# Optimum Selection Combining for $M$-QAM on Fading Channels 

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#### Abstract

In this paper, we present the optimum selection combining (SC) scheme for $M$-QAM which minimizes the average bit error rate on fading channels. We show that the selection combining scheme where each bit in a QAM symbol selects the diversity branch with the largest magnitude of the log-likelihood ratio (LLR) of that bit is optimum in the sense that it minimizes the average bit error rate (BER). In this optimum SC scheme, different bits in a given QAM symbol may select different diversity branches (since the largest LLRs for different bits may occur on different diversity branches), and hence its complexity is high. However, this scheme provides the best possible BER performance for $M$-QAM with selection combining, and can serve as a benchmark to compare the performance of other SC schemes (e.g., selection based on maximum SNR). We compare the BER performance of this optimum SC scheme with other SC schemes where the diversity selection is done based on maximum SNR and maximum symbol LLR.


Keywords - M-QAM, bit log-likelihood ratio, selection combining.

## I. Introduction

Multilevel quadrature amplitude modulation ( $M-\mathrm{QAM}$ ) is an attractive modulation scheme for wireless communications due to the high spectral efficiency it provides [1]. Diversity reception is a well known technique for mitigating the effects of fading on wireless channels [3],[2]. Typical diversitycombining schemes include maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), and generalized selection combining (GSC). Selection combining is the simplest of all, as it processes only one of the diversity branches. In this paper, we are concerned with selection combining for $M$-QAM.
The diversity branch selection in SC schemes can be done in several ways. One way is to choose the diversity branch with the largest instantaneous SNR. It is known that choosing the diversity branch with the maximum SNR is not the optimum. An alternate way is to choose the branch with the largest magnitude of the log-likelihood ratio (LLR) of the transmitted symbol (we call this as the 'symbol LLR' - SLLR), as proposed in [4], where the authors show that choosing the branch with the largest SLLR minimizes the symbol error rate (SER) for $M$-ary signals.
We, in this paper, obtain the optimum selection combining scheme for $M$-QAM which minimizes the average bit error rate (BER), rather than minimizing the SER. In our scheme, we compute the LLR for each bit in a given QAM symbol (we

[^0]call this as the 'bit LLR' - BLLR) on each diversity branch. For a given bit in a QAM symbol, the diversity branch having the largest magnitude of the BLLR is chosen. We show that the above BLLR based diversity branch selection minimizes the average BER for $M-\mathrm{QAM}$, and hence is optimum. In this optimum SC scheme, it can be noted that different bits in a given QAM symbol may select different diversity branches (since the largest LLRs for different bits may occur on different diversity branches), and hence its complexity is high, i.e., the scheme needs all the $L$ receive RF chains to be present for the bits to choose their respective best antennas. We however note that this scheme provides the best possible BER performance for $M$-QAM with selection combining, and can serve as a benchmark to compare the performance of other SC schemes (e.g., selection based on maximum SNR).
We present a BER performance comparison of the BLLR based optimum SC scheme with other SC schemes where the diversity branch selection is done based on maximum SNR and maximum symbol LLR. We show that, for 16-QAM with one transmit antenna and $L$ receive antennas, at a BER of $10^{-2}$, maximum SLLR based SC performance is away from the BLLR based optimum SC performance by 0.9 dB for $L=2$, by 1.4 dB for $L=3$, and by 1.6 dB for $L=4$. Likewise, the maximum SNR based SC performance is away from the optimum SC performance by 1.4 dB for $L=2$, by 2.1 dB for $L=3$, and by 2.6 dB for $L=4$. We also provide similar comparisons for 16-QAM with two transmit antennas using Alamouti code [5] and $L$ receive antennas. For 16QAM with two transmit antennas and $L$ receive antennas, the SLLR based SC performance is away from the optimum SC performance by 1.1 dB for $L=2$, by 1.6 dB for $L=3$, and by 1.9 dB for $L=4$ at a BER of $10^{-2}$. We present similar performance comparison for 32-QAM as well. Although the results are shown only for 16- and 32-QAM in this paper, the method for BLLR derivation can be extended for any $M$-ary QAM.
The rest of the paper is organized as follows. In Section 2, we derive BLLR expressions for 16-QAM on a given receive antenna in a system with one-Tx/two-Tx antennas. In Section 3, we show that the SC scheme that chooses the branch with the largest BLLR minimizes the BER, and hence is optimum. Simulation results of the BER performance of the optimum SC scheme in comparison with the performance of other SC schemes are presented in Section 4. Conclusions are given in Section 5.


Fig. 1. 16-QAM Constellation

## II. Bit Log-Likelihood Ratios

In this section, we derive expressions for the BLLRs for 16QAM (i.e., $M=16)^{1}$ scheme shown in Fig. 1 , where 4 bits $\left(r_{1}, r_{2}, r_{3}, r_{4}\right)$ are mapped on to a complex symbol $a=a_{I}+$ $j a_{Q}$. The horizontal/vertical line pieces in Fig. 1 denote that all bits under these lines take the value 1 , and the rest take the value 0 . For example, the symbol with coordinates $(-3 d, 3 d)$ maps the 4-bit combination $r_{1}=1, r_{2}=0, r_{3}=r_{4}=1$.

## A. 1-Tx and L-Rx Antennas

First, consider the case of one transmit antenna and $L$ receive antennas. Assuming that the transmitted symbol $a$ undergoes multiplicative and independent fading on each diversity path (the fading is assumed to be slow, frequency non-selective and remain constant over one symbol interval on each diversity path), the received signal $y_{l}$ at the $l^{t h}$ receive antenna corresponding to the transmitted symbol $a$ can be written as

$$
\begin{equation*}
y_{l}=h_{l} a+n_{l}, \quad l=0, \cdots, L-1 \tag{1}
\end{equation*}
$$

where $h_{l}, l=0, \cdots, L-1$, is the complex channel coefficient on the $l^{t h}$ receive antenna with $E\left\{\left\|h_{l}\right\|^{2}\right\}=1$ and the r.v's $h_{l}$ 's are assumed to be i.i.d, and $n_{l}=n_{l I}+j n_{l Q}$ is a complex Gaussian noise r.v of zero mean and variance $\sigma^{2} / 2$ per dimension.
We define the log-likelihood ratio of bit $r_{i}, i=1,2,3,4$, of the received symbol on the $l^{t h}$ antenna, $L L R_{l}\left(r_{i}\right)$, as [6]

$$
\begin{equation*}
L L R_{l}\left(r_{i}\right)=\log \left(\frac{\operatorname{Pr}\left\{r_{i}=1 \mid y_{l}, h_{l}\right\}}{\operatorname{Pr}\left\{r_{i}=0 \mid y_{l}, h_{l}\right\}}\right) . \tag{2}
\end{equation*}
$$

Clearly, the optimum decision rule for the $l^{\text {th }}$ branch is to decide $\hat{r_{i}}=1$ if $L L R_{l}\left(r_{i}\right) \geq 0$, and 0 otherwise. Define two set partitions, $S_{i}^{(1)}$ and $S_{i}^{(0)}$, such that $S_{i}^{(1)}$ comprises

[^1]symbols with $r_{i}=1$ and $S_{i}^{(0)}$ comprises symbols with $r_{i}=0$ in the constellation. Then, from (2), we have
\[

$$
\begin{equation*}
L L R_{l}\left(r_{i}\right)=\log \left(\frac{\sum_{\alpha \in S_{i}^{(1)}} \operatorname{Pr}\left\{a=\alpha \mid y_{l}, h_{l}\right\}}{\sum_{\beta \in S_{i}^{(0)}} \operatorname{Pr}\left\{a=\beta \mid y_{l}, h_{l}\right\}}\right) \tag{3}
\end{equation*}
$$

\]

Assume that all the symbols are equally likely and that fading is independent of the transmitted symbols. Using Bayes' rule, we then have

$$
\begin{equation*}
L L R_{l}\left(r_{i}\right)=\log \left(\frac{\sum_{\alpha \in S_{i}^{(1)}} f_{y_{l} \mid h_{l}, a}\left(y_{l} \mid h_{l}, a=\alpha\right)}{\sum_{\beta \in S_{i}^{(0)}} f_{y_{l} \mid h_{l}, a}\left(y_{l} \mid h_{l}, a=\beta\right)}\right) \tag{4}
\end{equation*}
$$

Since $f_{y_{l} \mid h_{l}, a}\left\{y_{l} \mid h_{l}, a=\alpha\right\}=\frac{1}{\pi \sigma^{2}} \exp \left(\frac{-1}{\sigma^{2}}\left\|y_{l}-h_{l} \alpha\right\|^{2}\right)$,
can be written as

$$
\begin{equation*}
L L R_{l}\left(r_{i}\right)=\log \left(\frac{\sum_{\alpha \in S_{i}^{(1)}} \exp \left(\frac{-1}{\sigma^{2}}\left\|y_{l}-h_{l} \alpha\right\|^{2}\right)}{\sum_{\beta \in S_{i}^{(0)}} \exp \left(\frac{-1}{\sigma^{2}}\left\|y_{l}-h_{l} \beta\right\|^{2}\right)}\right) \tag{5}
\end{equation*}
$$

Using $\log \left(\sum_{j} \exp \left(-X_{j}\right)\right) \approx-\min _{j}\left(X_{j}\right)$, which is a good approximation [7], we can approximate (5) as
$L L R_{l}\left(r_{i}\right)=\frac{1}{\sigma^{2}}\left[\min _{\beta \in S_{i}^{(0)}}\left\|y_{l}-h_{l} \beta\right\|^{2}-\min _{\alpha \in S_{i}^{(1)}}\left\|y_{l}-h_{l} \alpha\right\|^{2}\right]$.
Define $z_{l}$ as

$$
\begin{equation*}
z_{l} \triangleq \frac{y_{l}}{h_{l}}=a+\frac{n_{l}}{h_{l}}=a+\widehat{n_{l}}, \tag{7}
\end{equation*}
$$

where $\widehat{n_{l}}$ is complex Gaussian with variance $\sigma^{2} /\left\|h_{l}\right\|^{2}$. Using (7) in (6), and normalizing $L L R_{l}\left(r_{i}\right)$ by $4 / \sigma^{2}$, we get
$L L R_{l}\left(r_{i}\right)=\frac{\left\|h_{l}\right\|^{2}}{4}\left[\min _{\beta \in S_{i}^{(0)}}\left\|z_{l}-\beta\right\|^{2}-\min _{\alpha \in S_{i}^{(1)}}\left\|z_{l}-\alpha\right\|^{2}\right]$.
Further simplification of (8) gives

$$
\begin{align*}
L L R_{l}\left(r_{i}\right)= & \frac{\left\|h_{l}\right\|^{2}}{4}\left[\min _{\beta \in S_{i}^{(0)}}\left\{\|\beta\|^{2}-2 z_{l I} \beta_{I}-2 z_{l Q} \beta_{Q}\right\}\right. \\
& \left.-\min _{\alpha \in S_{i}^{(1)}}\left\{\|\alpha\|^{2}-2 z_{l I} \alpha_{I}-2 z_{l Q} \alpha_{Q}\right\}\right], \tag{9}
\end{align*}
$$

where $z_{l}=z_{l I}+j z_{l Q}, \alpha=\alpha_{I}+j \alpha_{Q}$ and $\beta=\beta_{I}+j \beta_{Q}$. Note that the set partitions $S_{i}^{(1)}$ and $S_{i}^{(0)}$ are delimited by horizontal or vertical boundaries. As a consequence, two symbols in different sets closest to the received symbol always lie either on the same row (if the delimiting boundaries are vertical) or on the same column (if the delimiting boundaries are horizontal). Using the above fact, the LLRs for bit $r_{1}, r_{2}, r_{3}$ and $r_{4}$ are given by

$$
L L R_{l}\left(r_{1}\right)= \begin{cases}-\left\|h_{l}\right\|^{2} z_{l I} d & \left|z_{l I}\right| \leq 2 d \\ 2\left\|h_{l}\right\|^{2} d\left(d-z_{l I}\right) & z_{l I}>2 d \\ -2\left\|h_{l}\right\|^{2} d\left(d+z_{l I}\right) & z_{l I}<-2 d\end{cases}
$$

$$
\left.\begin{array}{l}
L L R_{l}\left(r_{2}\right)= \begin{cases}-\left\|h_{l}\right\|^{2} z_{l Q} d & \left|z_{l Q}\right| \leq 2 d \\
2\left\|h_{l}\right\|^{2} d\left(d-z_{l Q}\right) & z_{l Q}>2 d \\
-2\left\|h_{l}\right\|^{2} d\left(d+z_{l Q}\right) & z_{l Q}<-2 d\end{cases} \\
L L R_{l}\left(r_{3}\right)=\left\|h_{l}\right\|^{2} d\left(\left|z_{l I}\right|-2 d\right) \\
L L R_{l}\left(r_{4}\right) \tag{10}
\end{array}\right\}\left\|h_{l}\right\|^{2} d\left(\left|z_{l Q}\right|-2 d\right) \quad \text {. }
$$

where $2 d$ is the minimum distance between pairs of signal points.

## B. 2-Tx and L-Rx Antennas

Next, we consider the case of two transmit antennas and $L$ receive antennas. During a given symbol interval, two symbols are transmitted simultaneously on the two antennas using Alamouti code [5]. Let $a_{1},-a_{2}^{*}$ be the symbols transmitted on the first and the second transmit antennas, respectively, during a symbol interval. During the next symbol interval, $a_{2}, a_{1}^{*}$ are transmitted on the first and the second transmit antennas, respectively [5]. We denote the fading coefficients as follows: $h_{2 l-1}$ represents the fading coefficient from transmit antenna 1 to receive antenna $l, l=1, \cdots, L$, and $h_{2 l}$ represents the fading coefficient from transmit antenna 2 to receive antenna $l, i=1, \cdots, L$. Let $y_{2 l-1}$ and $y_{2 l}, l=1, \cdots, L$ be the received signals at the $l^{t h}$ antenna during two successive symbol intervals, respectively. Assuming that the channel remain constant over two consecutive symbol intervals, the received signals during the two consecutive symbol intervals can be written as

$$
\begin{align*}
y_{2 l-1} & =a_{1} h_{2 l-1}-a_{2}^{*} h_{2 l}+n_{2 l-1} \\
y_{2 l} & =a_{2} h_{2 l-1}+a_{1}^{*} h_{2 l}+n_{2 l} \tag{11}
\end{align*}
$$

where $\left\{h_{2 l-1}\right\}_{l=1}^{L}$ and $\left\{h_{2 l}\right\}_{l=1}^{L}$ are the complex fading coefficients and $n_{2 l-1}$ and $n_{2 l}$ are complex Gaussian random variables of zero mean and variance $\sigma^{2}$. Assuming perfect knowledge of the fading coefficients at the receiver, we form $\hat{a}_{1 l}$ and $\hat{a}_{2 l}$, for the $l^{t h}$ receive branch as

$$
\begin{align*}
& \hat{a}_{1 l}=\left(h_{2 l-1}^{*} y_{2 l-1}+h_{2 l} y_{2 l}^{*}\right) \\
& \hat{a}_{2 l}=\left(h_{2 l-1}^{*} y_{2 l}-h_{2 l} y_{2 l-1}^{*}\right) . \tag{12}
\end{align*}
$$

After further simplification, $\hat{a}_{1 l}$ and $\hat{a}_{2 l}$ can be rewritten as

$$
\begin{align*}
& \hat{a}_{1 l}=\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) a_{1}+\zeta_{1} \\
& \hat{a}_{2 l}=\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) a_{2}+\zeta_{2} \tag{13}
\end{align*}
$$

where $\zeta_{1}$ and $\zeta_{2}$ are complex Gaussian random variables with of mean and variance $\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) \sigma^{2}$.
Following similar steps as in the case of one transmit and $L$ receive antennas above, the LLR of bits $r_{i}, i=1,2,3,4$ of symbol $a_{j}, j=1,2$ on the $l^{\text {th }}$ antenna, $L L R_{l}^{a_{j}}\left(r_{i}\right)$, for the two transmit and $L$ receive antennas, can be derived as
$L L R_{l}^{a_{j}}\left(r_{1}\right)= \begin{cases}-\left\{\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right\} \hat{z}_{j l}^{I} d & \left|\hat{z}_{j l}^{I}\right| \leq 2 d \\ 2\left\{\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right\} d\left(d-\hat{z}_{j l}^{I}\right) & \hat{z}_{j l}^{I}>2 d \\ -2\left\{\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right\} d\left(d+\hat{z}_{j l}^{I}\right) & \hat{z}_{j l}^{I}<-2 d,\end{cases}$

$$
\begin{gather*}
L L R_{l}^{a_{j}}\left(r_{2}\right)= \begin{cases}-\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) \hat{z}_{j l}^{Q} d & \left|\hat{z}_{j l}^{Q}\right| \leq 2 d \\
2\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) d\left(d-\hat{z}_{j l}^{Q}\right) & \hat{z}_{j l}^{Q}>2 d \\
-2\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) d\left(d+\hat{z}_{j l}^{Q}\right) & \hat{z}_{j l}^{Q}<-2 d,\end{cases} \\
L L R_{l}^{a_{j}}\left(r_{3}\right)=\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) d\left(\left|\hat{z}_{j l}^{I}\right|-2 d\right), \\
L L R_{l}^{a_{j}}\left(r_{4}\right)=\left(\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}\right) d\left(\left|\hat{z}_{j l}^{Q}\right|-2 d\right) . \tag{14}
\end{gather*}
$$

In the above equations, $\hat{z}_{j l}^{I}$ and $\hat{z}_{j l}^{Q}$ are the real and imaginary parts of $\hat{z}_{j l}$, respectively, where $\hat{z}_{j l}$ is given by

$$
\begin{equation*}
\hat{z}_{j l}=\frac{\hat{a}_{j l}}{\left\|h_{2 l-1}\right\|^{2}+\left\|h_{2 l}\right\|^{2}} \tag{15}
\end{equation*}
$$

It is noted that the LLRs of the various bits in any $M$-QAM constellation of order $M$ and for any arbitrary mapping of bits to the $M$-QAM symbols can be derived following similar steps given above for 16-QAM.

## III. BLLR based Optimum SC

In this section, we derive the rule for optimal selection combining so as to minimize the BER of each of the bits forming the QAM symbol. We prove that in order to minimize the BER of bit $r_{i}$, we must select the diversity branch which has the largest $\left|L L R_{l}\left(r_{i}\right)\right|$. The proof is as follows.
The BER for bit $r_{i}, P_{b i}$, is given as

$$
\begin{equation*}
P_{b i}=1-\int_{\mathbf{y}, \mathbf{h}} \operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid \mathbf{y}, \mathbf{h}\right\} f_{\mathbf{y}, \mathbf{h}} d \mathbf{y} d \mathbf{h} \tag{16}
\end{equation*}
$$

where $\mathbf{y}=\left(y_{0}, y_{2}, \cdots, y_{L-1}\right), \mathbf{h}=\left(h_{0}, h_{2}, \cdots, h_{L-1}\right)$, and $f_{\mathbf{y}, \mathbf{h}}$ is the joint probability density function of $\mathbf{y}, \mathbf{h}$. It follows from the above equation that $P_{b i}$ in minimized by maximizing $\operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid \mathbf{y}, \mathbf{h}\right\}$ for all $\mathbf{y}, \mathbf{h}$. Now,

$$
\begin{align*}
\operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid \mathbf{y}, \mathbf{h}\right\}= & \sum_{l=0}^{L-1} \operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid l^{t h} \text { branch selected, } \mathbf{y}, \mathbf{h}\right\} \\
& \cdot \operatorname{Pr}\left\{l^{t h} \text { branch selected } \mid \mathbf{y}, \mathbf{h}\right\} \\
= & \sum_{l=0}^{L-1} \operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid y_{l}, h_{l}\right\} \\
& \cdot \operatorname{Pr}\left\{l^{t h} \text { branch selected } \mid \mathbf{y}, \mathbf{h}\right\} \\
\leq & \max _{l} \operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid y_{l}, h_{l}\right\} \tag{17}
\end{align*}
$$

Note that $P_{b i}$ is minimized by selecting the branch that provides the maximum $\operatorname{Pr}\left\{\hat{r}_{i}=r_{i} \mid y_{l}, h_{l}\right\}$, or, equivalently, selecting the branch that provides the minimum $\operatorname{Pr}\left\{\hat{r}_{i} \neq r_{i} \mid y_{l}, h_{l}\right\}$, which can be written as

$$
\begin{align*}
\operatorname{Pr}\left\{\hat{r}_{i} \neq r_{i} \mid y_{l}, h_{l}\right\}= & \operatorname{Pr}\left\{\hat{r}_{i}=1, r_{i}=0 \mid y_{l}, h_{l}\right\} \\
& +\operatorname{Pr}\left\{\hat{r}_{i}=0, r_{i}=1 \mid y_{l}, h_{l}\right\} \\
= & \operatorname{Pr}\left\{L L R_{l}\left(r_{i}\right) \geq 0, r_{i}=0 \mid y_{l}, h_{l}\right\} \\
+ & \operatorname{Pr}\left\{L L R_{l}\left(r_{i}\right)<0, r_{i}=1 \mid y_{l}, h_{l}\right\} . \tag{18}
\end{align*}
$$

If $L L R_{l}\left(r_{i}\right) \geq 0$, then

$$
\begin{align*}
\operatorname{Pr}\left\{\hat{r_{i}} \neq r_{i} \mid y_{l}, h_{l}\right\} & =\operatorname{Pr}\left\{r_{i}=0 \mid y_{l}, h_{l}\right\} \\
& =\frac{1}{1+e^{L L R_{l}\left(r_{i}\right)}} . \tag{19}
\end{align*}
$$

If $L L R_{l}\left(r_{i}\right)<0$, then

$$
\begin{align*}
\operatorname{Pr}\left\{\hat{r_{i}} \neq r_{i} \mid y_{l}, h_{l}\right\} & =\operatorname{Pr}\left\{r_{i}=1 \mid y_{l}, h_{l}\right\} \\
& =\frac{1}{1+e^{-L L R_{l}\left(r_{i}\right)}} \tag{20}
\end{align*}
$$

Hence, we have

$$
\begin{equation*}
\operatorname{Pr}\left\{\hat{r}_{i} \neq r_{i} \mid y_{l}, h_{l}\right\} \quad=\frac{1}{1+e^{\left|L L R_{l}\left(r_{i}\right)\right|}} \tag{21}
\end{equation*}
$$

Therefore, to minimize $\operatorname{Pr}\left\{\hat{r}_{i} \neq r_{i} \mid y_{l}, h_{l}\right\}$, we need to maximize the denominator in (21), or, equivalently, maximize the term, $\left|L L R_{l}\left(r_{i}\right)\right|$. Hence, by selecting the branch that provides the largest magnitude of $L L R_{l}\left(r_{i}\right)$, we minimize the BER, $P_{b i}$, and hence minimize the average BER.

It is noted that different bits in a given symbol may choose different antennas, since the largest BLLRs for different bits may occur on different antennas, and hence will require that all the $L$ receive RF chains are present for the bits to choose their respective best antennas. This scheme however provides the best possible BER performance of $M$-QAM with selection combining, and can serve as a benchmark to compare the performance of other SC schemes (as illustrated in the next section).

## IV. Simulation Results

In this section, we present the simulated BER performance of the BLLR optimum SC scheme derived in the previous section in comparison with the performance of other SC schemes where the diversity branch selection is done based on maximum SNR (i.e., choose the branch with largest instantaneous SNR) and maximum SLLR (i.e., choose the branch with the largest magnitude of the symbol LLR). The channel gain coefficients $h_{l}$ 's are taken to be i.i.d complex Gaussian (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E\left\{\left\|h_{l}\right\|^{2}\right\}=1$. Figure 2 shows the simulated average BER performance as function of average SNR per branch for the following $a$ ) BLLR based optimum SC scheme, $b$ ) SNR based SC scheme, and c) SLLR based optimum SC scheme, for 16QAM with one transmit and $L=1,2,3,4$ receive antennas. From Fig. 2, it is observed that, at a BER of $10^{-2}$, the SLLR based SC performance is away from the BLLR based optimum SC performance by 0.9 dB for $L=2$, by 1.4 dB for $L=3$, and by 1.6 dB for $L=4$. Likewise, the SNR based SC performance is away from the optimum SC performance by 1.4 dB for $L=2$, by 2.1 dB for $L=3$, and by 2.6 dB for $L=4$. Since both the SNR based SC as well as the SLLR based SC have the same complexity (i.e., only one of the diversity branches needs to be processed in both cases), SLLR


Fig. 2. Comparison of various selection combining schemes for 16-QAM for 1 Tx antenna and $L=1,2,3,4 \mathrm{Rx}$ antennas - BLLR based optimum SC, SLLR based SC, and SNR based SC.
based SC is preferred over SNR based SC since it achieves BER performance closer to that of the BLLR based optimum SC

Figure 3 shows similar comparison for 16-QAM with two transmit antennas using Alamouti code and $L$ receive antennas. It is pointed out that the plots corresponding to the SLLR based selection in this figure has been obtained by deriving the expressions for the symbol LLRs for the two transmit and $L$ receive antennas case (i.e., by extending derivation in [4] to the 2 Tx antennas case using Alamouti code). From Fig. 3, it is observed that, for 2-Tx and $L=4 \mathrm{Rx}$ antennas, the SLLR based SC performance is away from the BLLR based optimum SC performance by 1.1 dB for $L=2$, by 1.6 dB for $L=3$, and by 1.9 dB for $L=4$, at a BER of $10^{-2}$. Similarly, the SNR based SC performance is away from the optimum SC performance by 1.5 dB for $L=2$, by 2.6 for $L=3$, and by 3.1 for $L=4$, at a BER of $10^{-2}$.
We also derived the BLLR expressions for 32-QAM (derivation not given in this paper), and evaluated the BER performance of the three SC schemes for 32-QAM. Figure 4 shows the BER performance of the three SC schemes for 32-QAM for 1-Tx and $L=2,4 \mathrm{Rx}$ antennas. It can be observed that for 32-QAM, $L=4$, at a BER of $10^{-2}$, the SLLR based SC is worse by 1.6 dB and the SNR based SC by 2.5 dB compared to the BLLR based optimum SC.

## V. Conclusions

We presented the optimum selection combining (SC) scheme for $M$-QAM which minimize the average bit error rate (BER) on fading channels. We showed that the SC scheme which chooses the diversity branch with the largest magnitude of the log-likelihood ratios (LLRs) of the individual bits in the QAM symbol minimizes the BER, and hence is optimum. It was pointed out that the complexity of this optimum SC scheme is higher since different bits in a given QAM symbol


Fig. 3. Comparison of various selection combining schemes for 16-QAM for 2 Tx antennas using Alamouti code and $L=1,2,3,4 \mathrm{Rx}$ antennas BLLR based optimum SC, SLLR based SC, and SNR based SC.


Fig. 4. Comparison of various selection combining schemes for 32-QAM for 1 Tx antenna and $L=2,4 \mathrm{Rx}$ antennas - BLLR based optimum SC, SLLR based SC, and SNR based SC.
may select different diversity branches and since the largest LLRs for different bits may occur on different diversity branches. However, this scheme provides the best possible BER performance for $M$-QAM with selection combining, and can serve as a benchmark to compare the performance of other SC schemes. We presented a BER performance comparison of this optimum SC scheme with other SC schemes where the diversity selection is done based on maximum SNR and maximum symbol LLR.

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[^1]:    ${ }^{1}$ BLLR expressions for other values of $M$ can be derived likewise.

