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Precoder Optimization in Cognitive Radio with Interference Constraints

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***Abstract*—We consider precoding strategies at the secondary base station (SBS) in a cognitive radio network with interference constraints at the primary users (PUs). Precoding strategies at the SBS which satisfy interference constraints at the PUs in cognitive radio networks have not been adequately addressed in the literature so far. In this paper, we consider two scenarios: i) when the primary base station (PBS) data is not available at SBS, and ii) when the PBS data is made available at the SBS. We derive the optimum MMSE and Tomlinson-Harashima precoding (THP) matrix filters at the SBS *which satisfy the interference constraints at the PUs* for the former case. For the latter case, we propose a precoding scheme at the SBS which performs *pre-cancellation of the PBS data*, followed by THP on the pre-cancelled data. The optimum precoding matrix filters are computed through an iterative search. To illustrate the robustness of the proposed approach against *imperfect CSI at the SBS*, we then derive robust precoding filters under imperfect CSI for the latter case. Simulation results show that the proposed optimum precoders achieve good bit error performance at the secondary users while meeting the interference constraints at the PUs.**

keywords: Cognitive radio, interference constraints, precoding, pre-cancellation, primary/secondary base stations, primary/secondary users.

I. Introduction

Growth in high data rate wireless applications and services is steadily driving the spectrum demand. Cognitive radio (CR) techniques are widely recognized as powerful means to enhance the utilization efficiency of the allotted spectrum [1]–[4]. Three different CR paradigms, namely, *underlay model*, *overlay model*, and *interweave model* are being researched [1]. In underlay model, the secondary network does not have the knowledge of the primary network data, whereas, in overlay model, secondary network has the knowledge of primary network data which can be exploited for transmit side pre-processing at the secondary network. In this paper, we consider both underlay and overlay CR models, and propose precoding schemes at the secondary network which satisfy interference constraints at the primary network.

Interference management techniques at the secondary network through sophisticated transmit side signal processing is getting increased attention [5]–[7]. In [5], optimal power control policies considering interference power constraint in terms of maximizing the ergodic capacity of the secondary user (SU) are studied. In [6], robust linear precoders are proposed for multiple-input single output (MISO) CR networks based on worst-case design criteria. Robust transceiver optimization in MISO CR networks, aiming at minimizing the worst-case per-user mean square error is reported in [7]. In the above studies, the primary network interference to the SUs is ignored.

In this paper, we consider a CR network where there are multiple primary users (PU) served by a primary base station (PBS), multiple SUs served by a secondary base station (SBS), and transmissions to PUs and SUs occur simultaneously (as shown in Fig. 1). In this scenario, while the primary network (which serves licensed users) would perform the normal signal processing at the transmitter and receivers, the burden of limiting the interference from the secondary network to the PUs and ensuring good error performance at the SUs is left to the SBS. We propose to achieve this objective through efficient transmit precoding strategies implemented at the SBS.

We consider two different scenarios: i) when the PBS data is not available at SBS (underlay model), and ii) when the PBS data is made available at the SBS (overlay model). We derive optimum minimum mean squared error (MMSE) and Tomlinson-Harashima precoding (THP) matrix filters at the SBS which satisfy the interference constraints at the PUs for the underlay case. For the overlay case, we propose a precoding strategy at the SBS based on two key ideas: a) performing pre-cancellation of PBS data at the SBS, and b) performing THP on the pre-cancelled data to limit the increase in signal power due to pre-cancellation. We derive optimum THP matrix filters at the SBS which satisfy the interference constraints at the PUs. The optimum precoding filters are computed through an iterative search. To our knowledge, THP precoder optimization at the SBS with PBS data pre-cancellation along with interference constraints at the PUs has not been reported so far.

Channel state information (CSI) is crucial in CR precoding. In particular, CSI can be imperfect due to estimation errors, feedback errors and feedback delays. Precoder designs that do not take into account the errors/uncertainties in CSI will fail to meet the design targets in the presence of imperfections in CSI. Robust optimization methods, which take into account the CSI imperfections by way of incorporating the CSI error variance or the size of uncertainty region in the optimization, can alleviate this problem. In this paper, we present a robust optimization of the THP precoder in the overlay model and illustrate the robustness of the proposed approach in the presence of imperfect CSI at the SBS.

The rest of this paper is organized as follows. The system model is presented in Section II. The proposed optimum precoders for underlay and overlay models are presented in

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Sections III and IV, respectively. Robust optimization of THP with imperfect CSI in overlay model is also presented in Section IV. Simulation results and discussions are presented in Section V. Conclusions are presented in Section VI.

II. System Model

We consider a CR network shown in Fig. 1. A PBS having R transmit antennas serves N PUs on the downlink, $R \geq N$. The PUs are equipped with one receive antenna each. In addition, a SBS having M transmit antennas communicates with K SUs on its downlink, $M \geq K$. The SUs are also assumed to have one receive antenna each[1].

Let $\mathbf{x}_p = [x_{p1}, x_{p2}, \cdots, x_{pN}]^T$ denote[2] the data symbol vector that needs to be communicated from the PBS to the PUs, where x_{pi} is the data symbol intended for the ith PU, and let $\mathbf{R}_{\mathbf{x}_p} = \mathbb{E}\ [\mathbf{x}_p\mathbf{x}_p^H] = \sigma^2_{\mathbf{x}_p}\mathbf{I}_N$ Likewise, let $\mathbf{x}_s = [x_{s1}, x_{s2}, \cdots, x_{sK}]^T$ denote the data symbol vector that SBS wants to send to the SUs, where x_{si} is the ith SU's data symbol and let $\mathbf{R}_{\mathbf{x}_s} = \mathbb{E}\ [\mathbf{x}_s\mathbf{x}_s^H] = \sigma^2_{\mathbf{x}_s}\mathbf{I}_K$.

The PBS transmits $\mathbf{x}_p$ using MMSE precoding[3] which is received by the PUs. On the other hand, the SBS performs linear or non-linear precoding subject to a constraint on the interference caused to the PUs.

Let $\mathbf{H}_{pp} = [h_{ij}]$ denote the $N \times R$ channel gain matrix from the PBS to the PUs, where h_{ij} is the channel gain from the jth transmit antenna of the PBS, $j = 1, 2, \cdots, R$, to the ith PU's receive antenna, $i = 1, 2, \cdots, N$. The channel gains are assumed to be independent circularly symmetric complex Gaussian (CSCG) random variables with variance $\sigma^2_{\mathbf{H}_{pp}}$, i.e., $\mathcal{CN}(0, \sigma^2_{\mathbf{H}_{pp}})$. Similarly, let $\mathbf{H}_{ps}$ denote the $K \times R$ channel matrix from the PBS to the SUs, with channel variance $\sigma^2_{\mathbf{H}_{ps}}$. Further, let $\mathbf{H}_{sp}$ of size $N \times M$ denote the channel matrix from SBS to PUs, and $\mathbf{H}_{ss}$ of size $K \times M$ denote the channel matrix from SBS to SUs, with channel variances $\sigma^2_{\mathbf{H}_{sp}}$ and $\sigma^2_{\mathbf{H}_{ss}}$, respectively. We assume that $\mathbf{H}_{pp}$ is known at the PBS. $\mathbf{H}_{pp}$ can be estimated by the PUs and fed back to the PBS. We further assume that $\mathbf{H}_{sp}$ and $\mathbf{H}_{ss}$ are known at the SBS. $\mathbf{H}_{ss}$ can be estimated by the SUs and fed back to the SBS. In systems where channel reciprocity holds, $\mathbf{H}_{sp}$ can be estimated by the SBS using the transmissions from the PUs. Initially, we assume perfect knowledge of CSI, and later we relax this assumption in Sec. IV-A.

Let $\mathbf{W}_p$ denote the $R \times N$ MMSE precoding matrix at the PBS, under transmit power constraint $P_p = \text{tr}(\mathbf{W}_p\mathbf{R}_{\mathbf{x}_p}\mathbf{W}_p^H)$. Let $\mathbf{z}_p = \mathbf{W}_p\mathbf{x}_p$ denote the $R \times 1$ vector transmitted from the PBS. Under these assumptions, the interference caused at the SUs due to the PBS transmission is $\mathbf{H}_{ps}\mathbf{W}_p\mathbf{x}_p$.

[1]The precoding strategies we propose in this paper can be extended to the case of multiple receive antennas at the PUs and SUs as well.

[2]We use the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[\cdot]^T$ denotes the transpose operation, $[\cdot]^H$ denotes the Hermitian operation, $\text{tr}(\cdot)$ denotes the trace operation, and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. $\mathbf{I}_n$ denotes $n \times n$ identity matrix.

[3]Although we consider MMSE precoding at the PBS, other precoding strategies like ZF and THP at the PBS can also be considered.

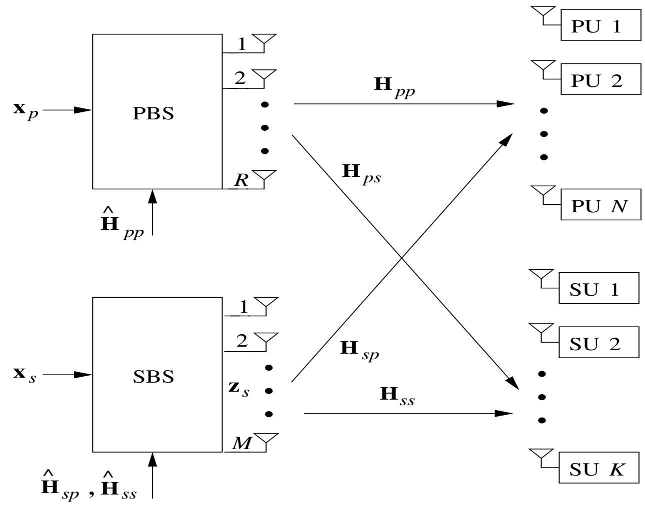

Fig. 1. Cognitive network system model.

III. Proposed SBS Precoders for Underlay Model

The $K \times 1$ received vector $\mathbf{y}_s$ at the SUs, and the $N \times 1$ received vector $\mathbf{y}_p$ at the PUs, can be written as

$$\mathbf{y}_s = \mathbf{H}_{ss}\mathbf{W}_s\mathbf{x}_s + \underbrace{\mathbf{H}_{ps}\mathbf{W}_p\mathbf{x}_p}_{interference\, from\, PBS} + \mathbf{n}_s, \quad (1)$$

$$\mathbf{y}_p = \mathbf{H}_{pp}\mathbf{W}_p\mathbf{x}_p + \underbrace{\mathbf{H}_{sp}\mathbf{W}_s\mathbf{x}_s}_{interference\, from\, SBS} + \mathbf{n}_p, \quad (2)$$

where $\mathbf{n}_s$, $\mathbf{n}_p$ are CSCG random noise vectors with covariance matrix $\sigma_n^2\mathbf{I}$. At the SUs, $\mathbf{y}_s$ is scaled by β^{-1}, where β^{-1} represents the scaling factor used at the SU. This is then used to form the estimate $\widehat{\mathbf{x}}_s$ of the transmitted vector $\mathbf{x}_s$.

A. MMSE Precoder with Interference Constraints (MMSE-IC)

Let $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_N]^T$, where θ_i is the interference energy constraint at the ith PU. The SBS does MMSE precoding towards the SUs, under a transmit power constraint P_s, such that the interference at the ith PU does not exceed θ_i. Since the knowledge of $\mathbf{H}_{ps}\mathbf{W}_p\mathbf{x}_p$ is not known at SBS, we can consider the expectation of the MSE w.r.t $\mathbf{H}_{ps}$ and $\mathbf{x}_p$. The MSE can be expressed as

$$\begin{aligned}\mathbb{E}_{\mathbf{H}_{ps}}\left[\|\mathbf{x}_s - \beta^{-1}\mathbf{y}_s\|^2\right] = {} & \beta^{-2}(\text{tr}(\mathbf{H}_{ss}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H\mathbf{H}_{ss}^H \\ & + \mathbf{R}_{\mathbf{n}_s}) + KP_p\,\sigma^2_{\mathbf{H}_{ps}}) + K\sigma^2_{\mathbf{x}_s} \\ & - 2\beta^{-1}\Re(\text{tr}(\mathbf{H}_{ss}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s})), \quad (3)\end{aligned}$$

where $\mathbf{R}_{\mathbf{n}_s} = \mathbb{E}\ [\mathbf{n}_s\mathbf{n}_s^H]$.

Now, the interference seen at the ith PU from the SBS is given by $[\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{ii}$, where $[\mathbf{X}]_{ij}$ denotes the i,jth entry of the matrix $\mathbf{X}$. The optimization problem under MMSE-IC can be formulated as follows:

$$\begin{aligned}\{\mathbf{W}_s^{opt}, \beta_{opt}\} = {} & \underset{\{\mathbf{W}_s,\ \beta\}}{\arg\min}\ \mathbb{E}_{\mathbf{H}_{ps}}\left[\|\mathbf{x}_s - \beta^{-1}\mathbf{y}_s\|^2\right] \\ \text{s.t.} \quad & \text{tr}(\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H) = P_s \\ & [\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{jj} < \theta_j \ \ j = 1, \cdots, N. \quad (4)\end{aligned}$$

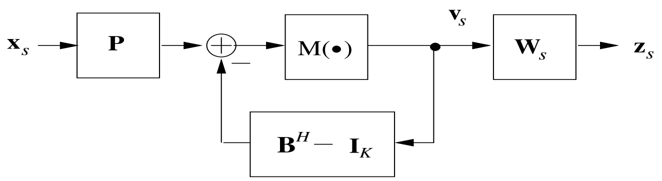

Fig. 2. Proposed THP precoder at the SBS with interference constraints.

The above problem represents an inequality constrained optimization problem. The interference constraint at the ith PU is said to be an active or a binding constraint, if $[\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{ii} > \theta_i$. Otherwise, it is called an inactive constraint. The inactive constraints can be ignored without affecting the optimum solution [11]. In solving the above optimization problem, we adopt the following strategy. A normal MMSE filter is first derived without the interference constraints [8], and is used to check whether the constraint at each PU is active. If all the constraints are inactive, then the normal MMSE filter is used in the precoding process. On the contrary, if the interference constraints are active at certain PUs, then only the set of active constraints are considered for the optimization problem. The inequality constraint in (4) can be converted to equality constraint, by setting $[\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{x}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{ii} = \theta_i$, for the PUs at which the constraint is active [11]. In the following, we derive the optimum filter under the assumption that the interference constraint at each PU is active. The extension to the case when the constraint is active at only a subset of the PUs is straightforward. By the method of Lagrangian multipliers, the solution to the above problem is given by

$$\tilde{\mathbf{W}}_s = (\mathbf{H}_{ss}^H\mathbf{H}_{ss} + \rho\mathbf{I}_M + \mathbf{H}_{sp}^H\mathbf{\Lambda}\mathbf{H}_{sp})^{-1} \quad (5)$$

$$\mathbf{W}_s^{opt} = \beta_{opt}\tilde{\mathbf{W}}_s, \quad (6)$$

$$\beta_{opt} = P_s/\mathrm{tr}(\tilde{\mathbf{W}}_s\mathbf{R}_{\mathbf{x}_s}\tilde{\mathbf{W}}_s^H), \quad (7)$$

where $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N)$ is a $N \times N$ diagonal matrix with i,ith entry equal to λ_i. ρ, $\lambda_1, \cdots, \lambda_N \in \mathbb{R}$ are scalar multipliers, such that the following conditions are satisfied:

$$\rho P_s + \sum_{i=1}^{N} \lambda_i \theta_i = \mathrm{tr}(\mathbf{R}_{\mathbf{n}_s}) + KP_p\,\sigma^2_{\mathbf{H}_{ps}}, \quad (8)$$

$$\lambda_i \geq 0, \; i = 1, \cdots, N. \quad (9)$$

Let $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]^T$. Unlike in a normal MMSE filter, here ρ and $\boldsymbol{\lambda}$ have to be found numerically. We obtain $\boldsymbol{\lambda}$ through a parallel bisection search till the interference constraints are satisfied at all PUs, and use it to compute ρ. These are used to compute the optimum filter $\mathbf{W}_s^{opt}$ in (6) and β_{opt} in (7).

B. THP Precoder with Interference Constraints (THP-IC)

Next, we propose a THP precoder at the SBS with interference constraints. The proposed scheme preforms THP precoding on the signal vector $\mathbf{x}_s$. The THP precoder comprises of a permutation matrix $\mathbf{P}$ of size $K \times K$, a backward matrix filter $\mathbf{B}$ of size $K \times K$, and a feedforward matrix filter $\mathbf{W}_s$ of size $M \times K$, as shown in Fig. 2. After permutation by $\mathbf{P}$, the signal vector is iteratively filtered by $\mathbf{B}^H$, which is a $K \times K$ lower triangular matrix with unit diagonal entries. It is then followed by the modulo operator to form the $K \times 1$ signal vector $\mathbf{v}_s$. The modulo operation is done to reduce the signal power increase due to the filtering with $\mathbf{B}^H$. The modulo operator for a complex variable c is defined as

$$\mathrm{M}(c) = c - \lfloor \Re(c)/a + 1/2 \rfloor a - j\lfloor \Im(c)/a + 1/2 \rfloor a, \quad (10)$$

where $j = \sqrt{-1}$, and a depends on the constellation; e.g., $a = 2\sqrt{2}$ for QPSK symbols [10]. Note that each symbol in $\mathbf{v}_s$ is an element of the set $\mathbb{M} = \{x+jy \mid x, y \in [-a/2, a/2)\}$. It is popularly assumed that both the real and imaginary parts of each symbol in $\mathbf{v}_s$ are distributed uniformly resulting in a variance of $\sigma^2_{\mathbf{v}_s} = a^2/6$. Since the modulo operator is not applied to the first component of $\mathbf{v}_s$, it has a variance of $\sigma^2_{\mathbf{x}_s}$. Consequently, $\mathbf{R}_{\mathbf{v}_s} = \mathbb{E}\,[\mathbf{v}_s\mathbf{v}_s^H]$ is given by $\mathbf{R}_{\mathbf{v}_s} = \mathrm{diag}(\sigma^2_{\mathbf{x}_s}, \sigma^2_{\mathbf{v}_s}, \cdots, \sigma^2_{\mathbf{v}_s})$. The vector $\mathbf{v}_s$ is filtered by the $M \times K$ feedforward matrix $\mathbf{W}_s$, and the resulting $M \times 1$ output vector $\mathbf{W}_s\mathbf{v}_s$ is transmitted using M transmit antennas. At the SUs, $\mathbf{y}_s$ is scaled by β^{-1} and then the modulo operator is applied to form the signal vector $\mathbf{r}_s$, i.e., $\mathbf{r}_s = \mathrm{M}(\beta^{-1}\mathbf{y}_s)$. The vector $\mathbf{r}_s$ is then used to form the estimate $\widehat{\mathbf{x}}_s$ of the transmitted vector $\mathbf{x}_s$. In the following subsection, we derive the matrix filters $\mathbf{B}$ and $\mathbf{W}_s$ for our system with interference constraints at PUs.

Optimum Design of Matrix Filters $\mathbf{B}$ *and* $\mathbf{W}_s$: For a given permutation matrix $\mathbf{P}$, we jointly optimize $\mathbf{B}^H$, $\mathbf{W}_s$ and β to minimize the MSE between $\widehat{\mathbf{x}}_s$ and $\mathbf{x}_s$, i.e., $\mathbb{E}\big[\|\mathbf{x}_s - \widehat{\mathbf{x}}_s\|^2\big]$. The presence of the modulo operator in Fig. 2 introduces non-linearity. The modulo operator at the precoder and at the receiver can be removed to get an equivalent linear representation of the system by making use of an auxiliary vector $\mathbf{t}_s$, chosen in such a way that the components of $\mathbf{v}_s$ remain unchanged [8,9]. Specifically, the vector $\mathbf{t}_s$ is introduced at the input of the permutation matrix of the precoder and also at the output of the SUs, as follows. At the precoder side, the vector $\mathbf{t}_s$ is added to $\mathbf{x}_s$ to form the new input vector to the permutation matrix $\mathbf{u}_s$, i.e., $\mathbf{u}_s = \mathbf{x}_s + \mathbf{t}_s$. At the SU receiver side, let $\widehat{\mathbf{u}}_s$ represent the output of the SUs scaled by β^{-1} in the equivalent linear model, i.e., $\widehat{\mathbf{u}}_s = \beta^{-1}\mathbf{y}_s$. Then, the corresponding modulo operator at the receive side is modeled by subtracting $\mathbf{t}_s$ from $\widehat{\mathbf{u}}_s$ to form the estimate $\widehat{\mathbf{x}}_s$, i.e., $\widehat{\mathbf{x}}_s = \widehat{\mathbf{u}}_s - \mathbf{t}_s$. Using the above equivalent linear model, the vector $\mathbf{v}_s$ can be written in terms of $\mathbf{u}_s$ as $\mathbf{v}_s = \mathbf{P}\mathbf{u}_s - (\mathbf{B}^H - \mathbf{I}_K)\mathbf{v}_s$. Solving for $\mathbf{u}_s$, we get $\mathbf{u}_s = \mathbf{P}^T\mathbf{B}^H\mathbf{v}_s$. Now, the MSE can be equivalently expressed in terms of $\mathbf{u}_s$ and $\widehat{\mathbf{u}}_s$ as

$$\begin{aligned}\mathbb{E}_{\mathbf{H}_{ps}}\big[\|\mathbf{u}_s - \widehat{\mathbf{u}}_s\|^2\big] = \; & \beta^{-2}(\mathrm{tr}(\mathbf{H}_{ss}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H\mathbf{H}_{ss}^H + \mathbf{R}_{\mathbf{n}_s}) \\ & + KP_p\,\sigma^2_{\mathbf{H}_{ps}}) + \mathrm{tr}(\mathbf{P}^T\mathbf{B}^H\mathbf{R}_{\mathbf{v}_s}\mathbf{B}\mathbf{P}) \\ & - 2\beta^{-1}\Re(\mathrm{tr}(\mathbf{H}_{ss}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{B}\mathbf{P})). \quad (11)\end{aligned}$$

The optimum matrix filters $\mathbf{B}$ and $\mathbf{W}_s$ need to satisfy the following constraints: i) the total transmit power at the SBS should be equal to P_s, ii) the feedback filter $\mathbf{B}^H$ must be unit lower triangular, so that $(\mathbf{B}^H - \mathbf{I}_K)$ is strictly lower triangular, which is necessary to ensure causality of the feedback loop, and iii) the interference at the ith PU should not exceed θ_i.

The optimization problem under the above constraints can be formulated as

$$\begin{aligned} \{\mathbf{W}_s^{opt}, \mathbf{B}_{opt}^H, \beta_{opt}\} &= \underset{\{\mathbf{W}_s, \mathbf{B}^H, \beta\}}{\arg\min} \ \mathbb{E}_{\mathbf{H}_{ps}}\left[\|\mathbf{u}_s - \widehat{\mathbf{u}}_s\|^2\right] \\ \text{s.t.} \quad & \text{tr}(\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H) = P_s \\ & \mathbf{S}_i \ (\mathbf{B}^H - \mathbf{I}_K)\mathbf{e}_i = \mathbf{0}_i \ \ i = 1, \cdots, K \\ & [\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{jj} < \theta_j \ \ j = 1, \cdots, N, \end{aligned} \quad (12)$$

where $\mathbf{e}_i$ is the ith column of $\mathbf{I}_K$, and $\mathbf{S}_i = [\mathbf{I}_i, \mathbf{0}_{i \times K-i}]$ is the $i \times K$ selection matrix. The above problem represents an inequality constrained optimization problem, and we use the technique developed in Sec. III-A to convert the problem to an equality constrained one. In the following, it is assumed that the interference constraint is active at each PU.

For a given $\mathbf{P}$, by the method of Lagrangian multipliers, the solution to the equality constraint problem is

$$\mathbf{B}_{opt}^H = \sum_{i=1}^{K} \mathbf{P}\mathbf{A}\mathbf{P}^T\mathbf{S}_i^T(\mathbf{S}_i\mathbf{P}\mathbf{A}\mathbf{P}^T\mathbf{S}_i^T)^{-1}\mathbf{S}_i\mathbf{e}_i\mathbf{e}_i^T, \quad (13)$$

$$\tilde{\mathbf{W}}_s = \sum_{i=1}^{K} \mathbf{C}\mathbf{H}_{ss}^H\mathbf{A}\mathbf{P}^T\mathbf{S}_i^T(\mathbf{S}_i\mathbf{P}\mathbf{A}\mathbf{P}^T\mathbf{S}_i^T)^{-1}\mathbf{S}_i\mathbf{e}_i\mathbf{e}_i^T, \quad (14)$$

$$\mathbf{W}_s^{opt} = \beta_{opt}\tilde{\mathbf{W}}_s, \quad (15)$$

$$\beta_{opt} = P_s/\text{tr}(\tilde{\mathbf{W}}_s\mathbf{R}_{\mathbf{v}_s}\tilde{\mathbf{W}}_s^H), \quad (16)$$

where $\mathbf{C} = (\mathbf{H}_{ss}^H\mathbf{H}_{ss} + \rho\mathbf{I}_M + \mathbf{H}_{sp}^H\mathbf{\Lambda}\mathbf{H}_{sp})^{-1}$ and $\mathbf{A} = (\mathbf{I}_K - \mathbf{H}_{ss}\mathbf{C}\mathbf{H}_{ss}^H)^{-1}$. ρ, $\mathbf{\Lambda}$ satisfy (8), (9) and are computed numerically as described in Sec. III-A. These are used to compute the optimum filters $\mathbf{B}_{opt}^H$, $\mathbf{W}_s^{opt}$ in (13), (15), respectively, and β_{opt} in (16). We refer to the above precoding as THP-IC.

Optimization over $\mathbf{P}$*:* In the above, we have obtained optimum $\mathbf{B}_{opt}^H$, $\mathbf{W}_s^{opt}$ and β_{opt} by minimizing the MSE for a fixed $\mathbf{P}$ with interference constraint $\boldsymbol{\theta}$. The MSE can be further reduced by optimizing over $\mathbf{P}$. Two efficient algorithms that optimize over $\mathbf{P}$ in a normal THP have been reported in [8,9]. However, the algorithm proposed in [9] cannot be used in our problem for the following reason. We find through simulations that, for values of θ_i of practical interest, $\rho \in \mathbb{R}^+$. For these values of ρ, $\mathbf{C}$ is positive definite, which makes $\mathbf{A}$ a non-positive definite matrix. Consequently, the algorithm in [9], which requires the corresponding $\mathbf{A}$ to be positive definite to solve for the normal THP filters, cannot be applied. Hence, we adopt the algorithm in [8] for optimizing over $\mathbf{P}$.

IV. Proposed SBS Precoders for Overlay Model

In this section, we assume that PBS transmit vector $\mathbf{z}_p = \mathbf{W}_p\mathbf{x}_p$ is non-causally made available at the SBS. The SBS pre-cancels $\mathbf{z}_p$ vector and performs THP precoding on the pre-cancelled data, subject to a constraint on the interference caused to the PUs. We further assume that in addition to $\mathbf{H}_{sp}$ and $\mathbf{H}_{ss}$, the channel matrix from the PBS to the SUs, $\mathbf{H}_{ps}$ is also known at the SBS. $\mathbf{H}_{ps}$ can be estimated by the SUs and fed back to the SBS.

The proposed scheme preforms THP precoding on the signal vector $\tilde{\mathbf{x}}_s$, which is formed by pre-subtracting the interference at the SUs due to PBS transmission, as

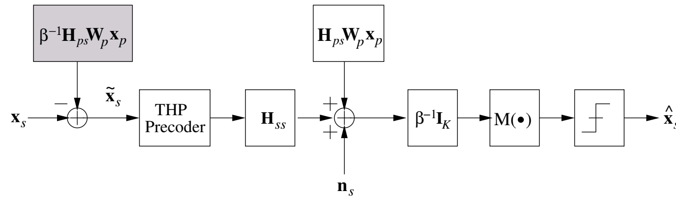

Fig. 3. THP at SBS with PBS data pre-cancellation (overlay model).

$$\tilde{\mathbf{x}}_s = \mathbf{x}_s - \beta^{-1}\mathbf{H}_{ps}\mathbf{z}_p, \quad (17)$$

The PBS interference is scaled by β^{-1} in (17) to compensate for the increase in signal power introduced by the feedforward filter $\mathbf{W}_s$. Unlike in a normal THP, due to PBS interference pre-subtraction in the proposed scheme, the modulo operator has to be applied to the first component of $\mathbf{v}_s$ also, which results in an equal variance of $\sigma_{\mathbf{v}_s}^2$ for all the components. Consequently, $\mathbf{R}_{\mathbf{v}_s} = \mathbb{E}\left[\mathbf{v}_s\mathbf{v}_s^H\right] = \sigma_{\mathbf{v}_s}^2\mathbf{I}_K$.

Thus, instead of $\mathbf{x}_s$, $\tilde{\mathbf{x}}_s$ is now fed to the permutation matrix $\mathbf{P}$ in Fig. 2. Apart from this difference, the design of the optimum filters is on the same lines as the one derived in Sec. III-B, and the optimum filters are given by (13), (15) and (16), with the exception that, ρ and $\mathbf{\Lambda}$ satisfy the following equations

$$\rho\, P_s + \sum_{i=1}^{N} \lambda_i\, \theta_i = \text{tr}(\mathbf{R}_{\mathbf{n}_s}), \quad (18)$$

$$\lambda_i \geq 0, \ i = 1, \cdots, N. \quad (19)$$

We refer to this scheme as THP-IC-PC (THP-IC with pre-cancellation). We provide below the analysis for the THP with PBS data pre-cancellation, where $\mathbf{X}^{(k)}$ denotes the kth column of the matrix $\mathbf{X}$. Let $q_{k,j} = \beta^{-1}\big(\mathbf{H}_{ss}^{H(k)}\big)^H\mathbf{W}_s^{(j)}v_{sj}$, $t_j = (\mathbf{B}^H - \mathbf{I}_K)^{(j)}v_{sj}$ and $i_p = \beta^{-1}\big(\mathbf{H}_{ps}^{H(k)}\big)^H\mathbf{z}_p$. The received symbol by the kth SU is

$$\begin{aligned} r_{sk} &= (q_{k,k} + \sum_{j\neq k} q_{k,j} + i_p + \beta^{-1}n_{sk}) \text{mod } a \\ &= (q_{k,k} + \sum_{j\neq k} q_{k,j} + i_p + \beta^{-1}n_{sk} + \sum_{j<k} t_j - \sum_{j<k} t_j) \text{mod } a \\ &= (q_{k,k} + \sum_{j\neq k} q_{k,j} + i_p + \beta^{-1}n_{sk} + \tilde{x}_{sk} - v_{sk} - \sum_{j<k} t_j) \text{mod } a \end{aligned}$$

where a is defined in (10) and $(\sum_{j<k} t_j)$ mod $a = (\tilde{x}_{sk} - v_{sk})$ mod a. Since $q_{k,k} \approx v_{sk}$ and using (17), the above equation can be further simplified as

$$r_{sk} = (x_{sk} + \sum_{j\neq k} q_{k,j} - \sum_{j<k} t_j + \beta^{-1}n_{sk}) \text{ mod } a \quad (20)$$

From the above equation, it can be seen that the interference from the PBS at the kth SU is cancelled out due to pre-cancellation at the SBS.

A. Robust Design of Optimum Filters with Imperfect CSI

In this subsection, we address the issue of imperfect CSI at the SBS. We consider the following stochastic error model for the CSI error. The true channel matrix $\mathbf{H}_{xy}$, is represented as

$$\mathbf{H}_{xy} = \widehat{\mathbf{H}}_{xy} + \mathbf{E}_{xy}, \qquad x, y \in \{p, s\}, \quad (21)$$

where $\widehat{\mathbf{H}}_{xy}$ is the estimated CSI and $\mathbf{E}_{xy}$ is the CSI error matrix. $\mathbf{E}_{xy}$ is assumed to be Gaussian distributed with zero mean and $\mathbb{E}[\mathbf{E}_{xy}\mathbf{E}_{xy}^H] = \sigma^2_{\mathbf{E}_{xy}}\mathbf{I}$ where $\mathbf{I}$ is an identity matrix of appropriate dimension. This statistical model is suitable for systems with uplink-downlink reciprocity. We further assume that the SBS has perfect knowledge of $\widehat{\mathbf{H}}_{ss}$ but has only the imperfect estimates $\widehat{\mathbf{H}}_{ps}$ and $\widehat{\mathbf{H}}_{sp}$.

The residual error in $\mathbf{r}_s$, neglecting the modulo loss is $\mathbf{E}_{ps}\mathbf{z}_p$. Since $\mathbf{E}_{ps}$ is a random matrix, the MSE is a random variable, and the expectation of the MSE is minimized in such cases. Now, the expected value of the MSE is same as (11) with the term $KP_p\,\sigma^2_{\mathbf{H}_{ps}}$ replaced by $KP_p\,\sigma^2_{\mathbf{E}_{ps}}/R$.

Due to uncertainty in the knowledge of $\mathbf{H}_{sp}$ at the SBS, the exact interference caused to the PUs by the SBS is not known. In such cases, the expected value of the interference caused to the PUs is made to be below a threshold. Taking the expected value of the interference caused to the ith PU w.r.t $\mathbf{E}_{sp}$

$$\mathbb{E}_{\mathbf{E}_{sp}}[\mathbf{H}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H\mathbf{H}_{sp}^H]_{ii} = [\widehat{\mathbf{H}}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H\hat{\mathbf{H}}_{sp}^H]_{ii} + P_s\sigma^2_{\mathbf{E}_{sp}}/M \qquad (22)$$

The optimization problem with considered CSI error model can be formulated similar to (12) with the third constraint replaced by the following constraint

$$[\widehat{\mathbf{H}}_{sp}\mathbf{W}_s\mathbf{R}_{\mathbf{v}_s}\mathbf{W}_s^H\hat{\mathbf{H}}_{sp}^H]_{jj} \;<\; \theta'_j \; j = 1,\cdots,N, \quad (23)$$

where $\theta'_j = \theta_j - P_s\sigma^2_{\mathbf{E}_{sp}}/M$. The optimum filters are given by (13), (15) and (16). ρ and $\mathbf{\Lambda}$ satisfy (8), (9) with the term $KP_p\,\sigma^2_{\mathbf{H}_{ps}}$ replaced by $KP_p\,\sigma^2_{\mathbf{E}_{ps}}/R$ in (8) and are computed numerically as described in Sec. III-A.

V. Simulation Results

In this section, we present the simulated bit error rate (BER) performance results at the SUs and interference energy at the PUs, achieved by the proposed precoding schemes. The average signal-to-noise ratio (SNR) at the SU receiver with a transmit energy constraint P_s at the SBS is $P_s/\text{tr}(\mathbf{R}_{\mathbf{n}_s})$, and the SNR at the PU receiver with a transmit energy constraint P_p at the PBS is $P_p/\text{tr}(\mathbf{R}_{\mathbf{n}_p})$, where $\mathbf{R}_{\mathbf{n}_p} = \mathbb{E}\,[\mathbf{n}_p\mathbf{n}_p^H]$. PU SNRs are fixed at 6 dB and modulation employed is QPSK. We consider a licensed primary network consisting of $N = 2$ PUs, and a PBS with $R = 2$ transmit antennas. The secondary network consists of a SBS equipped with $M = 4$ antennas, and $K = 2$ SUs. The channel variances are taken to be $\sigma^2_{\mathbf{H}_{pp}} = \sigma^2_{\mathbf{H}_{ss}} = \sigma^2_{\mathbf{H}_{ps}} = \sigma^2_{\mathbf{H}_{sp}} = 1$.

On the choice of θ_i: Let $SINR_i$ denote the received signal-to-interference-plus-noise ratio (SINR) at the ith PU with simultaneous transmissions from PBS and SBS. Let SNR_i denote the received SNR at the ith PU when the secondary network is absent. What constitutes an acceptable level of interference to the PUs can be the system operator's choice. One possible way of choosing θ_i would be to put a condition on the $SINR_i$. Here, we stipulate that at ith PU, the $SINR_i \geq 0.75\,SNR_i$, i.e., there is at most a degradation of 25% in the value of SNR_i. Solving for θ_i at the ith PU yields, $\theta_i \leq \sigma_n^2/3$. For the system parameters considered in the simulation, this evaluates to $\theta_i \approx -4.7$ dB $\forall i$. We will show the SU BER performance and interference caused to the PU at this interference temperature of -4.7 dB.

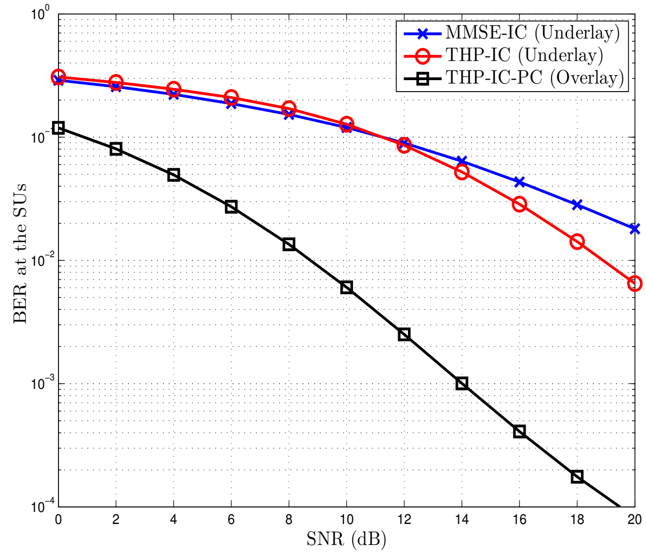

Fig. 4. Secondary User BER performance of the proposed precoding schemes at interference temperature $\theta_i = -4.7$ dB, $\forall i$. PU SNR = 6 dB. $R = N = 2$, $M = 4$, $K = 2$. QPSK modulation.

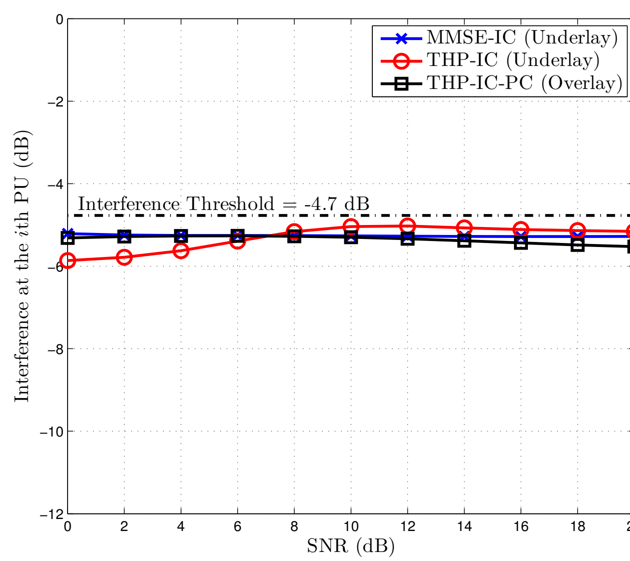

Fig. 5. Interference at the Primary User due to SBS transmission. Interference temperature $\theta_i = -4.7$ dB, $\forall i$. PU SNR = 6 dB. $R = N = 2$, $M = 4$, $K = 2$. QPSK modulation.

In Fig. 4, we plot the SU BER performance of the proposed precoding schemes in the considered system setting. From Fig. 4, it can be seen that the THP-IC performs significantly better than MMSE-IC at high SNRs. Also plotted in this figure is the performance of THP-IC-PC scheme which does PBS data pre-cancellation (overlay). It can be observed that the overlay scheme gives more than an order of magnitude improvement in BER performance over the underlay schemes without pre-cancellation.

In Fig. 5, we have plotted the interference energy at the PUs

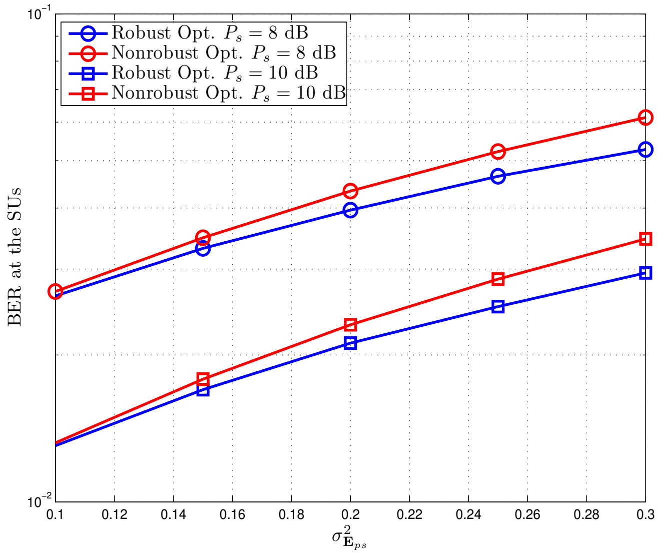

Fig. 6. Secondary User BER performance of robust and nonrobust precoding schemes with imperfect CSI for different values of $\sigma^2_{\mathbf{E}_{ps}}$. Interference temperature $\theta_i = -4.7$ dB, $\forall i$. PU SNR = 6 dB. $\sigma^2_{\mathbf{E}_{sp}} = 0.01$. $R = N = 2$, $M = 4$, $K = 2$. QPSK modulation.

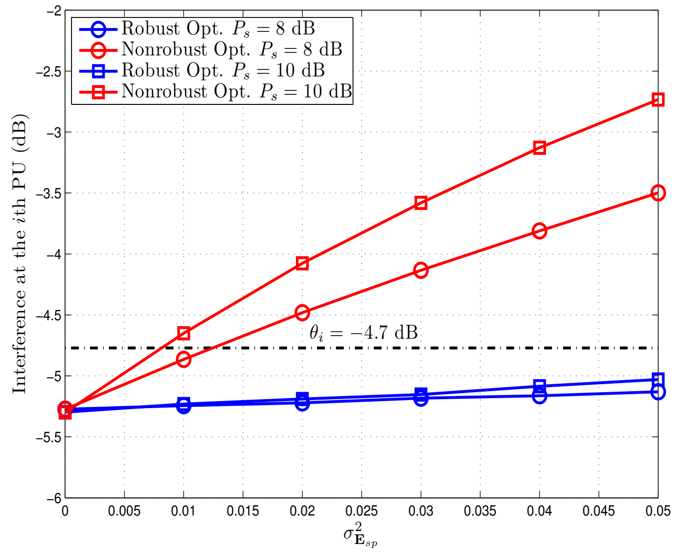

Fig. 7. Interference at the Primary User due to SBS transmission in robust and nonrobust precoding schemes for different values of $\sigma^2_{\mathbf{E}_{sp}}$. $\sigma^2_{\mathbf{E}_{ps}} = 0.1$, interference temperature $\theta_i = -4.7$ dB, $\forall i$. PU SNR = 6 dB. $R = N = 2$, $M = 4$, $K = 2$. QPSK modulation.

caused by the SBS transmission for different operating SNRs at the SUs. It can be seen that both the underlay and overlay schemes (without and with pre-cancellation) result in interference energies below the interference temperature of $\theta_i = -4.7$ dB, validating the proposed interference constrained precoder optimization.

In Figs. 6 and 7, we illustrate the robustness of the THP-IC-PC precoding scheme in the presence of CSI errors at the SBS. The robust precoder optimization is as per Sec. IV-A. The nonrobust precoder optimization is as per Sec. IV. In Fig. 6, we plot the BER at the SU as a function of $\sigma^2_{\mathbf{E}_{ps}}$ at PU SNR = 6 dB and $\sigma^2_{\mathbf{E}_{sp}} = 0.01$. Plots for different SBS transmit power, $P_s = 8$ dB and 10 dB, are shown. From Fig. 6, it can be seen that the proposed robust optimization results in better BER performance at the SU than with the nonrobust optimization. More importantly, in the presence of CSI errors, the PU interference constraints are not met if nonrobust optimization is employed. This can be observed in Fig. 7. Whereas, if the robust optimization is employed, then the PU constraints are comfortably met even in the presence of imperfect CSI (again as seen in Fig. 7).

VI. Conclusions

We investigated the problem of precoder optimization in CR networks for underlay and overlay models. First, in the underlay setting, we formulated the problem of SBS precoder design as an optimization problem with PU interference constraints assuming perfect CSI, and solved them using an iterative search. Next, in the overlay setting, we formulated and solved the SBS precoder optimization problem with pre-cancellation of PBS data, again assuming perfect CSI. While the proposed precoders met the PU interference constraints in both underlay and overlay settings, the BER performance at the SUs was shown to be superior in the overlay setting, which is attributed to the pre-cancellation of PBS data. Finally, we proposed a robust optimization scheme which met the PU interference constraints even in the presence of imperfect CSI, and achieved good SU BER performance as well.

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