Performance Analysis of a (3, L) Selection Combining Scheme for Binary NCFSK Signals on Rayleigh Fading Channels

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Abstract*— In a previous paper, we derived the optimum selection combining (OSC) scheme for binary noncoherent FSK (NCFSK) signals on independent (but not necessarily identically distributed) Rayleigh fading channels with L-antenna diversity reception. In the OSC scheme, the diversity branch having the largest magnitude of the logarithm of the ratio of the a posteriori probabilities (log-APP ratio – LAPPR) of the transmitted information bit is chosen. In this paper, we derive the bit error performance of a $(3, L)$ selection combining scheme for binary NCFSK signals which combines the three branches whose LAPPR magnitudes are the largest among the available L branches in i.i.d Rayleigh fading. Numerical results for this $(3, L)$ selection scheme show that, for $L = 5$, combining the three branches with the largest LAPPR magnitudes yields almost the full performance of the square-law combining of all the $L = 5$ branches. For $L = 7$, the performance of combining the three branches with the largest LAPPR magnitudes is just about 0.2 dB worse from the performance of square-law combining of all the $L = 7$ branches. We also compare the performance of the proposed $(3, L)$ selection scheme with the $(3, L)$ selection combining scheme of Chyi *et al.

I. Introduction

Diversity reception is a well known technique for mitigating the effects of fading in wireless communication systems [1]. Diversity reception can improve the wireless link quality and reduce the link budget. Combining of diversity signals can be done either coherently or noncoherently. Typical diversity combining schemes include maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), and generalized selection (hybrid EGC/SC) combining. In a (K, L) generalized selection combining (GSC) scheme, K out of the L available branches are chosen and combined [2]. In this paper, we are concerned with the performance of generalized selection combining of binary noncoherent FSK (NCFSK) signals on Rayleigh fading channels.

In a companion paper [3], we derived the optimum selection combining (OSC) scheme for noncoherent binary FSK signals on independent (but not necessarily identically distributed) Rayleigh fading channels with L-antenna diversity reception. In the OSC scheme of [3], the diversity branch having the largest magnitude of the logarithm of the ratio of the a posteriori probabilities (log-APP ratio – LAPPR) of the transmitted information bit is chosen. It was shown that choosing the branch with the largest magnitude of the LAPPR indeed minimizes the probability of bit error, and hence is optimum.

Our focus now in this paper is the performance analysis of a (K, L) GSC scheme for binary NCFSK signals which combines the K branches whose LAPPR magnitudes are the largest among the L available branches. Specifically, we derive the bit error probability expression for a $(3, L)$ GSC scheme on i.i.d Rayleigh fading channels. We compare the performance of this proposed $(3, L)$ GSC scheme with both the square-law combining of all the L branches as well as the $(3, L)$ selection scheme of Chyi *et al* [4]. For $L = 5$, the proposed combining of the three branches with the largest LAPPR magnitudes (i.e., (3, 5) GSC scheme) yields almost the full performance of the square-law combining of all the $L = 5$ branches. For $L = 7$, the performance of the proposed (3, 7) GSC scheme is just about 0.2 dB worse than the performance of square-law combining of all the $L = 7$ branches.

The rest of the paper is organized as follows. In Section II, we introduce the system model and derive the log-a posteriori probabilities ratio (LAPPR) of bit detection. In Section III, we derive the bit error probability expression for the proposed $(3, L)$GS C scheme. In Section IV, we derive the bit error probability expression for the $(3, L)$ selection combining scheme of Chyi *et al.* Results and discussions are presented in Section V. Conclusions are given in Section VI.

II. System Model

We assume that the transmitted symbols are BFSK modulated with $\underline{s}_0 = [1, 0]^T$ and $\underline{s}_1 = [0, 1]^T$ denoting the BFSK symbols, associated with the messages m_0 and m_1, respectively. The complex orthonormal basis functions $\phi_1(t) = exp(j2\pi f_1 t)$ and $\phi_2(t) = exp(j2\pi f_2 t)$ represent the transmitted information symbol $\underline{s}_m = [s_{m,x}, s_{m,y}]$. That is, $s_m(t) = s_{m,x}\phi_1(t) + s_{m,y}\phi_2(t)$.

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The transmitted symbols are passed through a fading channel and noise gets added to them at the receiver front end. We assume that the fading process is slow, frequency non-selective and remains constant over one symbol interval. Assuming perfect symbol timing at the receiver, the equivalent low pass representation of the received symbols, after the non-coherent demodulation as shown in Fig. 1, is given by [5]

$$\underline{r}_c^{(l)} = \alpha^{(l)}\underline{s}_i \cos\theta^{(l)} + \underline{n}_c^{(l)}, \quad (1)$$

$$\underline{r}_s^{(l)} = \alpha^{(l)}\underline{s}_i \sin\theta^{(l)} + \underline{n}_s^{(l)}, \qquad l = 1, 2, \ldots, L, \quad (2)$$

where $\underline{s}_i$ is the transmitted BFSK signal point corresponding to the message m_i, $i \in \{0,1\}$, $\underline{r}_c^{(l)}$ and $\underline{r}_s^{(l)}$ denote the vector valued outputs of the quadrature demodulators at the l^{th} antenna. The random phase on the l^{th} antenna path, $\theta^{(l)}$, is distributed uniformly over $[0, 2\pi]$. $\alpha^{(l)}$ is the random fade experienced by the transmitted symbol $\underline{s}_i$ on the l^{th} antenna path. The $\alpha^{(l)}$'s are assumed to be independent Rayleigh random variables with a density function on the l^{th} antenna given by $f_{\alpha^{(l)}}(a) = \frac{2a}{\Omega_l} e^{-\frac{a^2}{\Omega_l}}$, $a \geq 0$. The second moment of $\alpha^{(l)}$ is set to Ω_l (i.e., $E([\alpha^{(l)}]^2) = \Omega_l$), and $\underline{n}_c^{(l)}$ and $\underline{n}_s^{(l)}$ denote the in phase and quadrature phase noise vectors on l^{th} antenna path whose components have zero mean and variance $\sigma^2 = N_0/2E_b$, where E_b/N_0 is the SNR per bit. It is noted that the $\underline{r}_c^{(l)}$, $\underline{r}_s^{(l)}$, $\underline{s}_i$, $\underline{n}_c^{(l)}$, $\underline{n}_s^{(l)}$ are vectors each having two dimensions. With the assumption of signal point $\underline{s}_1$ being transmitted corresponding to the data bit '1', we have

$$\underline{r}_c^{(l)} = [r_{c,x}^{(l)}, r_{c,y}^{(l)}] = [n_{c,x}^{(l)}, \alpha^{(l)}\cos\theta^{(l)} + n_{c,y}^{(l)}], \quad (3)$$

$$\underline{r}_s^{(l)} = [r_{s,x}^{(l)}, r_{s,y}^{(l)}] = [n_{s,x}^{(l)}, \alpha^{(l)}\sin\theta^{(l)} + n_{s,y}^{(l)}]. \quad (4)$$

The LAPPR of the transmitted information symbol $\underline{s}_i$ on the l^{th} antenna path is given by

$$LAPPR^{(l)} \triangleq \Lambda^{(l)} = \log\left(\frac{\text{Prob}(\underline{s}_i = \underline{s}_1 | \underline{r}_c^{(l)}, \underline{r}_s^{(l)})}{\text{Prob}(\underline{s}_i = \underline{s}_0 | \underline{r}_c^{(l)}, \underline{r}_s^{(l)})}\right). \quad (5)$$

For equally probable message signals,

$$\Lambda^{(l)} = \log\left(\frac{f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)} | \underline{s}_i = \underline{s}_1)}{f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)} | \underline{s}_i = \underline{s}_0)}\right). \quad (6)$$

The quantity $f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)} | \underline{s}_i = \underline{s}_m)$, $m \in \{0,1\}$, can be calculated as follows [6]:

$$f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_i = \underline{s}_m) = E_{\alpha^{(l)}}\left\{E_{\theta^{(l)}}\left[f_{\underline{r}_c^{(l)}\underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_m, \alpha^{(l)}, \theta^{(l)})\right]\right\}, \quad (7)$$

where $E_{\alpha^{(l)}}[\cdot]$ and $E_{\theta^{(l)}}[\cdot]$ denote the expectation operations with respect to $\alpha^{(l)}$ and $\theta^{(l)}$, respectively. The quantity $f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_m, \alpha^{(l)}, \theta^{(l)})$ can be calculated as

$$= f_{\underline{n}_c^{(l)}}(\underline{x} - \alpha^{(l)}\underline{s}_m \cos\theta^{(l)}) f_{\underline{n}_s^{(l)}}(\underline{y} - \alpha^{(l)}\underline{s}_m \sin\theta^{(l)})$$
$$\sim e^{-[\alpha^{(l)}]^2 \frac{E_b}{N_0}} e^{\frac{2E_b}{N_0}\left(\underline{x}\cdot\underline{s}_m \alpha^{(l)}\cos\theta^{(l)} + \underline{y}\cdot\underline{s}_m \alpha^{(l)}\sin\theta^{(l)}\right)}$$
$$\sim e^{-[\alpha^{(l)}]^2 \frac{E_b}{N_0}} e^{\frac{2E_b X_m^{(l)}}{N_0}\alpha^{(l)}\cos\left(\theta^{(l)} - \phi_m^{(l)}\right)}, \quad (8)$$

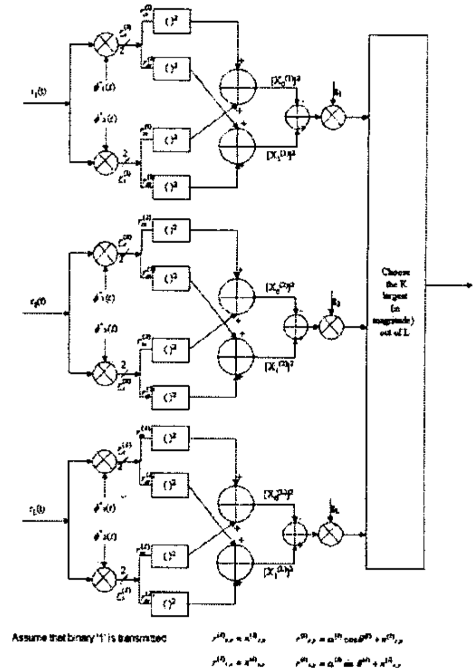

Fig. 1. Proposed (K, L) GSC Scheme for Binary NCFSK Signals.

where $X_m^{(l)} = \sqrt{(\underline{x}\cdot\underline{s}_m)^2 + (\underline{y}\cdot\underline{s}_m)^2}$ and $\phi_m^{(l)} = \tan^{-1}\left(\frac{\underline{y}\cdot\underline{s}_m}{\underline{x}\cdot\underline{s}_m}\right)$. From (8), $f_{\underline{r}^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_m, \alpha^{(l)})$ is equal to

$$= E_{\theta^{(l)}}\left(\frac{E_b}{\pi N_0} e^{-(2+[\alpha^{(l)}]^2)\frac{E_b}{N_0}} e^{\frac{2X_m^{(l)}E_b}{N_0}\alpha^{(l)}\cos(\theta^{(l)} - \phi_m^{(l)})}\right)$$
$$\sim e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} I_0\left(2\alpha^{(l)}\frac{X_m^{(l)}E_b}{N_0}\right), \quad (9)$$

where $I_0(\cdot)$ is the modified Bessel function of the zeroth order and first kind [5]. Finally, we can obtain $f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_m)$, from (9), as

$$f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y} | \underline{s}_m) \sim E_{\alpha^{(l)}}\left(e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} I_0\left(2\alpha^{(l)}\frac{X_m^{(l)}E_b}{N_0}\right)\right)$$
$$\sim \frac{1}{1+\Omega_l\gamma} e^{\frac{[X_m^{(l)}]^2\Omega_l\gamma^2}{1+\Omega_l\gamma}}, \quad (10)$$

where $\gamma = E_b/N_0$. Finally, substituting (10) in (6) and scaling by γ^2, we obtain

$$\Lambda^{(l)} = g_l\left([X_1^{(l)}]^2 - [X_0^{(l)}]^2\right), \quad (11)$$

where g_l is the weighting factor on l^{th} antenna path and is given by

$$g_l = \frac{\Omega_l}{1+\Omega_l\gamma}. \quad (12)$$

For i.i.d Rayleigh fading, $\Omega_1 = \Omega_2 = \ldots = \Omega_L = 1$. We choose the K diversity branches whose magnitude of the LAPPRs in (6) are the largest among the L available branches. The special cases of $K = 1$ and $K = L$ correspond to the optimum SC scheme presented in [3] and the traditional square-law combining of all the L diversity branches, respectively.

III. Proposed (K, L) Selection Combining

In this section, we consider the derivation of the bit error probability expression for combining K branches whose LAPPR magnitudes are the largest among the available L branches. We restrict the analysis to the case of i.i.d Rayleigh fading, as the analysis appears to be prohibitively difficult for independent fading channels. The decision statistic for the (K,L)GS C scheme is the sum of the first K log-likelihood ratios arranged in the decreasing order of their (absolute) values, and is given by

$$Z^{GSC} = \sum_{k=1}^{K} \Lambda_{(k)}, \qquad |\Lambda_{(1)}| \geq |\Lambda_{(2)}| \geq \cdots \geq |\Lambda_{(K)}|, \quad (13)$$

where $\Lambda_{(1)}, \Lambda_{(2)}, \ldots, \Lambda_{(L)}$ are the order statistics [7] of the random variables $\Lambda^{(1)}, \Lambda^{(2)}, \ldots, \Lambda^{(L)}$. Note that the notation used here is that $\Lambda^{(1)}, \Lambda^{(2)} \ldots, \Lambda^{(L)}$ are the original log-likelihood ratios on each antenna path defined in (6), whereas $\Lambda_{(1)}, \Lambda_{(2)} \ldots, \Lambda_{(L)}$ are the log-likelihood ratios obtained by sorting the original log-likelihood ratios in decreasing order of absolute magnitude. Also, for a later derivation in Sec. III-A, we define $\Lambda_{[1]}, \Lambda_{[2]}, \ldots, \Lambda_{[L]}$ to be another order statistic of the original log-likelihood ratios $\Lambda^{(1)}, \Lambda^{(2)}, \ldots, \Lambda^{(L)}$ such that $\Lambda_{[1]} \geq \Lambda_{[2]}, \ldots, \Lambda_{[L]}$. For example, for $L = 3$, if $\Lambda^{(1)} = -7$, $\Lambda^{(2)} = 6$ and $\Lambda^{(3)} = -9$, then $\Lambda_{(1)} = -9$, $\Lambda_{(2)} = -7$ and $\Lambda_{(3)} = 6$, whereas $\Lambda_{[1]} = 6$, $\Lambda_{[2]} = -7$ and $\Lambda_{[3]} = -9$.

As shown in [8], the decision statistic Z^{GSC} does not change its sign for $K = 2$, because $|\Lambda_{(1)}| \geq |\Lambda_{(2)}|$. In other words, the sign of the decision statistic Z^{GSC} is same as that of the optimum SC (OSC) receiver in [3], which implies that the bit error probability of (2,L) GSC scheme is same as the bit error probability of the OSC scheme. One difficulty in analyzing the performance of the above (K,L) GSC receiver for any K, for a given,L is that the decision regions over which the error probability is to be evaluated grow exponentially as a function of K [8]. In what follows, we derive the error probability expression for any L, but K is restricted to 3, i.e., for the (3,L) GSC scheme.

Assuming that a binary '1' is transmitted, a decision error occurs under the following conditions.

1) $\mathcal{R}_1$: $\Lambda_{(1)} < 0$ and $\Lambda_{(2)} < 0$
2) $\mathcal{R}_2$: $\Lambda_{(1)} < 0, \Lambda_{(2)} > 0$, and $\Lambda_{(3)} < 0$
3) $\mathcal{R}_3$: $\Lambda_{(1)} < 0$, $\Lambda_{(2)} > 0$, $\Lambda_{(3)} > 0$, and $\left(\Lambda_{(1)} + \Lambda_{(2)} + \Lambda_{(3)}\right) < 0$
4) $\mathcal{R}_4$: $\Lambda_{(1)} > 0$, $\Lambda_{(2)} < 0$, $\Lambda_{(3)} < 0$, and $\left(\Lambda_{(1)} + \Lambda_{(2)} + \Lambda_{(3)}\right) < 0$.

It is to be noted that the above error events are mutually exclusive and therefore the probability of error is given by

$$P_e^{(3,L)GSC} = \Pr(\mathcal{R}_1) + \Pr(\mathcal{R}_2) + \Pr(\mathcal{R}_3) + \Pr(\mathcal{R}_4). \quad (14)$$

In the following sub-section, we derive the probability of the error events $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$.

A. Derivation of $P_e^{(3,L)GSC}$

The error events $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4$ can be further simplified as follows:

$$\begin{aligned} \Pr(\mathcal{R}_1) &= \Pr(\Lambda_{(1)} < 0,\ \Lambda_{(2)} < 0) \\ &= \Pr(\Lambda_{[1]} < 0) + \Pr(\Lambda_{[1]} > 0,\ \Lambda_{[L-1]} < -\Lambda_{[1]}) \\ &= \int_{x=-\infty}^{0} f_{\Lambda_{[1]}}(x)\,dx \\ &\quad + \int_{x=0}^{\infty} \int_{y=-\infty}^{-x} f_{\Lambda_{[1]},\Lambda_{[L-1]}}(x,y)\,dx\,dy. \end{aligned} \quad (15)$$

From order statistics [7], we have

$$f_{\Lambda_{[1]}}(x) = \frac{L!}{(L-1)!} f_\Lambda(x)[F_\Lambda(x)]^{L-1}, \quad (16)$$

and

$$f_{\Lambda_{[1]},\Lambda_{[L-1]}}(x,y) = \frac{L!}{(L-3)!} F_\Lambda(y) f_\Lambda(y) f_\Lambda(x) \left[F_\Lambda(x) - F_\Lambda(y)\right]^{L-3} \quad (17)$$

Substituting (16) and (17) in (15) and performing the integration, we obtain

$$\begin{aligned} \Pr(\mathcal{R}_1) = & \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^L + \frac{L!}{(L-3)!} \sum_{k=0}^{L-3} \sum_{j=0}^{L-3-k} (-1)^{k+j} \binom{L-3}{k} \\ & \cdot \binom{L-3-k}{j} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{k+2} \frac{\alpha}{k+2} \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^j \\ & \cdot \frac{1}{\lambda_1(j+1)+\lambda_2(k+2)}. \end{aligned} \quad (18)$$

To derive $\Pr(\mathcal{R}_2)$, we simplify the expression for $\Pr(\mathcal{R}_2)$ as follows:

$$\begin{aligned} \Pr(\mathcal{R}_2) &= \Pr(\Lambda_{(1)} < 0, \Lambda_{(2)} > 0, \Lambda_{(3)} < 0) \\ &= \Pr(\Lambda_{[1]} + \Lambda_{[L]} < 0, \Lambda_{[1]} + \Lambda_{[L-1]} > 0, \\ &\quad \Lambda_{[2]} + \Lambda_{[L-1]} < 0, \Lambda_{[2]} > 0) + \\ &\quad \Pr(\Lambda_{[1]} + \Lambda_{[L]} < 0, \Lambda_{[1]} + \Lambda_{[L-1]} > 0, \Lambda_{[2]} < 0) \\ &= \int_{x=0}^{\infty} \int_{z=-x}^{0} \int_{y=0}^{-z} \int_{w=-\infty}^{-x} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\,dw\,dy\,dz\,dx \\ &\quad + \int_{x=0}^{\infty} \int_{z=-x}^{0} \int_{y=z}^{0} \int_{w=-\infty}^{-x} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\,dw\,dy\,dz\,dx. \end{aligned} \quad (19)$$

From [7], we have

$$f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w) = \frac{L!}{(L-4)!} f_\Lambda(x) f_\Lambda(y) f_\Lambda(z) f_\Lambda(w) \cdot [F_\Lambda(y) - F_\Lambda(z)]^{L-4}. \quad (20)$$

Substituting (20) in (19) and performing the integration, we obtain

$$\Pr(\mathcal{R}_2) = \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{L-4-k} (-1)^{j+k} \binom{L-4}{k} \binom{L-4-k}{j} \cdot \frac{\lambda_1^k \lambda_2^{j-1}}{(\lambda_1+\lambda_2)^{k+j+1}} \cdot \frac{1}{\lambda_1+\lambda_2(k+2)} \frac{1}{\lambda_2(k+2)+\lambda_1(j+2)} + \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} (-1)^k \cdot \binom{L-4}{k} \cdot \frac{\lambda_1^{L-4}}{\lambda_2^2(\lambda_1+\lambda_2)^{L-3}} \frac{1}{\lambda_1+\lambda_2(k+2)} \frac{1}{\lambda_1+\lambda_2(L-1)}. \quad (21)$$

To derive $\Pr(\mathcal{R}_3)$, the expression $\Pr(\mathcal{R}_3)$ can be simplified as follows:

$$\begin{aligned} \Pr(\mathcal{R}_3) &= \Pr(\Lambda_{(1)} < 0, \Lambda_{(2)} > 0, \Lambda_{(3)} > 0, \\ &\quad \Lambda_{(1)} + \Lambda_{(2)} + \Lambda_{(3)} < 0) \\ &= \Pr(\Lambda_{[1]} + \Lambda_{[L]} < 0, \Lambda_{[1]} + \Lambda_{[2]} + \Lambda_{[L]} < 0, \Lambda_{[L-1]} > 0) + \\ &\quad \Pr(\Lambda_{[1]} + Lambda_{[L]} < 0, \Lambda_{[1]} + \Lambda_{[2]} + \Lambda_{[L]} < 0, \\ &\quad \Lambda_{[2]} + \Lambda_{[L-1]} > 0, \Lambda_{[L-1]} < 0) \\ &= \int_{x=0}^{\infty} \int_{y=0}^{x} \int_{z=0}^{y} \int_{w=-\infty}^{-x-y} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\, dw\, dz\, dy\, dx \\ &\quad + \int_{x=0}^{\infty} \int_{y=0}^{x} \int_{z=-y}^{0} \int_{w=-\infty}^{-x-y} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\, dw\, dz\, dy\, dx. \end{aligned} \quad (22)$$

Again, substituting (20) in (22) and integrating, we get

$$\Pr(\mathcal{R}_3) = \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{k} \sum_{l=0}^{L-4-k} \binom{L-4}{k} \binom{L-4-k}{l} \binom{k}{j} \cdot \frac{\lambda_2^{l+j-1}}{(\lambda_1+\lambda_2)^{l+j+1}} \frac{1}{\lambda_1(l+2)+2\lambda_2} \frac{1}{\lambda_1(3+l+j)+2\lambda_2} + \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{L-4-k} (-1)^{k+j} \binom{L-4}{k} \binom{L-4-k}{j} \frac{\lambda_1^k \lambda_2^{j-1}}{(\lambda_1+\lambda_2)^{k+j+1}} \cdot \frac{1}{\lambda_1(j+2)+2\lambda_2} \frac{1}{\lambda_1(j+2)+\lambda_2(k+3)}. \quad (23)$$

Finally, the expression for $\Pr(\mathcal{R}_4)$ can be obtained as

$$\begin{aligned} \Pr(\mathcal{R}_4) &= \Pr(\Lambda_{(1)} > 0, \Lambda_{(2)} < 0, \Lambda_{(3)} < 0, \Lambda_{(1)} + \Lambda_{(2)} + \Lambda_{(3)} < 0) \\ &= \Pr(\Lambda_{[1]} + \Lambda_{[L]} > 0, \Lambda_{[2]} + \Lambda_{[L-1]} < 0, \\ &\quad \Lambda_{[1]} + \Lambda_{[L-1]} + \Lambda_{[L]} < 0, \Lambda_{[2]} > 0) + \\ &\quad \Pr(\Lambda_{[1]} + \Lambda_{[L]} > 0, \Lambda_{[1]} + \Lambda_{[L-1]} + \Lambda_{[L]} < 0, \Lambda_{[2]} < 0) \\ &= \int_{w=-\infty}^{0} \int_{z=w}^{0} \int_{y=0}^{-z} \int_{x=-w}^{-z-w} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\, dx\, dy\, dz\, dw \\ &\quad + \int_{w=-\infty}^{0} \int_{z=w}^{0} \int_{y=z}^{0} \int_{x=-w}^{-z-w} f_{\Lambda_{[1]},\Lambda_{[2]},\Lambda_{[L-1]},\Lambda_{[L]}}(x,y,z,w)\, dx\, dy\, dz\, dw. \end{aligned} \quad (24)$$

Substituting (20) in (24) and integrating, we obtain

$$\Pr(\mathcal{R}_4) = \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{L-4-k} (-1)^{j+k} \binom{L-4}{k} \binom{L-4-k}{j} \cdot \frac{\lambda_1^{k-1} \lambda_2^j}{(\lambda_1+\lambda_2)^{k+j+1}(j+1)} \left[\frac{1}{A(k)} - \frac{1}{B(j,k)}\right] + \frac{L!\alpha^4}{(L-4)!} \sum_{k=0}^{L-4} (-1)^k \binom{L-4}{k} \cdot \frac{\lambda_1^{L-3-1}}{\lambda_2(L-3-k)(\lambda_1+\lambda_2)^{L-3}} \left[\frac{1}{A(k)} - \frac{1}{C}\right]. \quad (25)$$

where $A(k) = (\lambda_1 + \lambda_2(k+2))(2\lambda_1 + \lambda_2(k+2))$, $B(j,k) = (\lambda_1(j+2) + \lambda_2(k+2))(\lambda_1(j+3) + \lambda_2(k+2))$, and $C = (\lambda_1 + \lambda_2(L-1))(2\lambda_1 + \lambda_2(L-1))$. Combining (18), (21), (23) and (25), we obtain the final expression for $P_e^{(3,L)GSC}$.

IV. Chyi's (K, L) Selection Combining

In [4], Chyi *et al* studied a selection combining scheme in which the diversity branch with the largest square-law detector output is chosen. But [4] does not consider a (K, L)selection combining scheme which combines the K branches whose square-law detector outputs are the largest among the L available branch outputs. In order to compare with our proposed (K, L) GSC scheme, we, in this section, derive the bit error probability expression for the (K, L) selection combining scheme of Chyi *et al.*

We denote $X_1^{1:L}, X_1^{2:L}, \ldots, X_1^{L:L}$ and $X_0^{1:L}, X_0^{2:L}, \ldots, X_0^{L:L}$ as the order statistics of $X_1^{(1)}, X_1^{(2)}, \ldots, X_1^{(L)}$ and $X_0^{(1)}, X_0^{(2)}, \ldots, X_0^{(L)}$, arranged in decreasing order, respectively. Further, define $U = \sum_{j=1}^{K} [X_1^{j:L}]^2$ and $V = \sum_{j=1}^{K} [X_0^{j:L}]^2$. The (K, L) Chyi scheme compares the two decision statistics U and V in the bit detection process. Assuming that the bit '1' is transmitted, the bit error probability, $P_e^{(K,L)Chyi}$, is given by

$$\begin{aligned} P_e^{(K,L)Chyi} &= \text{Prob}(U < V) = \text{Prob}(U - V < 0) \\ &= F_{U-V}(0), \end{aligned} \quad (26)$$

where $F_{U-V}(\cdot)$ is the cumulative distribution function (CDF) of the random variable $U - V$. We use the Laplace transform approach in [9] and the moment generating functions of the random variables U and V to simplify the above equation. From [9], (26) can be simplified as

$$\begin{aligned} P_e^{(K,L)Chyi} &= \frac{1}{2} - \frac{1}{\pi} \int_{t=0}^{\infty} \frac{\text{Imag}\left[\mathcal{L}_{U-V}(-jt)\right]}{t} dt \\ &= \frac{1}{2} - \frac{2}{\pi} \int_{\theta=0}^{\frac{\pi}{2}} \frac{\text{Imag}\left[\mathcal{L}_{U-V}(-j\tan\theta)\right]}{\sin 2\theta} d\theta. \end{aligned} \quad (27)$$

Here, $\mathcal{L}_Z(s) = E[e^{-sZ}]$ is the Laplace transform of the random variable Z and Imag$[\mathcal{W}]$ the imaginary component of complex number $\mathcal{W}$. Since U and V are independent, we have $\mathcal{L}_{U-V}(s) = E[e^{-s(U-V)}] = \mathcal{L}_U(s)\mathcal{L}_V(-s)$. Observe that each $[X_1^{(l)}]^2$ is exponentially distributed with mean, $\mu_U = \frac{1+\gamma}{\gamma}$ and each $[X_0^{(l)}]^2$ is exponentially distributed with mean, $\mu_V = \frac{1}{\gamma}$. With this and from [2], the Laplace transforms of U and V are given by

$$\mathcal{L}_U(s) = \left(\frac{1}{1+\mu_U s}\right)^K \prod_{l=K+1}^{L} \frac{1}{1+\frac{\mu_U K s}{l}}, \quad (28)$$

$$\mathcal{L}_V(s) = \left(\frac{1}{1+\mu_V s}\right)^K \prod_{l=K+1}^{L} \frac{1}{1+\frac{\mu_V K s}{l}}, \quad (29)$$

respectively. Substituting (28) and (29) in (27) and performing numerical integration, we can evaluate the expression $P_e^{(K,L)Chyi}$.

V. Results and Discussion

Fig. 2 shows the performance of the proposed (3,L) GSC scheme for i.i.d Rayleigh fading when $L = 5$, 6 and 7. The performances of both the optimum selection combining $(1, L)$ scheme proposed in [3] as well as the scheme which square-law combines all the L branches are also plotted for comparison. It is observed that the proposed $(3, L)$ GSC scheme performs better than the OSC scheme in [3], and performs slightly poorer compared the square-law combining of all the L branches. For $L = 5$, the $(3, L)$ GSC scheme yields almost the same performance as the L-branch square-law combiner. For $L = 7$, the $(3, L)$ GSC scheme performs just about 0.2 dB worse than the performance of square-law combining of all the $L = 7$ branches. In Fig. 3, the performance of both the proposed $(3, L)$ GSC scheme as well as Chyi's $(3, L)$ scheme are plotted for $L = 5$. The performances of OSC and the square-law combining of all L branches are also shown. It can be seen that the $(3, L)$ GSC scheme performs slightly better than Chyi's $(3, L)$ scheme, and that both the schemes perform very close to the square law combining of all $L = 5$ branches.

VI. Conclusions

We derived the bit error performance of a $(3, L)$ selection combining scheme for binary NCFSK signals which combines the three branches whose LAPPR magnitudes are the largest among the available L branches in i.i.d Rayleigh fading. For $L = 5$, we showed that combining the three branches with the largest LAPPR magnitudes yields almost the full performance of the square-law combining of all the $L = 5$ branches. For $L = 7$, it was shown that the performance of combining the three branches with the largest LAPPR magnitudes is just about 0.2 dB worse than the performance of square-law combining of all the $L = 7$ branches. We also compared the performance of the proposed $(3, L)$ selection scheme with the $(3, L)$ selection combining scheme of Chyi.

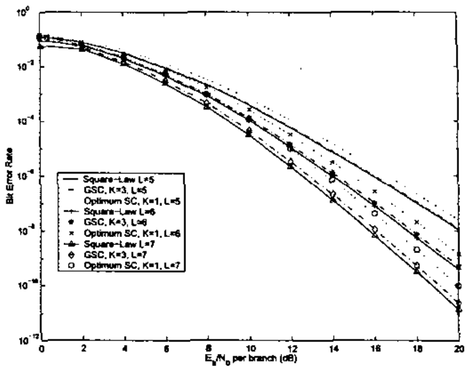

Fig. 2. Bit error performance of the proposed (3,L) GSC scheme on i.i.d Rayleigh fading for $L = 5$, 6 and 7.

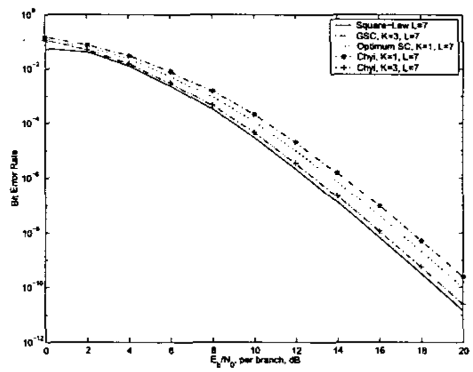

Fig. 3. Bit error performance of the proposed (3,L)G SC scheme and the Chyi's (3, L) scheme on i.i.d Rayleigh fading for $L = 5$.

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