Performance Analysis of Decorrelating Detector on Diversity Channels with Imperfect Channel Estimates

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Abstract— **In this paper, we present a performance analysis of the bit error performance of detectors in multiuser systems considering diversity reception with imperfect channel estimation. The detectors we consider are the decorrelating (DC) detector the conventional matched filter (MF) detector. We consider a pilot-based channel estimation scheme in flat as well as diversity fading channels, and analytically quantify the degradation in the bit error performance of the DC and the MF detectors due to imperfect channel estimates. We show that, while imperfect channel estimates degrade performance compared to perfect channel estimates, the degradation in the bit error performance can be compensated by using more number of receive antennas.**

Keywords – Decorrelating detector, imperfect channel estimation, receive diversity.

I. Introduction

In code-division multiple access (CDMA) systems, interference from other users limits the performance [1]. Multiuser detectors can help to alleviate the performance degradation due to other user interference [2]. They help to alleviate the near-far effect in CDMA and can lead to considerable improvement in system capacity. Owing to their ability to improve the capacity of wireless systems, multiuser detection has been included as an option in the current third generation (3G) cellular standards.

Multiuser detectors have been analyzed in considerable detail in literature [2],[3]. In particular, channel estimation for multiuser detectors is a topic widely discussed in recent literature [4]-[7]. In [4], Liu *et al* studied the decorrelating detector with imperfect channel estimates using a data-driven scheme on a flat fading channel without considering receive diversity. In [5], Stonjanovic and Zvonar considered error probability of an adaptive multiuser diversity receiver considering the impact of channel estimation errors, again using a data-driven channel estimation scheme. In [6], Xu studied the performance of the minimum mean-square-error (MMSE) detector with imperfect channel estimates. Often, channel estimation can be effectively performed using pilot symbols (e.g., in 3G, pilot symbols are sent on both forward and reverse links to enable such channel estimation).

In this paper, we are interested in the performance analysis of detectors in multiuser systems considering diversity reception and imperfect channel estimation. The detectors we consider in this paper are the decorrelating (DC) detector and the conventional matched filter (MF) detector. We consider a pilot-symbol based channel estimation scheme. We present a bit error performance analysis of the DC and MF detectors with diversity reception and imperfect channel estimates, and analytically quantify the degradation in the bit error performance of these detectors due to imperfect channel estimates compared to perfect channel estimates. We show that, while imperfect channel estimates degrade performance compared to perfect channel estimates, the degradation in the bit error performance can be compensated by using more number of receive antennas.

The rest of the paper is organized as follows. The system model and the channel estimation technique considered are presented in Section II. Section III presents the performance analysis of the DC and the MF detectors on flat fading channels. Section IV presents the analysis for receive diversity. Section V provides numerical results and Section VI provides the conclusion.

II. System Model

Consider a synchronous multiuser scheme with $K = 2^m$ users. Let $\mathbf{y} = (y_1, y_2, ..., y_K)$ denote the received signal vector at the output of the matched filters at the receiver. The received signal vector $\mathbf{y}$ can be written in the form

$$\mathbf{y} = \mathbf{CHb} + \mathbf{n}, \tag{1}$$

where the user correlation matrix $\mathbf{C}$ is given by

$$\mathbf{C} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{bmatrix}, \tag{2}$$

where ρ_{ij} is the correlation coefficient between the signature waveforms of the ith and the jth users. The channel coefficient matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_K \end{bmatrix}, \tag{3}$$

where h_k denotes the channel fade coefficient for user k. The fade coefficients h_k's are assumed to be i.i.d complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with

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zero mean and $E\left[h_{kI}^2\right] = E\left[h_{kQ}^2\right] = 1$, where h_{kI} and h_{kQ} are the real and imaginary parts of h_k. The data vector $\mathbf{b}$ is given by

$$\mathbf{b} = \left[\begin{array}{cccc} A_1 b_1 & A_2 b_2 & \cdots & A_K b_K \end{array} \right]^T, \qquad (4)$$

where A_k denotes the transmit amplitude, $b_k \in \{+1, -1\}$ denotes the data bit of the kth user and $[\cdot]^T$ denotes the transpose operator. The noise vector $\mathbf{n}$ is given by

$$\mathbf{n} = \left[\begin{array}{cccc} n_1^* & n_2^* & \cdots & n_K^* \end{array} \right]^H, \qquad (5)$$

where n_k denotes the additive noise component of the kth user, which is assumed to be complex Gaussian with zero mean with $E[n_k n_j^*] = 2\sigma^2$ when $j = k$ and $E[n_k n_j^*] = 2\sigma^2 \rho_{kj}$ when $j \neq k$. Here $[\cdot]^H$ denotes the Hermitian operator and $(\cdot)^*$ denotes the complex conjugate.

A. Channel Estimation

We assume that users employ Hadamard sequences as pilot symbols. so that the transmit pilot sequence matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{H}_K, \qquad (6)$$

where

$$\mathbf{H}_{2N} = \left[\begin{array}{cc} H_N & H_N \\ H_N & -H_N \end{array} \right], \qquad (7)$$

and

$$H_2 = \left[\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array} \right]. \qquad (8)$$

The received pilot symbol matrix $\mathbf{C_p}$ is then given by

$$\mathbf{C_p} = \mathbf{CHB} + \mathbf{N}, \qquad (9)$$

where

$$\mathbf{N} = \left[\begin{array}{cccc} n_{11} & n_{12} & \cdots & n_{1K} \\ n_{21} & n_{22} & \cdots & n_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ n_{K1} & n_{K2} & \cdots & n_{KK} \end{array} \right]. \qquad (10)$$

In the above, n_{kl} denotes the additive noise component for the kth user at the lth time instant. It is assumed as Gaussian with zero mean, and

$$E[n_{kl} n_{ji}^*] = \begin{cases} 2\Omega^2 & k = j, l = i \\ 2\Omega^2 \rho_{jk} & k \neq j, l = i \\ 0 & l \neq i. \end{cases} \qquad (11)$$

An estimate of the channel matrix can be obtained as

$$\widehat{\mathbf{H}} i = \mathbf{C}^{-1} \mathbf{C}_p \mathbf{B}^{-1}, \qquad (12)$$

which gives an imperfect estimate of the channel matrix $\mathbf{H}$. We use this estimated channel matrix $\widehat{\mathbf{H}}$ at the receiver.

III. Performance Analysis

In this section, we analyze the probability of bit error for the decorrelating and the matched filter detectors using the estimated channel matrix $\widehat{\mathbf{H}}$.

A. Decorrelating Detector

In the case of the decorrelating detector, the receiver does the decorrelating operations as follows:

$$\widehat{\mathbf{y}}_{DC} = \widehat{\mathbf{H}}^H \mathbf{C}^{-1} \mathbf{y}, \qquad (13)$$

where $\mathbf{y}$ is given by (1). The bit estimate for the kth user is then given by

$$\widehat{b}_k = \operatorname{sgn}\Big(\mathbf{e}_k^T \operatorname{Re}\big(\widehat{\mathbf{y}}_{DC}\big)\Big). \qquad (14)$$

where $\mathbf{e}_k$ is a unit vector with a 1 in the kth position and 0 otherwise.

B. Matched Filter Detector

In the case of the MF detector, the receiver does the following operation:

$$\widehat{\mathbf{y}}_{MF} = \widehat{\mathbf{H}}^H \mathbf{y}, \qquad (15)$$

where $\mathbf{y}$ is given by (1). The bit estimate for the kth user is then given by

$$\widehat{b}_k = \operatorname{sgn}\Big(\mathbf{e}_k^T \operatorname{Re}\big(\widehat{\mathbf{y}}_{MF}\big)\Big). \qquad (16)$$

C. Probability of bit error

Taking user 1 as the desired user, the bit decision for the desired user is

$$\widehat{b_1} = \operatorname{sgn}\Big(e_1^T \operatorname{Re}\big(\widehat{\mathbf{y}}\big)\Big), \qquad (17)$$

where $\widehat{\mathbf{y}}$ is given by (13) and (15) for the DC detector and the MF detector, respectively. We note that the real part of the received vector can be written in the form

$$\mathrm{Re}(\widehat{\mathbf{y}}) = \mathbf{V}^H\mathbf{Q}\mathbf{V}, \tag{18}$$

where the $\mathbf{V}$ matrix is given by

$$\mathbf{V} = \begin{bmatrix} h_1^* & \cdots & h_K^* & N_1 & \cdots & N_K & n_1^* & \cdots & n_K^* \end{bmatrix}^H, \tag{19}$$

where, for the $\mathbf{B}$ matrix so selected,

$$N_k = n_{k1}^* + n_{k2}^* + \cdots + n_{kK}^*. \tag{20}$$

The $\mathbf{Q}$ matrix is defined as follows. Let

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & \mathbf{0}_{(K-1)\times K} & & \end{bmatrix}_{K\times K}, \tag{21}$$

$$\mathbf{S} = diag\{A_1b_1 \;\; \cdots \;\; A_Kb_K\}, \tag{22}$$

$$\mathbf{N} = \frac{1}{2}\begin{bmatrix} \mathbf{C}^{-1}(1,1) & \mathbf{C}^{-1}(1,2) & \cdots & \mathbf{C}^{-1}(1,K) \\ & \mathbf{0}_{(K-1)\times K} & & \end{bmatrix}, \tag{23}$$

and

$$\mathbf{F} = \begin{bmatrix} A_1b_1 & \cdots & \frac{A_Kb_K\rho_{1K}}{2} \\ \vdots & \mathbf{0}_{(K-1)\times(K-1)} & \\ \frac{A_Kb_K\rho_{1K}}{2} & & \end{bmatrix}. \tag{24}$$

Using the above definitions, the $\mathbf{Q}$ matrix for the decorrelating detector is given by

$$\mathbf{Q}_{DC} = \begin{bmatrix} A_1b_1\mathbf{M} & \frac{\mathbf{S}\mathbf{C}^{-1}}{2K} & \mathbf{N} \\ \frac{\mathbf{S}\mathbf{C}^{-1}}{2K} & \mathbf{0}_{K\times K} & \frac{\mathbf{C}^{-1}(\mathbf{C}^{-1})^T}{2K} \\ \mathbf{N}^T & \frac{\mathbf{C}^{-1}(\mathbf{C}^{-1})^T}{2K} & \mathbf{0}_{K\times K} \end{bmatrix}. \tag{25}$$

The $\mathbf{Q}$ matrix for the MF detector is given by

$$\mathbf{Q}_{MF} = \begin{bmatrix} \mathbf{F} & \frac{\mathbf{S}}{2K} & \frac{\mathbf{M}}{2} \\ \frac{\mathbf{S}}{2K} & \mathbf{0}_{K\times K} & \frac{(\mathbf{C}^{-1})^T}{2K} \\ \frac{\mathbf{M}}{2} & \frac{(\mathbf{C}^{-1})^T}{2K} & \mathbf{0}_{K\times K} \end{bmatrix}. \tag{26}$$

Let $\mathbf{I}_K$ denote the identity matrix of size $K \times K$. The covariance matrix of $\mathbf{V}$ is given by

$$\mathbf{L} = \begin{bmatrix} 2\mathbf{I}_K & \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times K} & 2K\Omega^2\mathbf{C} & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} & 2\sigma^2\mathbf{C} \end{bmatrix}. \tag{27}$$

The characteristic function of (18) can be obtained as [9],[10]

$$\psi(iw) = \prod_{j=1}^{P} \frac{1}{1 - iw\lambda_j}, \tag{28}$$

where λ_j's are the eigen values of the matrix $\mathbf{LQ}$ and P is the number of eigen values of $\mathbf{LQ}$.

The probability of error for user 1 $\Pr\{\hat{b}_1 \neq b_1\}$ is the same for all possible values of $b_1, b_2, \cdots, b_K$. Therefore, the bit error probability is given by

$$\begin{aligned} P_e &= \Pr\left\{\widehat{b_1} \neq b_1\right\} \\ &= \Pr\left\{\widehat{b_1} \neq b_1 \,|\, b_1 = 1, b_2 = 1, \cdots, b_K = 1\right\} \\ &= \Pr\left\{\mathrm{Re}(\widehat{y}) < 0 \,|\, b_1 = 1, b_2 = 1, \cdots, b_K = 1\right\} \\ &= \Pr\left\{\mathbf{V}^H\mathbf{Q}\mathbf{V} < 0 \,|\, b_1 = 1, b_2 = 1, \cdots, b_K = 1\right\}. \end{aligned} \tag{29}$$

From (28) and (29), we obtain

$$P_e = \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\left(\prod_{j=1}^{P}\frac{1}{1 - iw\lambda_j}\right)e^{-iwx}dwdx. \tag{30}$$

Ignoring the positions where $\lambda_j = 0$ since the product term is unaltered, the above integral can be evaluated by splitting the product term in (30) into partial fractions. Let the number of distinct eigen values be Z. Let the multiplicity of eigen value λ_i be K_i. Splitting the product term into partial fractions, we get

$$P_e = \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\sum_{\substack{l=1\\ \lambda_l\neq 0}}^{Z}\sum_{j=1}^{K_l}\frac{A_l^{(j)}}{(1 - iw\lambda_l)^j}e^{-iwx}dwdx. \tag{31}$$

Using Eqns. 3.382 ET 1 118(3) and 118(4) in [8], it can be shown that

$$P_e = \sum_{\substack{l=1\\ \lambda_l<0}}^{Z}\sum_{j=1}^{K_l} A_l^{(j)}. \tag{32}$$

For the case of distinct eigen values, $A_i^{(1)}$'s in the above equation can be calculated as

$$A_i^{(1)} = \prod_{\substack{j=1\\ j\neq i\\ \lambda_j\neq 0}}^{P}\frac{\frac{1}{\lambda_j}}{\frac{1}{\lambda_j} - \frac{1}{\lambda_i}}. \tag{33}$$

IV. Diversity Reception

In this section, we consider diversity reception with L receive antennas. The received signal vector at the output of the jth receive antenna, $\mathbf{y}_j$, is given by

$$\mathbf{y}_j = \mathbf{C}\mathbf{H}_j\mathbf{b} + \mathbf{n}_j, \quad (34)$$

where the definitions of $\mathbf{C}$ and $\mathbf{b}$ are as in Section II. $\mathbf{H}_j$ and $\mathbf{n}_j$ are defined in a similar manner as (3) and (5), respectively, with the entries denoting the channel fade coefficients and the additive noise components on each path. The fade coefficients and the additive noise components on one path are assumed to be independent of the corresponding variables on every other path. The procedure for estimating the $\mathbf{H}_j$'s are same as before.

For the decorrelating detector, the resultant vector after decorrelation and combining is given by

$$\widehat{\mathbf{y}}_{DC} = \sum_{j=1}^{L} \widehat{\mathbf{H}}_j^H \mathbf{C}^{-1} \mathbf{y}_j. \quad (35)$$

For the MF detector, the resultant vector after combining is given by

$$\widehat{\mathbf{y}}_{MF} = \sum_{j=1}^{L} \widehat{\mathbf{H}}_j^H \mathbf{y}_j. \quad (36)$$

As before, the bit estimate is obtained as the sign of the real part of $\widehat{\mathbf{y}}$. The real part of $\widehat{\mathbf{y}}$ can be written in the form

$$\Re\{\widehat{\mathbf{y}}\} = \sum_{j=1}^{L} \mathbf{V}_j^H \mathbf{Q} \mathbf{V}_j, \quad (37)$$

where $\mathbf{V}_j$ and $\mathbf{Q}$ can be defined as in the previous section. The random vectors $\mathbf{V}_j$'s are independent. The characteristic function of the sum therefore is the product of the individual characteristic functions, which is given by

$$\psi(iw) = \left(\prod_{j=1}^{P} \frac{1}{1 - iw\lambda_j} \right)^L. \quad (38)$$

Let the number of distinct eigen values be Z and the multiplicity of λ_i be K_i as before. Splitting the product term into partial fractions, we get

$$P_e = \frac{1}{2\pi} \int_{-\infty}^{0} \int_{-\infty}^{\infty} \left(\sum_{\substack{l=1 \\ \lambda_l \neq 0}}^{Z} \sum_{j=1}^{NK_l} \frac{A_l^{(j)}}{(1 - iw\lambda_l)^j} \right) e^{-iwx} dw dx. \quad (39)$$

Using the same equations in [8] that were used to obtain (32), the resulting bit error probability is given by

$$P_e = \sum_{\substack{l=1 \\ \lambda_l < 0}}^{Z} \sum_{j=1}^{NK_l} A_l^{(j)}. \quad (40)$$

For the case of $N = 2$ and distinct eigen values,

$$A_j^{(1)} = \frac{-2}{\lambda_j} \sum_{\substack{k=1 \\ k \neq j \\ \lambda_k \neq 0}}^{P} \frac{\frac{1}{\lambda_k^2}}{\left(\frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right)^3} \prod_{\substack{m=1 \\ m \neq k \\ m \neq j \\ \lambda_m \neq 0}}^{P} \frac{\frac{1}{\lambda_m^2}}{\left(\frac{1}{\lambda_m} - \frac{1}{\lambda_j} \right)^2}, \quad (41)$$

and

$$A_j^{(2)} = \prod_{\substack{k=1 \\ k \neq j \\ \lambda_k \neq 0}}^{P} \frac{\frac{1}{\lambda_k}^2}{\left(\frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right)^2}. \quad (42)$$

V. Numerical Results

We computed the analytical bit error rate (BER) performance of the decorrelating and the MF detectors derived in the previous sections for different system parameter settings. We have also evaluated the performance through simulations and found that the analytical results closely match with the simulation results. Fig. 1 shows the analytical BER performance of the decorrelating detector without and with channel estimation errors for $K = 4, 8$ and $L = 1$. The single user case with perfect channel estimates is also shown for comparison. Although with perfect channel estimates the decorrelating detector completely eliminates the MAI and its performance degradation compared to the single user performance is only due to the noise-enhancement [2], with imperfect channel estimates its performance is found to degrade significantly compared to the perfect channel estimates as can be seen from Fig. 1. For example, when the number of users is 8 (i.e., $K = 8$), to achieve a BER of 5×10^{-2}, the imperfect channel estimates case requires about 7 dB more SNR compared to the perfect channel estimates case.

Fig. 2 shows the decorrelating detector performance without and with channel estimation errors for different number of receive antennas (i.e., for $L = 1, 2, 3$) for $K = 4$. It can be seen that while the performance for single receive antenna ($L = 1$) degrades with imperfect channel estimates compared to perfect channel estimates case, the performance of two receive antennas ($L = 2$) with imperfect channel estimates is slightly better than that of the single receive antenna ($L = 1$) with perfect channel estimates. For example, at an SNR of 20 dB, the BER for $L = 1$ *with perfect* channel estimates is about 3×10^{-3}, whereas the BER for $N = 2$ *with imperfect* channel estimates is about 2×10^{-3}. This implies that the degradation in performance due to imperfect channel estimation can be compensated by using more number of receive antennas.

Fig. 3 shows the performance of both the decorrelating detector as well as the MF detector without and with channel estimation errors for $K = 4$ and $L = 2$. It can be observed that at low SNRs the performance of the decorrelating detector with imperfect channel estimates is worse than the MF detector performance with perfect channel estimates. At high SNRs, however, the decorrelating detector with imperfect channel

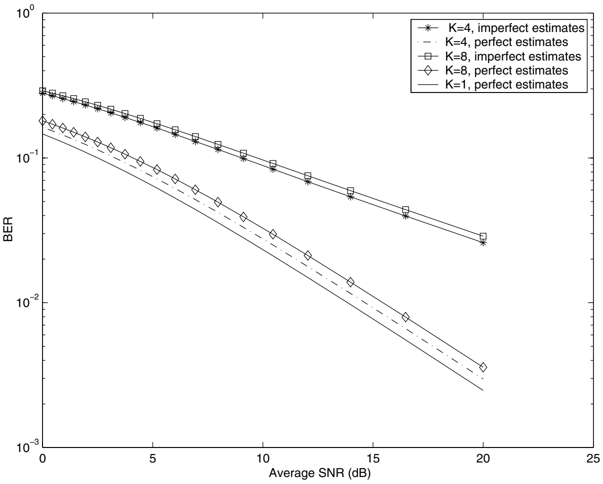

Fig. 1. BER performance of the decorrelating detector without and with channel estimation errors for $K = 4, 8$, $L = 1$.

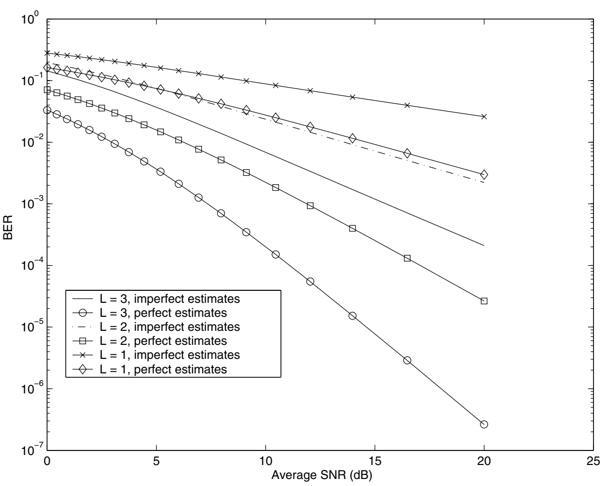

Fig. 2. BER performance of the decorrelating detector without and with channel estimation errors for $K = 4$, $L = 1, 2$, and 3.

estimates performs better than the MF with perfect channel estimates because of the high error floor experienced by the MF detector due to the MAI from other users.

VI. Conclusion

We presented the performance analysis of the decorrelating detector and the conventional matched filter detector in multiuser systems with diversity reception and imperfect channel estimation. We analytically quantified the degradation in the bit error performance of the decorrelating detector and the matched filter detector when the channel estimates in a pilot-based channel estimation scheme is imperfect. We showed that, while the imperfect channel estimates degrade performance compared to perfect channel estimates, the degradation in the bit error performance can be compensated by using more number of receive antennas.

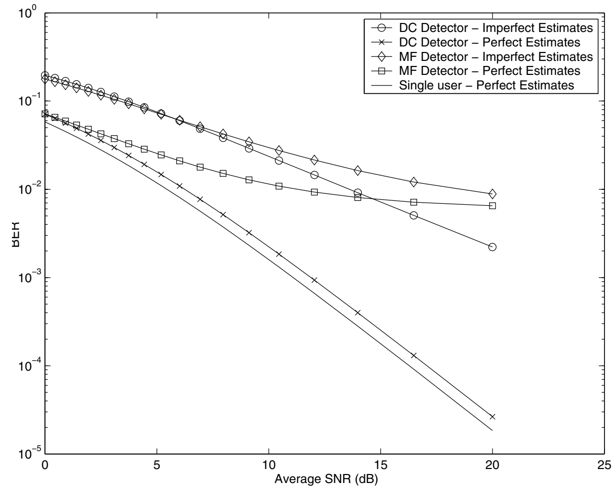

Fig. 3. BER performance of the decorrelating detector and the MF detector without and with channel estimation errors $K = 4$, $L = 2$.

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