A Hybrid RTS-BP Algorithm for Improved Detection of Large-MIMO M-QAM Signals

Tanumay Datta, N. Srinidhi, A. Chockalingam, and B. Sundar Rajan
Department of ECE, Indian Institute of Science, Bangalore 560012, INDIA

***Abstract*—Low-complexity near-optimal detection of large-MIMO signals has attracted recent research. Recently, we proposed a local neighborhood search algorithm, namely *reactive tabu search* (RTS) algorithm, as well as a factor-graph based *belief propagation* (BP) algorithm for low-complexity large-MIMO detection. The motivation for the present work arises from the following two observations on the above two algorithms: i) Although RTS achieved close to optimal performance for 4-QAM in large dimensions, significant performance improvement was still possible for higher-order QAM (e.g., 16-, 64-QAM). ii) BP also achieved near-optimal performance for large dimensions, but only for $\{\pm 1\}$ alphabet. In this paper, we improve the large-MIMO detection performance of higher-order QAM signals by using a hybrid algorithm that employs RTS and BP. In particular, motivated by the observation that when a detection error occurs at the RTS output, the least significant bits (LSB) of the symbols are mostly in error, we propose to first reconstruct and cancel the interference due to bits other than LSBs at the RTS output and feed the interference cancelled received signal to the BP algorithm to improve the reliability of the LSBs. The output of the BP is then fed back to RTS for the next iteration. Simulation results show that the proposed algorithm performs better than the RTS algorithm, and semi-definite relaxation (SDR) and Gaussian tree approximation (GTA) algorithms.**

I. Introduction

Multiple-input multiple-output (MIMO) systems with large number (e.g., tens) of transmit and receive antennas, referred to as 'large-MIMO systems,' are of interest because of the high capacities/spectral efficiencies theoretically predicted in these systems [1]. Research in low-complexity receive processing (e.g., MIMO detection) techniques that can lead to practical realization of large-MIMO systems is both nascent as well as promising. Evolution of WiFi standards (evolution from IEEE 802.11n to IEEE 802.11ac to achieve multi-gigabit rate transmissions in 5 GHz band) now considers 16×16 MIMO operation; see 16×16 MIMO indoor channel sounding measurements at 5 GHz reported in [2] for consideration in WiFi standards.

In the context of large-MIMO signal detection, in our recent works, we have shown that certain algorithms from machine learning/artificial intelligence achieve near-optimal detection performance in large-MIMO systems at low complexities [3]-[9][1]. In [3]-[5], a local neighborhood search based algorithm, namely, a *likelihood ascent search* (LAS) algorithm, was proposed and shown to achieve close to maximum-likelihood (ML) performance in MIMO systems with several tens of antennas (e.g., 32×32, 64×64 MIMO). Subsequently, in [6],[7], another local search algorithm, namely, *reactive tabu search* (RTS) algorithm, which performed better than LAS through the use of a local minima exit strategy was presented[2]. In [8], near-ML performance in a 50×50 MIMO system was demonstrated using a *Gibbs sampling* based detection algorithm, where the symbols take values from $\{\pm 1\}$. More recently, we, in [9], proposed a factor graph based *belief propagation* (BP) algorithm for large-MIMO detection, where we adopted a Gaussian approximation of the interference (GAI).

The motivation for the present work arises from the following two observations on the RTS and BP algorithms in [6],[7] and [9]: i) RTS works for general M-QAM. Although RTS was shown to achieve close to ML performance for 4-QAM in large dimensions, significant performance improvement was still possible for higher-order QAM (e.g., 16- and 64-QAM). ii) BP also was shown to achieve near-optimal performance for large dimensions, but only for $\{\pm 1\}$ alphabet. In this paper, we improve the large-MIMO detection performance of higher-order QAM signals by using a hybrid algorithm that employs RTS and BP. In particular, we observed that when a detection error occurs at the RTS output, the least significant bits (LSB) of the symbols are mostly in error. Motivated by this observation, we propose to first reconstruct and cancel the interference due to bits other than the LSBs at the RTS output and feed the interference cancelled received signal to the BP algorithm to improve the reliability of the LSBs. The output of the BP is then fed back to the RTS for the next iteration. Simulation results show that the proposed algorithm performs better RTS, and semi-definite relaxation (SDR) and Gaussian tree approximation (GTA) algorithms in [14],[15].

II. RTS and BP Algorithms for Large-MIMO Detection

Consider a $N_t \times N_r$ V-BLAST MIMO system whose received signal vector, $\mathbf{y}_c \in \mathbb{C}^{N_r}$, is of the form

$$\mathbf{y}_c \quad = \quad \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c, \tag{1}$$

where $\mathbf{x}_c \in \mathbb{C}^{N_t}$ is the symbol vector transmitted, $\mathbf{H}_c \in \mathbb{C}^{N_r \times N_t}$ is the channel gain matrix, and $\mathbf{n}_c \in \mathbb{C}^{N_r}$ is the noise vector whose entries are modeled as i.i.d $\mathcal{CN}(0, \sigma^2)$. Assuming rich scattering, we model the entries of $\mathbf{H}_c$ as i.i.d

[1]Similar algorithms have been reported earlier in the context of multiuser detection in large CDMA systems.

[2]In [5],[7], we compared the performance and complexities of LAS and RTS algorithms with those of the sphere decoding (SD) variants in [10] and [11], and showed that, while LAS and RTS scale well for large dimensions, these SD variants do not scale well for large dimensions.

$\mathcal{CN}(0,1)$. Each element of $\mathbf{x}_c$ is an M-PAM or M-QAM symbol. M-PAM symbols take values from $\{A_m, m = 1, 2, \cdots, M\}$, where $A_m = (2m-1-M)$, and M-QAM is nothing but two PAMs in quadrature. As in [4], we convert (1) into a real-valued system model, given by

$$\mathbf{y} \;\; = \;\; \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{H} \in \mathbb{R}^{2N_r \times 2N_t}$, $\mathbf{y} \in \mathbb{R}^{2N_r}$, $\mathbf{x} \in \mathbb{R}^{2N_t}$, $\mathbf{n} \in \mathbb{R}^{2N_r}$. For M-QAM, $[x_1, \cdots, x_{N_t}]$ can viewed to be from an underlying M-PAM signal set, and so is $[x_{N_t+1}, \cdots, x_{2N_t}]$. Let $\mathbb{A}_i$ denote the M-PAM signal set from which x_i takes values, $i = 1, 2, \cdots, 2N_t$. Defining a $2N_t$-dimensional signal space $\mathbb{S}$ to be the Cartesian product of $\mathbb{A}_1$ to $\mathbb{A}_{2N_t}$, the ML solution vector, $\mathbf{x}_{ML}$, is given by

$$\mathbf{x}_{ML} \;\; = \;\; \mathop{\arg\min}_{\mathbf{x} \in \mathbb{S}} \; \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \tag{3}$$

whose complexity is exponential in N_t. The RTS algorithm in [6],[7] is a low-complexity algorithm, which minimizes the ML metric in (3) through a local neighborhood search.

A. RTS Algorithm

A detailed description of the RTS algorithm for large-MIMO detection is available in [6],[7]. The per-symbol complexity of RTS for detection of V-BLAST signals is $O(N_t N_r)$. Here, we present the 16- and 64-QAM performance of the RTS algorithm that motivates the current work.

1) Motivation of Current Work: Figure 1 shows the uncoded BER performance of RTS using the algorithm parameters optimized through simulations for 4-, 16-, and 64-QAM in a 32×32 V-BLAST system. As lower bounds on the error performance in MIMO, the SISO AWGN performance for 4-, 16-, and 64-QAM are also plotted. It can be seen that, in the case of 4-QAM, the RTS performance is just about 0.5 dB away from the SISO AWGN performance at 10^{-3} BER. However, the gap between RTS performance and SISO AWGN performance at 10^{-3} BER widens for 16-QAM and 64-QAM; the gap is 7.5 dB for 16-QAM and 16.5 dB for 64-QAM. This gap can be viewed as a potential indicator of the amount of improvement in performance possible further. A more appropriate indicator will be the gap between RTS performance and the ML performance. Since simulation of sphere decoding (SD) of 32×32 V-BLAST with 16- and 64-QAM (64 real dimensions) is computationally intensive, we do not show the SD (ML) performance. Nevertheless, the widening gap of RTS Performance from SISO AWGN performance for 16- and 64-QAM seen in Fig. 1 motivated us to explore improved algorithms to achieve better performance than RTS performance for higher-order QAM.

B. BP Algorithm Based on GAI

In [9], we presented a detection algorithm based on BP on factor graphs of MIMO systems. In (2), each entry of the vector $\mathbf{y}$ is treated as a function node (observation node), and each symbol, $x_i \in \{\pm 1\}$, as a variable node. A key

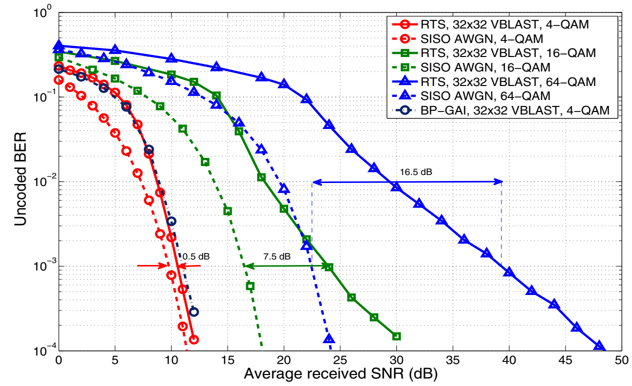

Fig. 1. Uncoded BER performance of RTS algorithm in 32×32 V-BLAST for 4-, 16-, 64-QAM. *Performance improvement is possible in 16-, 64-QAM.*

ingredient in the BP algorithm in [9], which contributes to its low complexity, is the Gaussian approximation of interference (GAI), where the interference + noise term, z_{ik}, in

$$y_i \;\; = \;\; h_{ik}x_k + \underbrace{\overbrace{\sum_{j=1, j\neq k}^{2N_t} h_{ij}x_j}^{interference} + n_i}_{\triangleq\, z_{ik}}, \tag{4}$$

is modeled as $\mathbb{CN}(\mu_{z_{ik}}, \sigma^2_{z_{ik}})$ with

$$\mu_{z_{ik}} = \sum_{j=1, j\neq k}^{N_t} h_{ij}\mathbb{E}(x_j),$$

and

$$\sigma^2_{z_{ik}} = \sum_{j=1, j\neq k}^{2N_t} |h_{ij}|^2 \,\mathrm{Var}(x_j) + \frac{\sigma^2}{2},$$

where h_{ij} is the (i,j)th element in $\mathbf{H}$. With x_i's $\in \{\pm 1\}$, the log-likelihood ratio (LLR) of x_k at observation node i, denoted by Λ_i^k, is

$$\begin{aligned} \Lambda_i^k \;\; &= \;\; \log \frac{p(y_i|\mathbf{H}, x_k = 1)}{p(y_i|\mathbf{H}, x_k = -1)} \\ &= \;\; \frac{2}{\sigma^2_{z_{ik}}} \Re\left(h^*_{ik}(y_i - \mu_{z_{ik}})\right). \end{aligned} \tag{5}$$

The LLR values computed at the observation nodes are passed to the variable nodes. Using these LLRs, the variable nodes compute the probabilities

$$p_i^{k+} \;\; \triangleq \;\; p_i(x_k = +1|\mathbf{y}) \;\; = \;\; \frac{\exp(\sum_{l\neq i} \Lambda_l^k)}{1 + \exp(\sum_{l\neq i} \Lambda_l^k)}, \tag{6}$$

and pass them back to the observation nodes. This message passing is carried out for a certain number of iterations. At the end, x_k is detected as

$$\widehat{x}_k \;\; = \;\; \mathrm{sgn}\Big(\sum_{i=1}^{2N_r} \Lambda_i^k\Big). \tag{7}$$

It has been shown in [9] that this BP algorithm with GAI, like LAS and RTS algorithms, exhibits 'large-system behavior,' where the bit error performance improves with increasing number of dimensions. In Fig. 1, the uncoded BER performance of this BP algorithm for 4-QAM (input data vector of size $2N_t$ with elements from $\{\pm1\}$) in 32×32 V-BLAST is also plotted. We can see that the performance is almost the same as that of RTS. In terms of complexity, the BP algorithm has the advantage of no need to compute an initial solution vector and $\mathbf{H}^T\mathbf{H}$, which is required in RTS. The per-symbol complexity of the BP algorithm for detection in V-BLAST is $O(N_t)$. A limitation with this BP approach is that it is not for general M-QAM. However, its good performance with $\{\pm1\}$ alphabet at lower complexities than RTS can be exploited to improve the higher-order QAM performance of RTS, as proposed in the following section.

III. Proposed Hybrid RTS-BP Algorithm for Large-MIMO Detection

In this section, we highlight the rationale behind the hybrid RTS-BP approach and present the proposed algorithm.

Why Hybrid RTS-BP?

The proposed hybrid RTS-BP approach is motivated by the the following observation we made in our RTS BER simulations. We observed that, at moderate to high SNRs, when an RTS output vector is in error, the least significant bits (LSB) of the data symbols are more likely to be in error than other bits. An analytical reasoning for this behavior can be given as follows.

Let $\mathbf{x}$ be the transmit vector and $\widehat{\mathbf{x}}$ be the corresponding output of the RTS detector. Let $\mathbb{A} = \{a_1, a_2, \cdots, a_M\}$ denote the M-PAM alphabet that x_i's take values from. Consider the symbol-to-bit mapping, where we can write the value of each entry of $\widehat{\mathbf{x}}$ as a linear combination of its constituent bits as

$$\widehat{x}_i \;=\; \sum_{j=0}^{N-1} 2^j\, \widehat{b}_i^{(j)}, \quad i = 1, \cdots, 2N_t, \tag{8}$$

where $N = \log_2 M$ and $\widehat{b}_i^{(j)} \in \{\pm1\}$. We note that the RTS algorithm outputs a local minima as the solution vector. So, $\widehat{\mathbf{x}}$, being a local minima, satisfies the following conditions:

$$\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|^2 \;\leq\; \|\mathbf{y} - \mathbf{H}(\widehat{\mathbf{x}} + \lambda_i \mathbf{e}_i)\|^2, \quad \forall i = 1, \cdots 2N_t, \tag{9}$$

where $\lambda_i = (a_q - \widehat{x}_i), q = 1, \cdots, M$, and $\mathbf{e}_i$ denotes the ith column of the identity matrix. Defining $\mathbf{F} \stackrel{\triangle}{=} \mathbf{H}^T\mathbf{H}$, $\mathbf{r} \stackrel{\triangle}{=} \mathbf{H}\widehat{\mathbf{x}}$, and denoting the ith column of $\mathbf{H}$ as $\mathbf{h}_i$, the conditions in (9) reduce to

$$2\lambda_i \mathbf{y}^T\mathbf{h}_i \;\leq\; 2\lambda_i \mathbf{r}^T\mathbf{h}_i + \lambda_i^2 f_{ii}, \tag{10}$$

where f_{ij} denotes the (i,j)th element of $\mathbf{F}$. Under moderate to high SNR conditions, ignoring the noise, (10) can be further reduced to

$$2(\mathbf{x} - \widehat{\mathbf{x}})^T\mathbf{f}_i \,\mathrm{sgn}(\lambda_i) \;\leq\; \lambda_i f_{ii}\,\mathrm{sgn}(\lambda_i), \tag{11}$$

where $\mathbf{f}_i$ denotes the ith column of $\mathbf{F}$. For Rayleigh fading, f_{ii} is chi-square distributed with $2N_t$ degrees of freedom with mean N_t. Approximating the distribution of f_{ij} to be normal with mean zero and variance $\frac{N_t}{4}$ for $i \neq j$ by central limit theorem, we can drop the $\mathrm{sgn}(\lambda_i)$ in (11). Using the fact that the minimum value of $|\lambda_i|$ is 2, (11) can be simplified as

$$\sum_{x_j \neq \widehat{x}_j} \Delta_j f_{ij} \;\leq\; f_{ii}, \tag{12}$$

where $\Delta_j = x_j - \widehat{x}_j$. Also, if $x_i = \widehat{x}_i$, by the normal approximation in the above

$$\sum_{x_j \neq \hat{x}_j} \Delta_j f_{ij} \;\sim\; \mathcal{N}\Big(0, \frac{N_t}{4} \sum_{x_j \neq \hat{x}_j} \Delta_j^2\Big). \tag{13}$$

Now, the LHS in (12) being normal with variance proportional to Δ_j^2 and the RHS being positive, it can be seen that Δ_i, $\forall i$ take smaller values with higher probability. Hence, the symbols of $\widehat{\mathbf{x}}$ are nearest Euclidean neighbors of their corresponding symbols of the global minima with high probability[3]. Now, because of the symbol-to-bit mapping in (8), $\widehat{x}_i$ will differ from its nearest Euclidean neighbors certainly in the LSB position, and may or may not differ in other bit positions. Consequently, the LSBs of the symbols in the RTS output $\widehat{\mathbf{x}}$ are least reliable.

The above observation then led us to consider improving the reliability of the LSBs of the RTS output using the BP algorithm in [9], and iterate between RTS and BP as follows.

Proposed Hybrid RTS-BP Algorithm:

Figure 2 shows the block schematic of the proposed hybrid RTS-BP algorithm. The following four steps constitute the proposed algorithm.

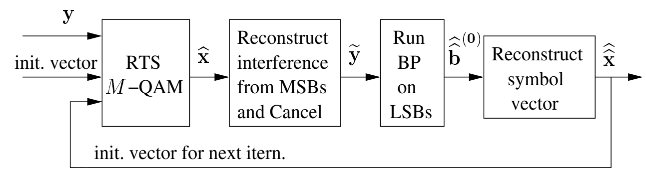

Fig. 2. Proposed hybrid RTS-BP algorithm.

- *Step 1:* Obtain $\widehat{\mathbf{x}}$ using the RTS algorithm. Obtain the output bits $\widehat{b}_i^{(j)}$, $i = 1, \cdots, 2N_t$, $j = 0, \cdots, N-1$, from $\widehat{\mathbf{x}}$ and (8).
- *Step 2:* Using the $\widehat{b}_i^{(j)}$'s from Step 1, reconstruct the interference from all bits other than the LSBs (i.e., interference from all bits other than $\widehat{b}_i^{(0)}$'s) as

$$\widetilde{\mathbf{I}} \;=\; \sum_{j=1}^{N-1} 2^j\, \mathbf{H}\,\widehat{\mathbf{b}}^{(j)}, \tag{14}$$

where $\widehat{\mathbf{b}}^{(j)} = \big[\widehat{b}_1^{(j)}, \widehat{b}_2^{(j)}, \ldots, \widehat{b}_{2N_t}^{(j)}\big]^T$. Cancel the reconstructed interference in (14) from $\mathbf{y}$ as

$$\widetilde{\mathbf{y}} \;=\; \mathbf{y} - \widetilde{\mathbf{I}}. \tag{15}$$

- *Step 3:* Run the BP-GAI algorithm in Sec. II-B on the vector $\widetilde{\mathbf{y}}$ in Step 2, and obtain an estimate of the

[3]Because x_i's and $\widehat{x}_i$'s take values from M-PAM alphabet, $\widehat{x}_i$ is said to be the Euclidean nearest neighbor of x_i if $|x_i - \widehat{x}_i| = 2$.

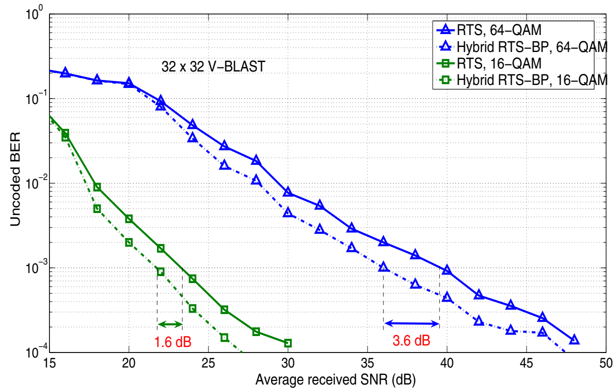

Fig. 3. Uncoded BER comparison between the proposed hybrid RTS-BP and the RTS for 16- and 64-QAM in 32×32 V-BLAST. *RTS-BP performs 3.6 dB better than RTS at* 10^{-3} *BER for 64-QAM.*

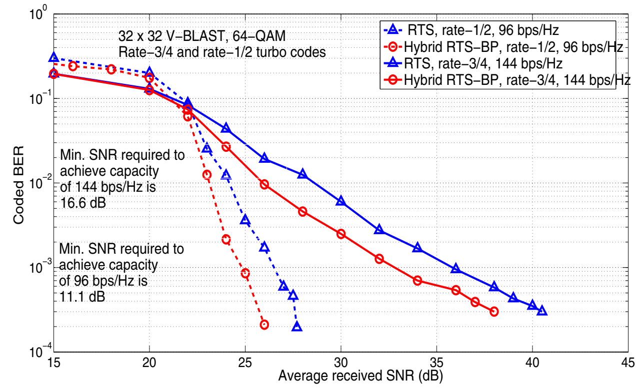

Fig. 4. Coded BER comparison between the proposed hybrid RTS-BP and the RTS in 32×32 V-BLAST with 64-QAM, i) rate-1/2 turbo code (96 bps/Hz), ii) rate-3/4 turbo code (144 bps/Hz).

LSBs. Denote this LSB output vector from BP as $\widehat{\widehat{\mathbf{b}}}^{(0)}$. Now, using $\widehat{\widehat{\mathbf{b}}}^{(0)}$ from the BP output, and the $\widehat{\mathbf{b}}^{(j)}$, $j = 1, \cdots, N-1$ from the RTS output in Step 1, reconstruct the symbol vector as

$$\widehat{\widehat{\mathbf{x}}} = \widehat{\widehat{\mathbf{b}}}^{(0)} + \sum_{j=1}^{N-1} 2^j \, \widehat{\mathbf{b}}^{(j)}. \qquad (16)$$

- *Step 4:* Repeat Steps 1 to 3 using $\widehat{\widehat{\mathbf{x}}}$ as the initial vector to the RTS algorithm.

The algorithm is stopped after a certain number of iterations between RTS and BP. Our simulations showed that two iterations between RTS and BP are adequate to achieve good improvement; more than two iterations resulted in only marginal improvement for the system parameters considered in the simulations. Since the complexity of BP part of RTS-BP is less than that of the RTS part, the order of complexity of RTS-BP is same as that of RTS.

IV. Performance of the Hybrid RTS-BP Detector

Here, we present the uncoded and coded BER performance of the proposed RTS-BP algorithm evaluated through simulations. Perfect knowledge of $\mathbf{H}$ is assumed at the receiver.

Performance in large V-BLAST Systems: Figure 3 shows the uncoded BER performance of 32×32 V-BLAST with 16- and 64-QAM. Performance of both RTS-BP as well as RTS are shown. It can be seen that, at an uncoded BER of 10^{-3}, RTS-BP performs better than RTS by about 3.6 dB for 64-QAM and by about 1.6 dB for 16-QAM. This illustrates the effectiveness of the proposed hybrid RTS-BP approach. Also, this improvement in uncoded BER is found to result in improved coded BER as well, as illustrated in Fig. 4. In Fig. 4, we have plotted the turbo coded BER of RTS-BP and RTS in 32×32 V-BLAST with 64-QAM for rate-1/2 (96 bps/Hz) and rate-3/4 (144 bps/Hz) turbo codes. It can be seen that, at a coded BER of 3×10^{-4}, RTS-BP performs better than RTS

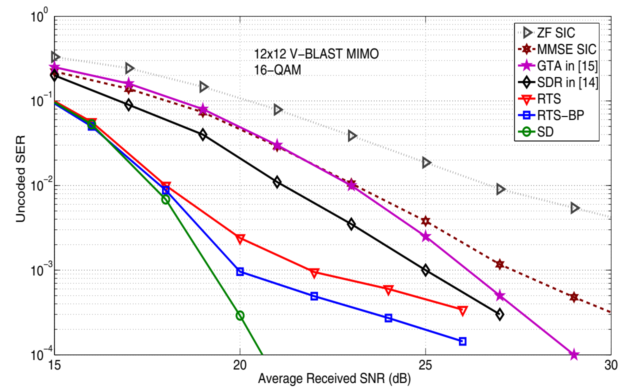

Fig. 5. Uncoded SER performance comparison of RTS-BP algorithm with other detectors for 12×12 V-BLAST MIMO with 16-QAM.

by about 1.5 dB at 96 bps/Hz and by about 2.5 dB at 144 bps/Hz.

Comparison with SDR and GTA algorithms: In Fig. 5, we present a comparison of the symbol error rate (SER) performance the RTS and RTS-BP algorithms with those of other low-complexity detection algorithms employing semi-definite relaxation (SDR) in [14] and Gaussian tree approximation (GTA) in [15] in a 12×12 V-BLAST MIMO system with 16-QAM. Sphere decoding (SD) and ZF-SIC/MMSE-SIC performances are also shown. We see that, while the performance of SDR and GTA are far from the SD performance, RTS-BP achieves closer to SD performance at a much less complexity than SD.

Performance in large non-orthogonal STBC MIMO systems: We also evaluated the BER performance of large non-orthogonal STBC MIMO systems with higher-order QAM using RTS-BP detection. Figure 6 shows the uncoded BER of 8×8 and 16×16 non-orthogonal STBC from cyclic division algebra [12] for 16-QAM. Here again, we can see that RTS-BP achieves better performance than RTS.

Performance in frequency-selective large V-BLAST systems: We note that the performance plots in Figs. 3 to 6 are for frequency-flat fading, which could be the fading scenario in

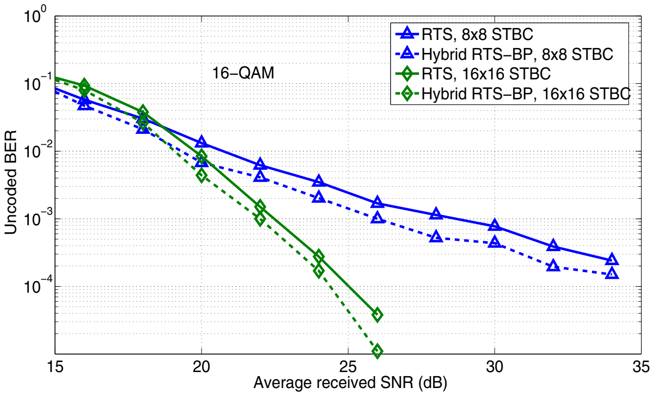

Fig. 6. Uncoded BER comparison between the hybrid RTS-BP and the RTS for 16-QAM in 8×8 and 16×16 non-orthogonal STBCs from CDA in [12].

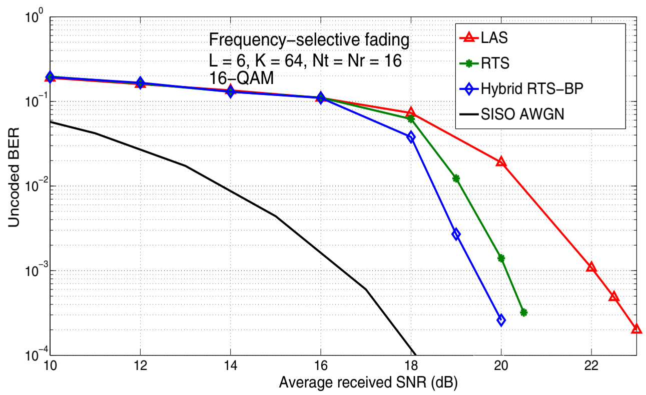

Fig. 7. Uncoded BER comparison between the hybrid RTS-BP, RTS, and LAS algorithms in 16×16 V-BLAST with 16-QAM in frequency-selective fading with $L = 6$, $K = 64$, and uniform power-delay profile.

MIMO-OFDM systems where a frequency-selective fading channel is converted to frequency-flat channels on multiple subcarriers. RTS-BP, RTS, and LAS algorithms, being suited to work well in large dimensions, can be applied to equalize signals in frequency-selective channels in large-MIMO systems. Following the equivalent real-valued system model of the form in (2) for frequency-selective MIMO systems developed in [13], we evaluated the performance of RTS-BP, RTS, and LAS algorithms in 16×16 V-BLAST with 16-QAM on a frequency selective channel with $L = 6$ equal energy multipath components and $K = 64$ symbols per frame. Figure 7 shows the superior performance of RTS-BP algorithm over RTS and LAS algorithms in this frequency-selective 16×16 MIMO system with 16-QAM.

V. Conclusions

We proposed a hybrid algorithm that exploited the good features of the RTS and BP algorithms to achieve improved bit error performance and nearness to capacity performance for M-QAM signals in large-MIMO systems at practically affordable low complexities. We illustrated the performance gains of the proposed hybrid approach over the RTS algorithm in flat-fading as well as frequency-selective fading for large V-BLAST and large non-orthogonal STBC MIMO systems. We note (e.g., from the performance plots for 64-QAM in Figs. 1 and 4) that further improvement in performance beyond what is achieved by the proposed hybrid RTS-BP algorithm could be possible. Exploring other algorithms based on this observation, we have proposed other variants of RTS including a layered RTS (LTS) algorithm [16], and a random-restart RTS (R3TS) algorithm [17] which achieved even closer to ML performance than the RTS-BP algorithm.

References

[1] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.,* vol. 10, no. 6, pp. 585-595, Nov. 1999.

[2] Gregory Breit et al, *802.11ac Channel Modeling*, doc. IEEE 802.11-09/0088r0, submission to Task Group TGac, 19 January 2009.

[3] K. Vishnu Vardhan, Saif K. Mohammed, A. Chockalingam, B. Sundar Rajan, "A low-complexity detector for large MIMO systems and multicarrier CDMA systems," *IEEE JSAC Spl. Iss. on Multiuser Detection for Adv. Commun. Sys. & Net.*, pp. 473-485, April 2008.

[4] Saif K. Mohammed, A. Chockalingam, and B. Sundar Rajan, "A low-complexity near-ML performance achieving algorithm for large-MIMO detection," *Proc. IEEE ISIT'2008*, Toronto, July 2008.

[5] Saif K. Mohammed, Ahmed Zaki, A. Chockalingam, B. Sundar Rajan, "High-rate space-time coded large-MIMO systems: Low-complexity detection and channel estimation," *IEEE Jl. Sel. Topics in Sig. Proc. (JSTSP): Spl. Iss. on Managing Complexity in Multiuser MIMO Systems*, vol. 3, no. 6, pp. 958-974, December 2009.

[6] N. Srinidhi, Saif K. Mohammed, A. Chockalingam, B. Sundar Rajan, "Low-complexity near-ML decoding of large non-orthogonal STBCs using reactive tabu search," *Proc. IEEE ISIT'2009*, Seoul, July 2009.

[7] N. Srinidhi, Saif K. Mohammed, A. Chockalingam, and B. Sundar Rajan, "Near-ML signal detection in large-dimension linear vector channels using reactive tabu search," Online arXiv:0911.4640v1 [cs.IT] 24 November 2009.

[8] M. Hansen, B. Hassibi, A. G. Dimakis, and W. Xu, "Near-optimal detection in MIMO systems using Gibbs sampling," *Proc. IEEE ICC'2009*, Honolulu, Hawaii, December 2009.

[9] Pritam Som, Tanumay Datta, A. Chockalingam, and S. Sundar Rajan, "Improved large-MIMO detection based on damped belief propagation," *Proc. IEEE ITW'2010*, Cairo, January 2010.

[10] Y. Wang and K. Roy, "A new reduced complexity sphere decoder with true lattice boundary awareness for multi-antenna systems," *IEEE ISCAS'2005*, vol. 5, pp. 4963-4966, May 2005.

[11] L. G. Barbero and J. S. Thompson, "Fixing the complexity of the sphere decoder for MIMO detection," *IEEE Trans. Wireless Commun.,* vol. 7, no. 6, pp. 2131-2142, June 2008.

[12] B. A. Sethuraman, B. Sundar Rajan, and V. Shashidhar, "Full-diversity high-rate space-time block codes from division algebras," *IEEE Trans. Inform. Theory*, pp. 2596-2616, October 2003.

[13] N. Srinidhi, Saif K. Mohammed, and A. Chockalingam, "A reactive tabu search based equalizer for severely delay-spread UWB MIMO-ISI channels," *Proc. IEEE GLOBECOM'2009*, December 2009.

[14] N. D. Sidiropoulos and Z.Q. Luo, "A semidefinite relaxation approach to MIMO detection for high-order QAM constellations," *IEEE Sig. Proc. Letters*, vol. 13, no. 9, pp. 525-528, September 2006.

[15] J. Goldberger and A. Leshem, "MIMO detection for high-order QAM based on a Gaussian tree approximation," Online arXiv:1001.5364v1[cs.IT] 29 Jan 2010.

[16] N. Srinidhi, Tanumay Datta, A. Chockalingam, B. Sundar Rajan, "Layered tabu search algorithm for large-MIMO detection and a lower bound on ML performance," *IEEE GLOBECOM'2010*, Miami, December 2010.

[17] Tanumay Datta, N. Srinidhi, A. Chockalingam, and B. Sundar Rajan, "Random-restart reactive tabu search algorithm for detection in large-MIMO systems," *to appear in IEEE Commun. Letters*, December 2010.