Pseudo-random Phase Precoded Spatial Modulation and Precoder Index Modulation

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***Abstract*—Spatial modulation (SM) is a transmission scheme that uses multiple transmit antennas but only one transmit RF chain. At each time instant, only one among the transmit antennas will be active and the others remain silent. The index of the active transmit antenna will also convey information bits in addition to the information bits conveyed through modulation symbols. Pseudo-random phase precoding (PRPP) is a technique that can achieve high diversity orders even in single antenna systems without the need for channel state information at the transmitter and transmit power control. In this paper, we exploit the advantages of both SM and PRPP simultaneously. We propose a *pseudo-random phase precoded SM* (PRPP-SM) scheme, where both the modulation bits and the antenna index bits are precoded by pseudo-random phases. The proposed PRPP-SM system gives significant performance gains over SM system without PRPP and PRPP system without SM. Since maximum likelihood (ML) detection becomes exponentially complex in large dimensions, we propose a low complexity local search based detection (LSD) algorithm suited for PRPP-SM systems with large precoder sizes. Our simulation results show that with 4 transmit antennas, 1 receive antenna, 5×20 pseudo-random phase precoder matrix and BPSK modulation, the performance of PRPP-SM using ML detection is better than SM without PRPP with ML detection by about 9 dB at 10^{-2} BER. This performance advantage gets even better for large precoding sizes. We also propose a *precoder index modulation* (PIM) scheme, which conveys additional information bits through the choice of a precoding matrix among a set of pre-determined PRPP matrices. Finally, combining the PIM and PRPP-SM schemes, we propose a PIM-SM scheme which conveys bits through both antenna index as well as precoder index.**

Keywords *– Multi-antenna systems, spatial modulation, pseudo-random phase precoding, local search based detection.*

I. Introduction

The link reliability in single-input-single-output (SISO) channels is poor due to fading. One way to improve the link reliability is to get diversity gains through the use of multiple antennas. Diversity gains can be achieved even in single-antenna systems using rotation coding [1] or transmit power control (TPC) [2]. TPC requires channel state information at the transmitter (CSIT). Whereas, rotation coding does not require CSIT. The idea in rotation coding is to use multiple channel uses and precode the transmit symbol vector using a phase precoder matrix without requiring more slots than the number of symbols precoded. A 2×2 phase precoder matrix with optimized phases is shown to achieve a diversity gain of two in SISO fading channels [1]. In [3], the rotation coding idea has been exploited for large precoder sizes. Instead of using optimized phases in the precoder matrix (solving for optimum phases for large precoder sizes is difficult), pseudo-random phases are used. Also, the issue of detection complexity at the receiver for large precoder sizes has been addressed by using the low complexity likelihood ascent search (LAS) algorithm in [4]. It has been shown that with pseudo-random phase precoding (PRPP) and LAS detection, near-exponential diversity is achieved in a SISO fading channel for large precoder sizes (e.g., 300×300 precoder matrix).

Recently, spatial modulation (SM) is getting increasingly popular for multi-antenna communications [5],[6]. SM requires less RF hardware, cost, and complexity. An advantage of SM over conventional modulation is that, for a given spectral efficiency, conventional modulation schemes require a larger modulation alphabet size than SM. For example, to achieve 3 bpcu spectral efficiency, conventional modulation with 1 transmit antenna and 1 transmit RF chain requires 8-QAM or 8-PSK, whereas SM with 4 transmit antennas and 1 transmit RF chain requires only BPSK. This possibility of using a smaller modulation alphabet size in SM, in turn, results in SNR gains (for a given probability of error performance) in favor of SM over conventional modulation [7],[8].

Here, we exploit the advantages of both SM and PRPP simultaneously. Our contributions in this paper are as follows.

- We propose a method to precode both the modulation bits and the antenna index bits in SM systems using PRPP. We refer to this system as PRPP-SM system. The novelty here is that while conventional PRPP system uses a square precoding matrix of size $p \times p$ (where p is the number of channel uses), in PRPP-SM we use a rectangular precoding matrix of size $p \times pn_t$ (where n_t is the number of transmit antennas) in order to precode the antenna index bits and the modulation bits.
- For small precoder sizes (e.g., 5×20), we demonstrate using ML detection that the PRPP-SM system significantly outperforms PRPP system (without SM) and SM system (without PRPP), for the same spectral efficiency.
- For large precoding sizes, we propose a low complexity detection algorithm based on local search. The novelty here is a suitable neighborhood definition that takes into account the antenna index bits in the PRPP-SM signals.
- Extending the idea of index modulation, we propose a new scheme, *precoder index modulation* (PIM), which conveys additional information bits through the choice of a precoding matrix among a set of pre-determined PRPP matrices. Then, combining the PIM and PRPP-SM schemes, we propose a PIM-SM scheme which conveys bits through antenna index as well as precoder index.

The rest of the paper is organized as follows. SM and PRPP are introduced in Section II. The PRPP-SM system and the

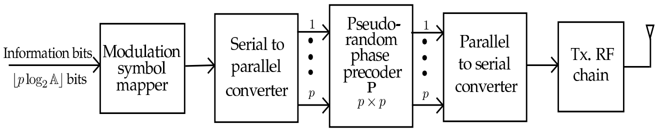


Fig. 1. PRPP transmitter.

local search based detection algorithm are presented in Section III. The PIM and PIM-SM schemes are presented in Section IV. Conclusions are presented in Section V.

II. PRPP and SM systems

In this section, we briefly introduce PRPP and SM systems.

A. PRPP system

Figure 1 shows the PRPP transmitter. It takes p modulated symbols and forms the symbol vector $\mathbf{s} \in \mathbb{A}^p$, where $\mathbb{A}$ is a conventional modulation alphabet. The symbol vector $\mathbf{s}$ is then precoded using a $p \times p$ precoding matrix $\mathbf{P}$ to get the transmit vector $\mathbf{Ps}$. The (r, c)th entry of the precoder matrix $\mathbf{P}$ is $\frac{1}{\sqrt{p}} e^{j\theta_{r,c}}$, where the phases $\{\theta_{r,c}\}$ are generated using a pseudo-random sequence generator. The seed of this random number generator is pre-shared among the transmitter and receiver. The precoded sequence $\mathbf{Ps}$ is transmitted through the channel, which is assumed to be frequency-flat fading. The channel fade coefficients are assumed to be i.i.d from one channel use to the other. At the receiver, after p channel uses, the received symbols are accumulated to form the $p \times 1$ received vector $\mathbf{y}$, given by

$$\mathbf{y} \;=\; \mathbf{DPs} + \mathbf{n} = \mathbf{Gs} + \mathbf{n}, \tag{1}$$

where $\mathbf{D} = \text{diag}\{h_{(1)}\, h_{(2)} \cdots h_{(p)}\}$, $\mathbf{G} = \mathbf{DP}$, $h_{(i)}$s are i.i.d. channel gains distributed as $\mathcal{CN}(0, 1)$, and $\mathbf{n}$ is the $p \times 1$ noise vector $[n_{(1)}\, n_{(2)} \cdots n_{(p)}]^T$, whose entries are distributed as $\mathcal{CN}(0, \sigma^2)$. The entries of $\mathbf{G}$ are uncorrelated and $\|\mathbf{D}\|_F = \|\mathbf{G}\|_F$. This creates a $p \times p$ virtual MIMO system. As the precoder size becomes large (e.g., $p = 300$) the performance of PRPP in SISO fading, using likelihood ascent search (LAS) detector in [4], approaches exponential diversity performance (i.e., close to SISO AWGN performance) [3]. This is illustrated in Fig. 2 which shows the performance of PRPP with BPSK modulation for $p = 50$ and 400 with SISO fading.

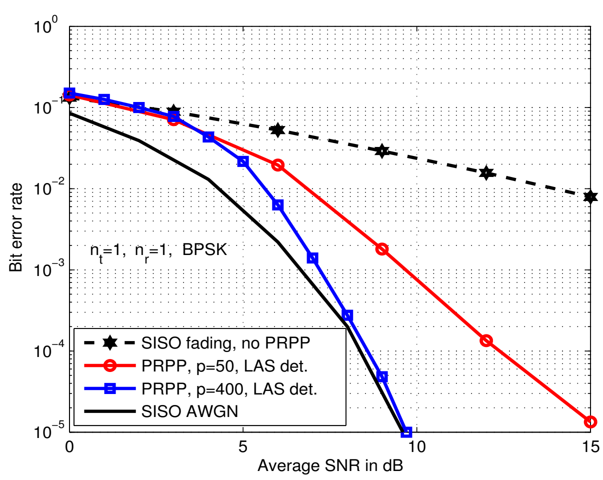


Fig. 2. Performance of PRPP in SISO fading with $p = 50, 400$, BPSK. and LAS detection.

B. SM system

The SM system uses n_t transmit antennas but only one transmit RF chain as shown in Fig. 3. The number of transmit RF chains, $n_{rf} = 1$. In a given channel use, the transmitter selects one of its n_t transmit antennas, and transmits a modulation symbol from the alphabet $\mathbb{A}$ on the selected antenna. The number of bits transmitted per channel use through the modulation symbol is $\lfloor \log_2 |\mathbb{A}| \rfloor$, and the number of bits conveyed per channel use through the index of the transmitting antenna is $\lfloor \log_2 n_t \rfloor$. Therefore, a total of $\lfloor \log_2 |\mathbb{A}| \rfloor + \lfloor \log_2 n_t \rfloor$ bits per channel use (bpcu) is conveyed. For example, in a system with $n_t = 2$, 8-QAM, the system throughput is 4 bpcu.

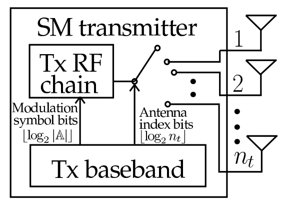


Fig. 3. SM transmitter with n_t antennas and one transmit RF chain.

The SM alphabet set for a fixed n_t and $\mathbb{A}$ is given by

$$\mathbb{S}_{n_t,\mathbb{A}} = \big\{\mathbf{x}_{j,l} : j = 1, \cdots, n_t, \quad l = 1, \cdots, |\mathbb{A}|\big\},$$
$$\text{s.t.} \;\; \mathbf{x}_{j,l} = [0, \cdots, 0, \underbrace{s_l}_{j\text{th coordinate}}, 0, \cdots, 0]^T, \quad s_l \in \mathbb{A}. \tag{2}$$

For example, for $n_t = 2$ and 4-QAM, $\mathbb{S}_{n_t,\mathbb{A}}$ is given by

$$\mathbb{S}_{2,\text{4-QAM}} = \Bigg\{ \begin{bmatrix} +1+j \\ 0 \end{bmatrix}, \begin{bmatrix} +1-j \\ 0 \end{bmatrix}, \begin{bmatrix} -1+j \\ 0 \end{bmatrix}, \begin{bmatrix} -1-j \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ +1+j \end{bmatrix}, \begin{bmatrix} 0 \\ +1-j \end{bmatrix}, \begin{bmatrix} 0 \\ -1+j \end{bmatrix}, \begin{bmatrix} 0 \\ -1-j \end{bmatrix} \Bigg\}. \tag{3}$$

Let $\mathbf{x} \in \mathbb{S}_{n_t,\mathbb{A}}$ denote the transmit vector. Let $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix, where $H_{i,j}$ denotes the channel gain from the jth transmit antenna to the ith receive antenna, assumed to be i.i.d and distributed as $\mathcal{CN}(0, 1)$. The received signal at the ith receive antenna is

$$y_i = H_{i,j} x_l + n_i, \qquad i = 1, \cdots, n_r, \tag{4}$$

where n_r is the number of receive antennas, x_l is the lth symbol in $\mathbb{A}$ transmitted by the jth antenna, and n_i is the noise component. The signals received at all the receive antennas can be written in vector form as

$$\mathbf{y} \;=\; \mathbf{Hx} + \mathbf{n}, \tag{5}$$

For this system model, the ML detection rule is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}_{n_t,\mathbb{A}}}{\text{argmin}} \; \|\mathbf{y} - \mathbf{Hx}\|^2. \tag{6}$$

III. Proposed PRPP-SM system

The proposed PRPP-SM transmitter consists of n_t transmit antennas and $n_{rf} = 1$ transmit RF chains as shown in Fig. 4. It takes p modulated symbols and forms the symbol vector $\mathbf{x}_s \in \mathbb{A}^p$, where $\mathbb{A}$ is the modulation alphabet. Let the matrix $\mathbf{A}$ of size $pn_t \times p$ denote the transmit antenna activation pattern, such that $\mathbf{Ax}_s \in \mathbb{S}^p_{n_t,\mathbb{A}}$, where $\mathbb{S}_{n_t,\mathbb{A}}$ is the SM signal set given by (2). The matrix $\mathbf{A}$ consists of p submatrices $\mathbf{A}_{(i)}$, $i = 1, \cdots, p$,

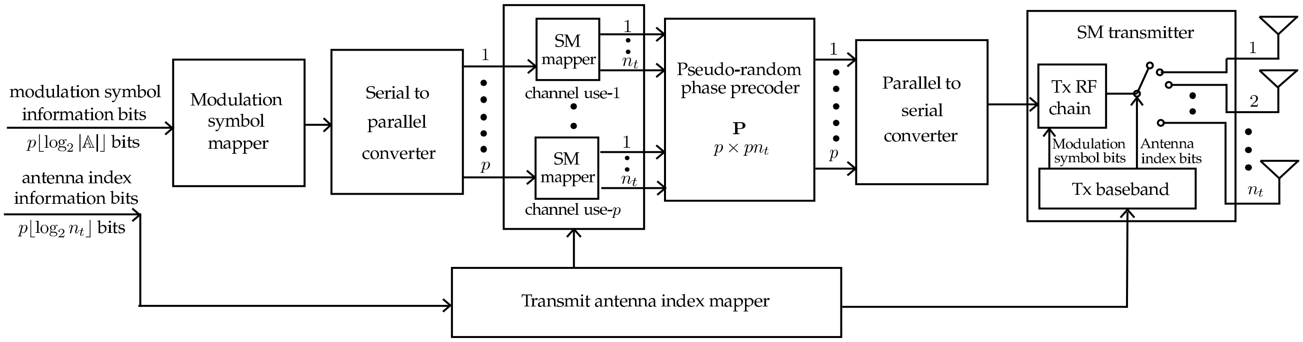


Fig. 4. Proposed PRPP-SM system.

each of size $n_t \times p$, such that $\mathbf{A} = [\mathbf{A}_{(1)}^T\, \mathbf{A}_{(2)}^T\, \cdots\, \mathbf{A}_{(p)}^T]^T$. The submatrix $\mathbf{A}_{(i)}$ is constructed as

$$\mathbf{A}_{(i)} = [\mathbf{0}_{(1)}\, \cdots\, \mathbf{0}_{(i-1)}\, \mathbf{a}_{(i)}\, \mathbf{0}_{(i+1)}\, \cdots\, \mathbf{0}_{(p)}], \tag{7}$$

where $\mathbf{0}_{(k)}$ is a $n_t \times 1$ vector of zeroes, and $\mathbf{a}_{(i)}$ is a $n_t \times 1$ vector constructed as $\mathbf{a}_{(i)} = [0 \cdots 0 \underbrace{1}_{j_i\text{th coordinate}} 0 \cdots 0]^T$, where j_i is the index of the active antenna during the ith channel use. Note that, with the above definitions, $\mathbf{A}_{(i)}\mathbf{x}_s \in \mathbb{S}_{n_t,\mathbb{A}}$. For example, in a system with $n_t = 2$ and $p = 3$, to activate antennas 1, 2, and 1 in three consecutive channel uses, respectively, the matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_{(1)}\\ \mathbf{A}_{(2)}\\ \mathbf{A}_{(3)}\end{bmatrix} = \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]^T. \tag{8}$$

Note that the indices of the non-zero rows in matrix $\mathbf{A}$ gives the support of the spatially modulated vector $\mathbf{A}\mathbf{x}_s \in \mathbb{S}_{n_t,\mathbb{A}}^p$. For example, in (8), the support given by $\mathbf{A}$ is $\{1, 4, 5\}$.

The $\mathbf{A}\mathbf{x}_s$ vector is then precoded as $\mathbf{P}\mathbf{A}\mathbf{x}_s$, using a rectangular precoder matrix $\mathbf{P}$ of size $p \times pn_t$. The (r, c)th entry of the $\mathbf{P}$ matrix is $\frac{1}{\sqrt{p}} e^{j\theta_{r,c}}$, where the phases $\{\theta_{r,c}\}$ are generated using a pseudo-random sequence generator, whose seed is pre-shared among the transmitter and receiver. The output of the precoder is transmitted on the selected antenna in each channel use[1].

Let n_r denote the number of receive antennas. The $pn_r \times 1$ received signal vector at the receiver is given by

$$\mathbf{y} = \mathbf{D}\mathbf{P}\mathbf{A}\mathbf{x}_s + \mathbf{n}, \tag{9}$$

where $\mathbf{D} = \text{diag}\{\mathbf{H}_{(1)}\, \mathbf{H}_{(2)}\, \cdots\, \mathbf{H}_{(p)}\}$, $\mathbf{H}_{(i)}$ is the $n_r \times n_t$ channel matrix of the ith channel use, the elements of $\mathbf{H}_{(i)}$ are i.i.d. complex Gaussian with zero mean and unit variance, $\mathbf{n}$ is the $pn_r \times 1$ noise vector $[\mathbf{n}_{(1)}^T\, \mathbf{n}_{(2)}^T\, \cdots\, \mathbf{n}_{(p)}^T]^T$, where the entries of $\mathbf{n}_{(i)}$ are distributed as $\mathbb{CN}(0, \sigma^2)$. Note that $\|\mathbf{D}\mathbf{P}\mathbf{A}\|_F = \|\mathbf{D}\mathbf{A}\|_F$. This creates a $pn_r \times p$ virtual MIMO system. For this system model, the ML detection rule is given by

$$\{\hat{\mathbf{x}}_s, \hat{\mathbf{A}}\} = \underset{\mathbf{x}_s \in \mathbb{A}^p, \forall \mathbf{A}}{\text{argmin}} \|\mathbf{y} - \mathbf{D}\mathbf{P}\mathbf{A}\mathbf{x}_s\|^2. \tag{10}$$

[1] *Remark:* In the proposed PRPP-SM system, precoding is applied to both the modulation bits as well as the antenna index bits, i.e., the transmit vector is $\mathbf{A}\mathbf{P}\mathbf{A}\mathbf{x}_s$. Instead, if only the modulation bits are precoded, the transmitted vector will be $\mathbf{A}\mathbf{P}\mathbf{x}_s$. When the antenna index bits are not precoded, the system fails to provide the diversity gain advantage of PRPP to the antenna index bits, and hence has a poor BER performance. The proposed PRPP-SM system, on the other hand, achieves very good performance because of the precoding of the antenna index bits as well.

The indices of the non-zero rows in $\hat{\mathbf{A}}$ and the entries of $\hat{\mathbf{x}}_s$ are demapped to obtain the information bits.

Note that the ML solution in (10) can be computed only for small precoder sizes because of its exponential complexity in p, i.e., $O((|\mathbb{A}|n_t)^p))$. In Section IV, we will establish the superiority of the PRPP-SM over conventional PRPP and SM systems using ML detection. For large precoder sizes, we propose a low complexity detection algorithm in the following subsection.

A. Proposed PRPP-SM detector

In this subsection, we propose a local search based detection (LSD) algorithm that achieves near-ML performance in PRPP-SM systems with large p at a low computational complexity. The LSD algorithm obtains a local minima in terms of the ML cost in a local neighborhood. In the proposed PRPP-SM system, a key requirement for local search is a suitable neighborhood definition that takes into account the antenna index bits also. We propose the following neighborhood definition for the local search. The set of neighbors of a given pair of $\{\mathbf{A}, \mathbf{x}_s\}$, denoted by $\mathcal{N}(\mathbf{A}, \mathbf{x}_s)$, is defined as the set of all pairs $\{\mathbf{A}', \mathbf{x}_s'\}$ that satisfies one of the following three conditions:

1) $\mathbf{x}_s = \mathbf{x}_s'$ and $\mathbf{A}_{(i)} \neq \mathbf{A}_{(i)}'$ for exactly a single index i.
2) $\mathbf{A} = \mathbf{A}'$ and $\mathbf{x}_s$ differs from $\mathbf{x}_s'$ in exactly one entry.
3) $\mathbf{A}_{(i)} \neq \mathbf{A}_{(i)}'$ for exactly a single index i, and for that index i, $x_s(i) \neq x_s'(i)$.

For a PRPP-SM system with $n_t = 2$, $p = 2$, and $\mathbb{A} = \{\pm 1\}$, an example of a neighborhood is

$$\mathcal{N}\left(\begin{bmatrix}1 & 0\\ 0 & 0\\ \hline 0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}+1\\ -1\end{bmatrix}\right) = \left\{\begin{array}{l} \left\{\begin{bmatrix}1 & 0\\ 0 & 0\\ \hline 0 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}+1\\ -1\end{bmatrix}\right\}, \left\{\begin{bmatrix}1 & 0\\ 0 & 0\\ \hline 0 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}+1\\ +1\end{bmatrix}\right\}, \\ \left\{\begin{bmatrix}1 & 0\\ 0 & 0\\ \hline 0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}-1\\ -1\end{bmatrix}\right\}, \left\{\begin{bmatrix}1 & 0\\ 0 & 0\\ \hline 0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}+1\\ +1\end{bmatrix}\right\}, \\ \left\{\begin{bmatrix}0 & 0\\ 1 & 0\\ \hline 0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}+1\\ -1\end{bmatrix}\right\}, \left\{\begin{bmatrix}0 & 0\\ 1 & 0\\ \hline 0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}-1\\ -1\end{bmatrix}\right\} \end{array}\right\}.$$

The proposed LSD algorithm starts with an initial solution $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$, which is also the current solution. Using the defined neighborhood, the algorithm considers all the neighbors of $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$ and searches for the neighbor with the least ML cost which also has a lower ML cost than the current solution. If such a neighbor is found, then this neighbor is designated as the current solution. This marks the completion of one iteration of the algorithm. Iterations are carried out until a local minima is reached (i.e., there is no neighbor better than the current solution). The solution corresponding to the local minima is declared as the final output $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\}$. This algorithm is listed in **Algorithm 1**.

Algorithm 1 Listing of the proposed LSD algorithm

1: **Input** : $\mathbf{y}, \mathbf{D}, \mathbf{P}$
2: Initial solution : $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$, $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\} = \{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$
3: Compute $\mathcal{N}(\hat{\mathbf{A}}, \hat{\mathbf{x}}_s)$
4: $\{\mathbf{A}^c, \mathbf{x}_s^c\} = \underset{\{\mathbf{B},\mathbf{z}\} \in \mathcal{N}(\hat{\mathbf{A}}, \hat{\mathbf{x}}_s)}{\text{argmin}} \|\mathbf{y} - \mathbf{DBPBz}\|^2$
5: **if** $\|\mathbf{y} - \mathbf{D}\mathbf{A}^c\mathbf{P}\mathbf{A}^c\mathbf{x}_s^c\|^2 < \|\mathbf{y} - \mathbf{D}\hat{\mathbf{A}}\mathbf{P}\hat{\mathbf{A}}\hat{\mathbf{x}}_s\|^2$ **then**
6: $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\} = \{\mathbf{A}^c, \mathbf{x}_s^c\}$
7: Go to step 3
8: **end if**
9: **Output** : $\{\hat{\mathbf{A}}, \hat{\mathbf{s}}\}$

Computing the initial support and initial solution $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$: The algorithm needs the initial support matrix $\mathbf{A}^{(0)}$ and the initial solution vector $\mathbf{x}_s^{(0)}$. The initial support matrix is obtained as follows. Obtain a $pn_t \times 1$ vector $\mathbf{v}$ through a minimum mean square error (MMSE) estimator as $\mathbf{v} = (\mathbf{D}^H\mathbf{D} + \sigma^2\mathbf{I})^{-1}\mathbf{D}^H\mathbf{y}$. The vector $\mathbf{v}$ consists of p subvectors $\mathbf{v}_{(i)}$, $i = 1, \cdots, p$, each of size $n_t \times 1$, such that $\mathbf{v} = [\mathbf{v}_{(1)}^T \, \mathbf{v}_{(2)}^T \cdots \mathbf{v}_{(p)}^T]^T$. The indices of the elements with the largest magnitude in each $\mathbf{v}_{(i)}$ are taken as the indices of the non-zero rows of $\mathbf{A}^{(0)}$. The initial solution vector is obtained through an MMSE estimator as

$$\mathbf{x}_s^{(0)} = \mathcal{Q}\left((\mathbf{F}^H\mathbf{F} + \sigma^2\mathbf{I})^{-1}\mathbf{F}^H\mathbf{y}\right),$$

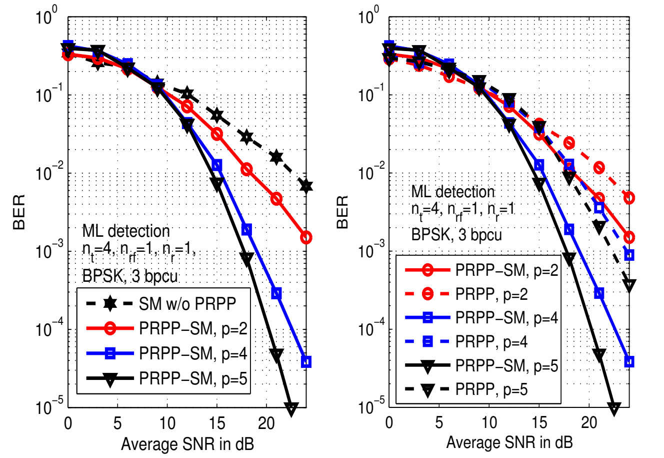
where $\mathbf{F} = \mathbf{D}\mathbf{A}^{(0)}\mathbf{P}\mathbf{A}^{(0)}$ and $\mathcal{Q}(.)$ denotes the Euclidean distance quantizer such that $\mathbf{x}_s^{(0)} \in \mathbb{A}^p$.

B. Simulation results

In this subsection, we present the simulation results on the BER performance of the proposed PRPP-SM system with ML detection (for small p) and LSD (for large p).

Figure 5(a) compares the BER performance of PRPP-SM against the performance of SM without PRPP at a spectral efficiency of 3 bpcu using ML detection. The BER plots for $n_t = 4, n_{rf} = 1, n_r = 1$, and BPSK modulation for different precoder sizes $p = 2, 4, 5$ are shown. It is observed that the performance of PRPP-SM is better than SM without PRPP by about 9 dB at $p = 5$ and 10^{-2} BER. This performance advantage in favor of the PRPP-SM system is due to the diversity gain offered by the pseudo-random phase precoding.

Figure 5(b) compares the performance of PRPP-SM against the performance of PRPP without SM (i.e., $n_t = 1$, $p > 1$) at a spectral efficiency of 3 bpcu using ML detection. Here, the PRPP-SM system has $n_t = 4, n_{rf} = 1, n_r = 1$, BPSK modulation, and the PRPP system without SM has $n_t = 1, n_r = 1$, 8-QAM with varying precoder sizes $p = 2, 4, 5$. It is observed that the performance of PRPP-SM is better than the PRPP system without SM by about 4 dB at $p = 5$ and 10^{-2} BER. This performance advantage in favor of PRPP-SM system is



(a) PRPP-SM vs SM (b) PRPP-SM vs PRPP

Fig. 5. Performance comparison of PRPP-SM system ($n_t = 4, n_{rf} = 1, n_r = 1$, BPSK) with ML detection with (a) SM system without PRPP using ML detector, and (b) PRPP system without SM ($n_t = 1, n_{rf} = 1, n_r = 1$, 8-QAM) with ML detection. System throughput is 3 bpcu.

mainly due to the SNR gain in using BPSK in PRPP-SM against using 8-QAM in PRPP with out SM for the same spectral efficiency of 3 bpcu.

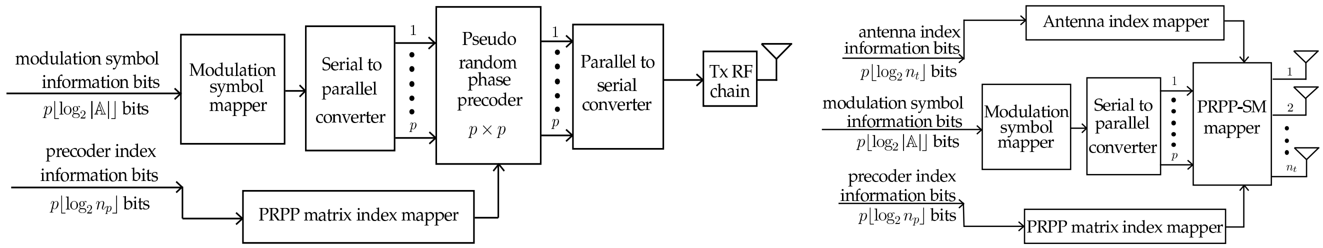
Figure 6 compares the performance of PRPP-SM using the proposed LSD algorithm against the performance of PRPP without SM using LAS detection in [4]. MMSE initial solution is used in both LSD and LAS algorithms. For PRPP-SM, we have used $n_t = 4, n_{rf} = 1, n_r = 8$, BPSK modulation. For PRPP without SM, we have used $n_t = 1, n_r = 8$, 8-QAM. Large precoder sizes are used here; the precoder sizes used in both the systems are $p = 10, 20, 70$. The spectral efficiency in both the systems is 3 bpcu. It is observed that the performance of PRPP-SM system is better than the PRPP system without SM by about 10 dB at $p = 70$ and 10^{-2} BER. This indicates the potential of PRPP-SM system to perform very well when large precoder sizes are employed.

IV. Proposed Precoder Index Modulation

In this section, we propose a precoder index modulation (PIM) scheme which conveys information bits through the choice of a precoding matrix among a set of pre-determined precoder matrices.

A. Precoder index modulation (PIM)

In the proposed PIM scheme, the transmitter has equal number of transmit antennas and RF chains. Consider the case of $n_t = n_{rf} = 1$. The idea here is to have a collection of precoder matrices each of size $p \times p$ and choose one among these matrices to precode p modulation symbols from an alphabet $\mathbb{A}$ in p channel uses. Call this collection of matrices as 'precoder set,' denoted by $\mathbb{P}$. Therefore, the number of bits conveyed per channel use through precoder indexing is $\frac{1}{p}\lfloor \log_2 |\mathbb{P}| \rfloor$. The total number of bits per channel use (including precoder index bits and modulation symbol bits)



(a) PIM system (b) PIM-SM system

Fig. 7. Proposed PIM and PIM-SM systems.

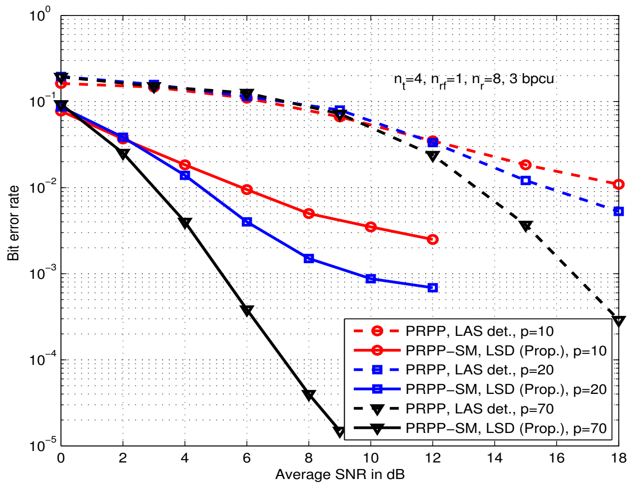


Fig. 6. Comparison of the BER performance of PRPP-SM ($n_t = 4, n_{rf} = 1, n_r = 8$, BPSK) using the proposed LSD algorithm with that of PRPP without SM ($n_t = n_{rf} = 1, n_r = 8$, 8-QAM) with LAS detection. System throughput is 3 bpcu.

is then given by

$$\frac{1}{p}\Big(\lfloor\log_2|\mathbb{P}|\rfloor + p\lfloor\log_2|\mathbb{A}|\rfloor\Big).$$

Construction of the precoder set: The precoder set $\mathbb{P} = \{\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_{|\mathbb{P}|}\}$ is constructed as follows. Let n denote the number of precoder index bits per channel use, i.e., $n \triangleq \frac{1}{p}\lfloor\log_2|\mathbb{P}|\rfloor$. Generate a PRPP matrix $\mathbf{Q}$ of size $p \times pn_p$, where $n_p = 2^n$. Note that the precoder set size $|\mathbb{P}| = (n_p)^p$. The matrix $\mathbf{Q}$ can be written as

$$\mathbf{Q} = [\mathbf{Q}_1 \ \mathbf{Q}_2 \ \cdots \ \mathbf{Q}_p],$$

where $\mathbf{Q}_i$s are sub-matrices each of size $p \times n_p$. Now, the p columns of a precoder matrix $\mathbf{P}_j$ are obtained by drawing one column from each $\mathbf{Q}_i$, $i = 1, 2, \cdots, p$. Each of such draws form one precoder matrix. Since there are n_p columns in each $\mathbf{Q}_i$, the number of such possible draws is $(n_p)^p$, which gives us all the matrices in the precoder set. For example, consider $p = 2$ and $n_p = 2$, and

$$\mathbf{Q} = \left[\begin{array}{cc|cc} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \end{array}\right].$$

The precoder set for this example is given by

$$\mathbb{P} = \left\{ \begin{bmatrix} q_{11} & q_{13} \\ q_{21} & q_{23} \end{bmatrix}, \begin{bmatrix} q_{11} & q_{14} \\ q_{21} & q_{24} \end{bmatrix}, \begin{bmatrix} q_{12} & q_{13} \\ q_{22} & q_{23} \end{bmatrix}, \begin{bmatrix} q_{12} & q_{14} \\ q_{22} & q_{24} \end{bmatrix} \right\}.$$

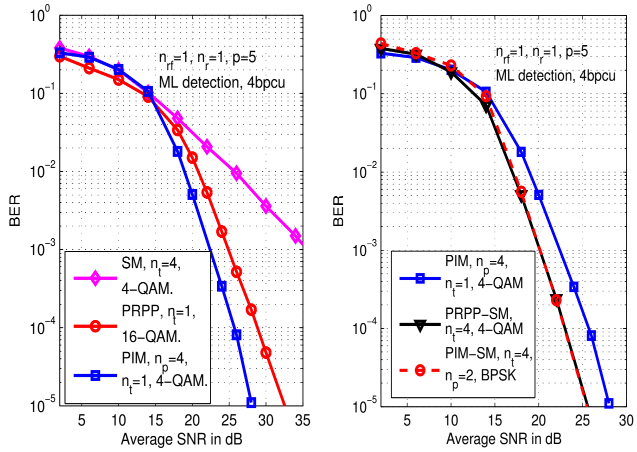
System model: The PIM transmitter is shown in Fig. 7(a). It takes $\lfloor\log_2|\mathbb{P}|\rfloor + p\lfloor\log_2|\mathbb{A}|\rfloor$ bits and encodes them as follows. The $p\lfloor\log_2|\mathbb{A}|\rfloor$ bits are used to obtain p modulation symbols. Let $\mathbf{x} \in \mathbb{A}^p$ denote the vector of these modulation symbols. The vector $\mathbf{x}$ is precoded by a precoder matrix $\mathbf{P}_j$ chosen from $\mathbb{P}$ whose index is given by the $\lfloor\log_2|\mathbb{P}|\rfloor$ bits. The transmitter then sends one precoded symbol in every channel use. The detection is performed after p channel uses. The $p \times 1$ received signal vector $\mathbf{y}$ in this system model can be written as

$$\mathbf{y} = \mathbf{D}\mathbf{P}_j\mathbf{x} + \mathbf{n}, \tag{11}$$

where $\mathbf{P}_j$ is the $p \times p$ PRPP matrix chosen from $\mathbb{P}$, and $\mathbf{D}$ is the channel matrix as described in Sec. III. The ML detection rule is given by

$$\{\hat{\mathbf{x}}, \hat{j}\} = \underset{\mathbf{x}\in\mathbb{A}^p,\, j=1,\cdots,|\mathbb{P}|}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{D}\mathbf{P}_j\mathbf{x}\|^2. \tag{12}$$

Simulation results: In Fig. 8(a), we present the BER performance of the PIM scheme with $n_t = 1, n_{rf} = 1, n_p = 4, n_r = 1, p = 5$ and 4-QAM, using ML detection. We compare this performance with the ML detection performance of PRPP system without SM for $n_t = 1, n_{rf} = 1, n_r = 1, p = 5$ and 16-QAM, and SM system without PRPP for $n_t = 4, n_{rf} = 1, n_r = 1$ and 4-QAM. Note that the spectral efficiency in all the three systems is 4 bpcu. We see that PIM outperforms PRPP without SM and SM without PRPP. For example, at 10^{-3} BER PIM is better than PRPP without SM by about 2.5 dB and better than SM without PRPP by about 12.5 dB. The PIM scheme achieves better performance than PRPP without SM because PIM can use a smaller sized (and hence more power efficient) modulation alphabet compared to PRPP without SM. Also, the reason behind the better performance of PIM compared to SM without PRPP is that PIM provides diversity gain due to precoding. Thus, PIM provides the benefits of both diversity advantage of PRPP and SNR advantage of SM. Note that PRPP-SM scheme in Sec. III also provides both these advantages, but the possibility of using smaller-sized modulation alphabet in PRPP-SM arises due to antenna indexing, whereas in PIM it arises due to precoder indexing. Therefore, PIM avoids the need to use multiple transmit antennas compared to PRPP-SM. This observations leads us to consider exploiting the antenna indexing in SM for further reduction in modulation alphabet size in PIM. We refer to such a PIM scheme that exploits both precoder indexing as well as antenna indexing as PIM-SM scheme. The proposed PIM-SM scheme is presented in the following subsection.



(a) PIM (b) PIM-SM

Fig. 8. BER performances of PIM ($n_t = 1, n_p = 4, p = 5$, 4-QAM), with (a) SM without PRPP ($n_t = 4, n_{rf} = 1$, 4-QAM) and PRPP without SM ($n_t = 1, p = 5$, 16-QAM), and (b) PRPP-SM ($n_t = 4, n_{rf} = 1, p = 5$, 4-QAM) and PIM-SM ($n_t = 4, n_{rf} = 1, n_p = 2, p = 5$, BPSK). System throughput is 4 bpcu.

B. Precoder index modulation with SM (PIM-SM)

The PIM-SM scheme uses n_t transmit antennas and one RF chain, so that $\lfloor \log_2 n_t \rfloor$ bits are conveyed as antenna index bits. These bits are in addition to the $\lfloor \log_2 n_p \rfloor + \lfloor \log_2 |\mathbb{A}| \rfloor$ bits conveyed in PIM. Therefore, the spectral efficiency of the PIM-SM scheme is

$$\lfloor \log_2 n_p \rfloor + \lfloor \log_2 n_t \rfloor + \lfloor \log_2 |\mathbb{A}| \rfloor \quad \text{bpcu}.$$

For immediate reference and comparison, the spectral efficiencies achieved by the different schemes and the bits that are precoded in these schemes are tabulated in Table I.

System model: This PIM-SM transmitter is illustrated in Fig. 7(b). The system model for the PIM-SM scheme can be written as

$$\mathbf{y} = \mathbf{DAPAP}_j \mathbf{x} + \mathbf{n}, \tag{13}$$

where $\mathbf{A}$ is the activation pattern matrix of the PRPP-SM scheme defined in Sec. III. The ML detection rule for PIM-SM can then be written as

$$\{\hat{\mathbf{x}}, \hat{\mathbf{A}}, \hat{j}\} = \underset{\mathbf{x} \in \mathbb{A}^p, \mathbf{A}, j=1,\cdots,|\mathbb{P}|}{\text{argmin}} \|\mathbf{y} - \mathbf{DAPAP}_j \mathbf{x}\|^2. \tag{14}$$

The indices of the non-zero rows in $\hat{\mathbf{A}}$, index $\hat{j}$, and the entries of $\hat{\mathbf{x}}$ are demapped to obtain the information bits.

Simulation results: In Fig. 8(b), we present the BER performance of PIM-SM $n_t = 4, n_{rf} = 1, n_r = 1, n_p = 2$, BPSK and $p = 5$, using ML detection. We also plot the ML detection performance of the PIM scheme (without SM) for $n_t = 1, n_{rf} = 1, n_r = 1, n_p = 4$, 4-QAM, and $p = 5$. We have also plotted the ML detection performance of PRPP-SM scheme with $n_t = 4, n_{rf} = 1, n_r = 1$, 4-QAM, and $p = 5$. Note that the spectral efficiency is 4 bpcu in all the three schemes. It can be seen that the PIM-SM scheme performs better than PIM scheme. This is because of the smaller-sized modulation alphabet in PIM-SM (BPSK) compared that in PIM (4-QAM). Also, PIM-SM performance is similar to that of PRPP-SM.

Scheme	Number of bits conveyed through			Precoding done on	
	Modulation symbols	Precoder index	Antenna index	Modulation symbols	Antenna index bits
PRPP	$\lfloor \log_2 \mathbb{A} \rfloor$	-	-	✓	-
SM	$\lfloor \log_2 \mathbb{A} \rfloor$	-	$\lfloor \log_2 n_t \rfloor$	×	×
PRPP-SM	$\lfloor \log_2 \mathbb{A} \rfloor$	-	$\lfloor \log_2 n_t \rfloor$	✓	✓
PIM	$\lfloor \log_2 \mathbb{A} \rfloor$	$\lfloor \log_2 n_p \rfloor$	-	✓	-
PIM-SM	$\lfloor \log_2 \mathbb{A} \rfloor$	$\lfloor \log_2 n_p \rfloor$	$\lfloor \log_2 n_t \rfloor$	✓	✓

TABLE I
MODES OF CONVEYING BITS AND PRECODING IN DIFFERENT SCHEMES.

V. CONCLUSIONS

We proposed a novel pseudo-random phase precoder based spatial modulation (PRPP-SM) scheme for fading channels. In the proposed scheme, we precoded both the modulation bits as well as the antenna index bits using a rectangular precoder matrix. Our simulation results showed that the proposed PRPP-SM scheme achieved diversity gains as well as SNR gains compared to conventional PRPP and SM systems. To facilitate the detection when large precoder sizes are used in the proposed PRPP-SM systems, we proposed a low complexity local search algorithm which used a neighborhood definition suited for taking into account the antenna index bits as well. We also proposed a precoder index modulation (PIM) scheme, which conveys additional information bits through the choice of a precoding matrix among a set of pre-determined PRPP matrices. Finally, combining the PIM and PRPP-SM schemes, we proposes a PIM-SM scheme which conveys bits through both antenna index as well as precoder index. We showed the performance of PIM and PIM-SM schemes for small precoder sizes using ML detection. Design of low complexity detection algorithms PIM and PIM-SM for large precoder sizes is open for future extension.

REFERENCES

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

[2] V. Sharma, K. Premkumar and R. N. Swamy, “Exponential diversity achieving spatio-temporal power allocation scheme for fading channels,” *IEEE Trans. Info. Theory*, vol. 54, no. 1, pp. 188-208, Jan. 2008.

[3] R. Annavajjala and P. V. Orlik, “Achieving near exponential diversity on uncoded low-dimensional MIMO, multi-user and multi-carrier systems without transmitter CSI,” *Proc. ITA 2011*, Jan. 2011.

[4] K. V. Vardhan, S. K. Mohammed, A. Chockalingam, and B. S. Rajan, “A low-complexity detector for large MIMO systems and multicarrier CDMA systems,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, pp. 473-485, Apr. 2008.

[5] R. Mesleh, H. Hass, S. Sinaovic, C. W. Ahn, “Spatial modulation,” *IEEE Trans. Veh. Tech.*, vol. 57, no. 4, pp. 2228-2241, Jul. 2008.

[6] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, “Spatial modulation for generalized MIMO: challenges, opportunities and implementation,” *Proceedings of the IEEE*, vol. 102, no. 1, pp. 56-103, Jan. 2014.

[7] N. Serafimovski1, S. Sinanovic, M. Di Renzo, and H. Haas, “Multiple access spatial modulation,” *EURASIP J. Wireless Commun. and Networking 2012*, 2012:299.

[8] P. Raviteja, T. Lakshmi Narasimhan, and A. Chockalingam, “Multiuser SM-MIMO versus massive MIMO: uplink performance comparison,” available online: arXiv:1311.1291 [cs.IT] 6 Nov 2013.