{(k)}\|^2,\\
&= \underset{\lambda\in\mathbb{Z}}{\text{arg min}}\; \lambda^2\mathbf{F}_{i,i} + \frac{2\lambda}{\tau}\tilde{\mathbf{u}}^{(k)^T}\mathbf{F}\mathbf{e}_i,\\
&= \underset{\lambda\in\mathbb{Z}}{\text{arg min}}\; \lambda^2\mathbf{F}_{(i,i)} + \frac{2\lambda}{\tau}{z_i}^{(k)}.
\end{aligned} \tag{19}$$

It can be shown that the exact solution to the minimization problem in (19) is given by

$$\lambda_i^{(k)} \;=\; -\text{sgn}({z_i}^{(k)}) \left\lfloor \frac{|{z_i}^{(k)}|}{\tau\mathbf{F}_{(i,i)}} \right\rceil, \tag{20}$$

where sgn(.) is the signum function and $\lfloor . \rceil$ is the rounding operator. It can also be shown that the value of $\Delta q_{i,opt}^{(k+1)}$ is always non-positive (proof omitted here for lack of space). This guarantees a monotonic descent in the value of $\|\mathbf{G}\tilde{\mathbf{u}}^{(k)}\|^2$ with every iteration until a local minima is reached; hence the name *norm descent search (NDS)* algorithm.

Though (19) gives a closed-form solution to $\lambda_i^{(k)}$, we have observed (in the simulations) that in cases when $\lambda_i^{(k)}$ is large, the algorithm tends to get trapped in some poor local minima early in the algorithm. In order to alleviate this phenomenon, we constrain the value of $\lambda_i^{(k)}$ to be within a set $\mathbb{S} = \{-s_{max}, -(s_{max}-1), \cdots, (s_{max}-1), s_{max}\}$, which is a finite subset of $\mathbb{Z}$, and s_{max} denotes the maximum absolute value in $\mathbb{S}$. For example, for 4-QAM, we have found (through simulations) the appropriate set $\mathbb{S}$ to be $\mathbb{S} = \{-1, 0, 1\}$. If $|\lambda_i^{(k)}| > s_{max}$, then $\lambda_i^{(k)}$ is set to 0 and so is $\Delta q_{i,opt}^{(k+1)}$. If $|\lambda_i^{(k)}| \le s_{max}$, then $\Delta q_{i,opt}^{(k+1)}$ is computed as per (17). We shall refer to this correction in $\lambda_i^{(k)}$ as λ-adjustment. In the $(k+1)$th iteration, we can therefore calculate $\Delta q_{i,opt}^{(k+1)}$ for $i = 1, 2, \cdots, 2N_u$. Given these values of $\lambda_i^{(k)}$, $i = 1, \cdots, 2N_u$, we update $\tilde{\mathbf{u}}^{(k)}$ as follows

$$\begin{aligned}
\tilde{\mathbf{u}}^{(k+1)} &= \tilde{\mathbf{u}}^{(k)} + \tau\,\lambda_j^{(k)}\mathbf{e}_j, \quad \text{where}\\
j &= \underset{i}{\text{arg min}}\; \Delta q_{i,opt}^{(k+1)}.
\end{aligned} \tag{21}$$

The values of $\lambda_j^{(k)}$ used in (21) are after the λ-adjustment described above. We also need to evaluate $\mathbf{z}^{(k+1)}$. From (18), we can write

$$\mathbf{z}^{(k+1)} - \mathbf{z}^{(k)} = \mathbf{F}(\tilde{\mathbf{u}}^{(k+1)} - \tilde{\mathbf{u}}^{(k)}). \qquad (22)$$

Using (21), we can rewrite (22) as

$$\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} + \tau\lambda_j^{(k)}\mathbf{f}_j. \qquad (23)$$

where $\mathbf{f}_j$ refers to the jth column of $\mathbf{F}$. Finally, the algorithm terminates after some iteration n if

$$\left\{ \min_i \ \Delta q_{i,opt}^{(n+1)} \right\} \geq 0. \qquad (24)$$

It is easy to see that the algorithm guarantees a monotonic descent in $\|\mathbf{G}\tilde{\mathbf{u}}^{(k)}\|^2$ with every iteration until a local minima is reached. Since $i)$ $\lambda_i^{(k)}$ can take values only from a finite integer valued set $\mathbb{S}$, and $ii)$ $\|\mathbf{G}\tilde{\mathbf{u}}^{(k)}\|^2$ has a global minima for perturbations with $\lambda_i^{(k)} \in \mathbb{S}$, we can see that the proposed algorithm will terminate in a finite number of iterations. The summary of the proposed NDS algorithm is given below.

---

1. Choose the set $\mathbb{S}$; $s_{max} = \max\limits_{s \in \mathbb{S}} \ s$
2. $\tilde{\mathbf{u}}^{(0)} = \mathbf{u}$; $\mathbf{F} = \mathbf{G}^T\mathbf{G}$; $k = 0$ (k is iteration index)
3. $\mathbf{z}^{(0)} = \mathbf{F}\tilde{\mathbf{u}}^{(0)}$; $\tau = 2|c_{max}| + \delta$
4. $nsymb = 2N_u$; ($nsymb$ is $2N_u$ for QAM and N_u for PAM)
5. for $i = 1, 2, \cdots, nsymb$
6. $\lambda_i^{(k)} = -\text{sgn}(z_i{}^{(k)})\lfloor \frac{|z_i{}^{(k)}|}{\tau \mathbf{F}_{(i,i)}} \rceil$
7. if $(|\lambda_i^{(k)}| > s_{max})$ $\lambda_i^{(k)} = 0$
8. $\Delta q_{i,opt}^{(k+1)} = {\lambda_i^{(k)}}^2 \tau^2 \mathbf{F}_{i,i} + 2\lambda_i^{(k)}\tau z_i{}^{(k)}$
9. end; (end of for in Step 5)
10. $\Delta q_{min} = \min\limits_i \Delta q_{i,opt}^{(k+1)}$
11. if $(\Delta q_{min} \geq 0)$ goto Step 16
12. $j = \arg\min\limits_i \Delta q_{i,opt}^{(k+1)}$
13. $\tilde{\mathbf{u}}^{(k+1)} = \tilde{\mathbf{u}}^{(k)} + \tau\lambda_j^{(k)}\mathbf{e}_j$
14. $\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} + \tau\lambda_j^{(k)}\mathbf{f}_j$
15. $k = k + 1$, goto Step 5
16. Terminate the algorithm

A. Complexity of the proposed NDS algorithm

The complexity of the proposed NDS algorithm in the above can be analyzed as follows. The per-symbol computation complexities of $\mathbf{G}^T\mathbf{G}$ in Step 2 and $\mathbf{z}^{(0)}$ in Step 3 are $O(N_uN_t)$ and $O(N_u)$, respectively. Step 5 to Step 15 is one basic iteration of the proposed algorithm, whose per-symbol complexity is constant. The mean number of iterations till the algorithm terminates, which we have obtained through simulations, has been found to be proportional to N_u (see Fig. 5); i.e., constant per-symbol complexity. Putting the above individual complexities together, the overall per-symbol complexity of the proposed NDS algorithm is $O(N_uN_t)$. This low-complexity feature makes practical precoding for large number of users (of the order of hundreds) to be feasible.

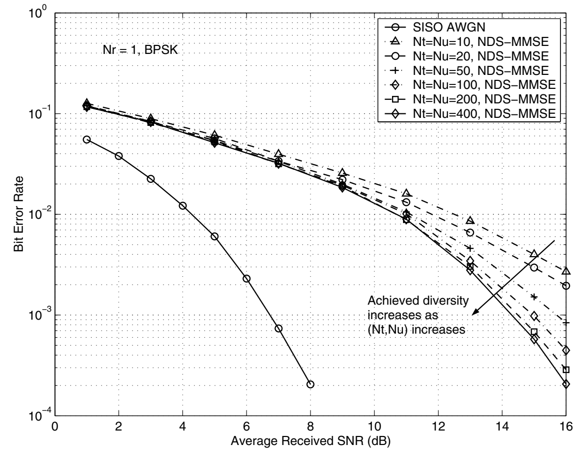

Fig. 3. Uncoded BER performance of the proposed NDS-MMSE precoder for different values of (N_t, N_u). $Nr = 1$, BPSK. Achieved diversity increases with increased (N_t, N_u).

IV. Results and Discussions

In this section, we present the uncoded and turbo-coded simulation results of the BER performance of the proposed precoder. In all our simulation results, we have taken the $\mathbf{G}$ matrix to be the MMSE precoding matrix. Hence, we refer to the proposed precoder as the NDS-MMSE precoder[4]. In Fig. 3, we illustrate the uncoded BER as a function of average received SNR with BPSK and $N_r = 1$, for different values of (N_t, N_u). In Fig. 3, perfect knowledge of the channel matrix is assumed. The performance of uncoded BPSK in SISO AWGN (given by $Q(\sqrt{SNR})$) is also plotted for comparison purposes. From Fig. 3, it can be observed that the proposed precoder achieves increased diversity with increasing (N_t, N_u); observe the slope of the BER curves from small to large (N_t, N_u). For example, an uncoded BER of 2×10^{-3} is achieved at an SNR of 16 dB for $N_t = N_u = 20$, whereas the same BER is achieved at an SNR of 13.5 dB for $N_t = N_u = 200$. This is due to the large system effect in the proposed precoder. For large (N_t, N_u) the BER curves show near-exponential fall (parallel to SISO AWGN curve) with some SNR loss from the SISO AWGN performance. The SNR loss is likely due to the reduced search space employed in the algorithm. We have observed similar performance behavior for 4-QAM as well.

We also evaluated the turbo coded BER performance of the proposed precoder without and with channel estimation errors. Figure 4 shows the coded BER performance of a system with $N_t = N_u = 300$, 4-QAM, rate-3/4 turbo code, and $N_r = 1$. The sum rate (sum-capacity) in this system is given by $300 \times 2 \times \frac{3}{4} = 450$ bits/channel use. The ergodic sum-capacity of the model in (1) is given by [9]

$$C_{\text{sum}} = E\left\{ \sup_{\mathbf{D} \in \mathcal{A}} \ \log\left(\left|\mathbf{I}_{N_t} + \rho\mathbf{H}_c^H\mathbf{D}\mathbf{H}_c\right|\right) \right\}, \qquad (25)$$

where $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix, $\mathcal{A}$ is the set of $N_u \times N_u$ diagonal matrices $\mathbf{D}$ with non-negative elements that sum to 1 (i.e., tr$(\mathbf{D}) = 1$), and ρ is the average SNR defined as

[4]If $\mathbf{G}$ is taken to be the ZF precoding matrix, then the resulting precoder is referred to as NDS-ZF precoder. However, we do not present the results of NDS-ZF precoder for lack of space.

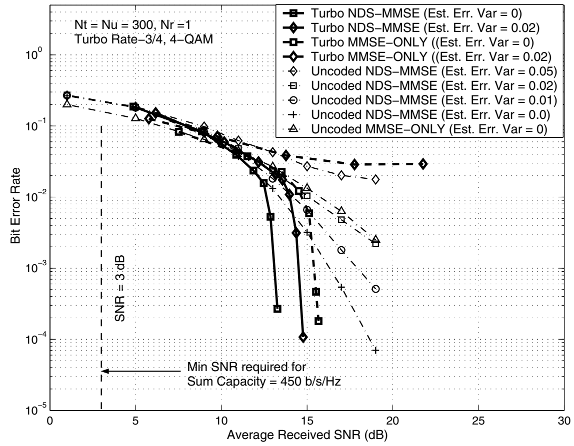

Fig. 4. Turbo coded BER performance of the proposed NDS-MMSE precoder without and with channel estimation errors. $N_t = N_u = 300$, $N_r = 1$, rate-3/4 turbo code, 4-QAM. Sum rate $= 300 \times 2 \times 3/4 = 450$ bits/channel use.

$1/\sigma^2$. We have evaluated the sum-capacity in (25) as a function of SNR for a $N_t = N_u = 300$ system through Monte-Carlo simulations, and obtained the minimum SNR required to achieve a sum-capacity of 450 bits/channel use. This limit SNR at 450 bits/channel use capacity (obtained to be 3 dB from simulations) is also shown in Fig. 4. To illustrate the effect of channel estimation errors on performance, we consider a channel estimation error model where the estimated channel matrix, $\widehat{\mathbf{H}}_c$, is taken to be $\widehat{\mathbf{H}}_c = \mathbf{H}_c + \Delta\mathbf{H}_c$, where $\Delta\mathbf{H}_c$ is the estimation error matrix, the entries of which are assumed to be i.i.d complex Gaussian with zero mean and variance σ_e^2. The values of σ_e^2 considered are $0, 0.01, 0.02$. Note that $\sigma_e^2 = 0$ corresponds to perfect channel estimation. The following observations can be made from Fig. 4.

- With perfect channel estimation (i.e., $\sigma_e^2 = 0$), the proposed NDS-MMSE precoder achieves vertical fall in turbo coded BER at about 13 dB (i.e., 10 dB away from the limit SNR at capacity). The linear MMSE precoder (without the proposed NDS algorithm), on the other hand, achieves the vertical fall only at about 16 dB. It is noted that the order of complexity for the NDS-MMSE and the linear MMSE are the same, with the proposed NDS-MMSE performing better than the linear MMSE.
- Interestingly, the robustness of the proposed NDS-MMSE precoder to imperfect channel estimation is superior compared to the linear MMSE. For example, for $\sigma_e^2 = 0.02$, the vertical fall occurs at about 15 dB for the NDS-MMSE, whereas for the linear MMSE vertical fall does not occur and a high error floor results (i.e., uncoded error rate with channel estimation errors is high in linear MMSE to an extent that even the turbo code is unable to avoid the error floor).

V. Conclusions

We proposed a low-complexity precoder for large multiuser MISO systems. The proposed precoder is suited, in terms of both complexity as well as performance, for large multiuser MISO systems with hundreds of downlink users. The proposed precoder was shown to be robust to channel estimation errors. We believe feasibility of low-complexity precoders, like the one we have proposed in this paper, can potentially trigger wide interest in the theory and implementation of large multiuser MISO systems.

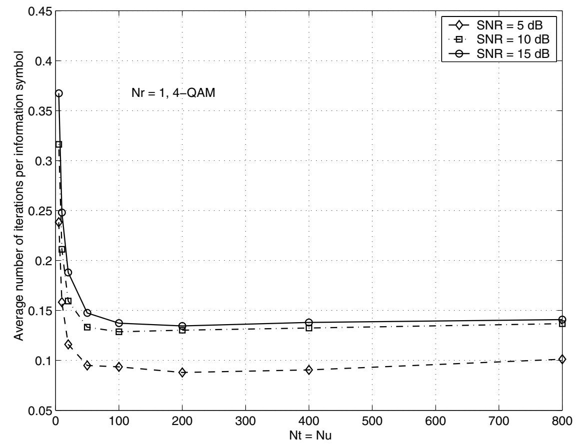

Fig. 5. Average number of iterations per information symbol till the NDS algorithm terminates as a function of (N_t, N_u). $N_r = 1$, 4-QAM.

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