Decode-and-Forward Cooperative Multicast with Space Shift Keying

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***Abstract*—In this paper, we consider space-shift keying (SSK) in dual-hop decode-and-forward (DF) cooperative *multicast* networks, where a source node communicates with *multiple destination nodes* with the help of relay nodes. We consider a topology consisting of a single source, two relays, and multiple destinations. For such a system, we propose a scheme to select a single relay among the two when SSK is used for transmission on each of the links in the cooperative system. We analyze the end-to-end average bit error probability (ABEP) of this system. For binary SSK, we derive an exact expression for the ABEP in closed-form. Analytical results exactly match the simulation results validating the analysis. For non-binary SSK, we derive an approximate ABEP expression, where the analytical ABEP results closely follow simulation results. We also derive the diversity order of the system through asymptotic ABEP analysis.**

Keywords: *Cooperative multicast, space-shift keying, decode-and-forward, ABEP analysis.*

I. Introduction

Conventional multi-antenna wireless systems employ multiple transmit radio frequency (RF) chains and transmit multiple transmit data streams simultaneously in order to boost spectral efficiency. However, the use of multiple transmit RF chains has several drawbacks, such as inter-antenna synchronization [1], backoff on individual power amplifiers [2], increased hardware complexity, size and cost. Therefore, multi-antenna transmission techniques that use fewer transmit RF chains are of interest. In this regard, an actively researched multi-antenna transmission technique is spatial modulation (SM) [3]. SM uses a multiple antenna array at the transmitter but only a single transmit RF chain [4]. Only one antenna in the array is activated at a time and the remaining antennas remain silent. The antenna to be activated is chosen based a group of information bits. On the activated antenna, a symbol from a modulation alphabet (e.g., QAM) is sent. Therefore, in SM, the index of the activated transmit antenna conveys information bits in addition to the information bits conveyed by conventional modulation alphabets like QAM.

Space shift keying (SSK) is a special case of SM [5]. SSK uses a one-to-one mapping between a group of l information bits and the spatial position (i.e., index) of the active transmitting antenna, which is chosen among the available $n_t = 2^l$ transmit antennas. On this chosen antenna, a signal known to the receiver, say +1, is sent. The remaining $n_t - 1$ antennas remain silent. By doing so, the problem of signal detection at the receiver becomes one of merely finding out which antenna is transmitting. This makes optimal detection of SSK signal less complex and the corresponding transceiver design simpler. The spectral efficiency of SSK is l bits/channel use (bpcu), which increases logarithmically with increasing n_t.

This work was supported in part by a gift from the Cisco University Research Program, a corporate advised fund of Silicon Valley Community Foundation.

The performance of SSK has been studied extensively in point-to-point communication links involving no relays [6]-[9]. In particular, works in [6]-[10] have shown through analysis and simulation that SSK can outperform conventional single-RF-chain communication systems. SSK has also been shown to be more energy efficient in point-to-point communication, especially at high bpcu [10]. Several recent works (e.g., [11]-[19]) have studied different aspects of SM and SSK in MIMO relay channels. These works on cooperative relaying with SSK, however, did not consider multicast scenarios.

In this paper, we consider SSK in DF relaying in a *multicast scenario*, which, to our knowledge, has not been reported before. In the considered system, a source node communicates with multiple destination nodes with the help of relay nodes. In practice, this setting could correspond to a base station (source node) sending a message to a group of multicast users (multiple destination nodes) through relays. Our motivation to consider multicast system with SSK arises from the works in [20]-[22], where outage performance and bit error performance for non-SSK type modulation (e.g., BPSK) have been studied under multicast system models. Our new contributions in this paper can be summarized as follows:

- proposal of a relay selection scheme for SSK in a two-hop two-relay multicast system.
- end-to-end average bit error probability (ABEP) analysis and derivation of exact closed-form ABEP expression for binary SSK.
- derivation of an approximate, yet accurate, ABEP expression for non-binary SSK.
- diversity order through asymptotic ABEP analysis.
- validation of ABEP and diversity analysis through simulation and numerical plots.

II. System Model

Consider a cooperative multicast network consisting of a source node S, two relay nodes R_1, R_2, and K (≥ 2) destination nodes $D_k, k = 1, \cdots, K$, as shown in Fig. 1. The source and relays are equipped with n_s transmit antennas each. The relay R_m $(m = 1, 2)$ and the destination D_k $(k = 1, \cdots, K)$ are equipped with n_{r_m} and n_{d_k} receive antennas, respectively. We denote the S-to-R_m, S-to-D_k, and R_m-to-D_k channel matrices as $\mathbf{H}_{sr_m}$, $\mathbf{H}_{sd_k}$, and $\mathbf{H}_{r_m d_k}$, respectively, whose entries are modeled as independent $\mathcal{CN}(0, \sigma^2_{sr_m})$, $\mathcal{CN}(0, \sigma^2_{sd_k})$, and $\mathcal{CN}(0, \sigma^2_{r_m d_k})$, respectively. $\sigma^2_{sr_m}$, $\sigma^2_{sd_k}$, and $\sigma^2_{r_m d_k}$ account for factors like path loss, shadowing in the corresponding links. The elements of the additive noise vectors in all the channels are modeled as i.i.d. $\mathcal{CN}(0, \sigma^2)$. Transmissions from the source and relays use

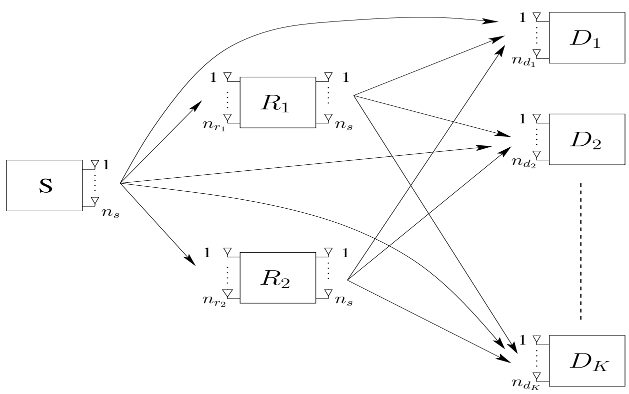

Fig. 1. Cooperative multicast relaying with SSK.

SSK, where the modulation alphabet for n_s transmit antennas is given by

$$\mathbb{S}_{n_s} \equiv \{\mathbf{s}_i : i = 1, \cdots, n_s\}, \tag{1}$$
$$\text{s.t.} \quad \mathbf{s}_i = [0, \cdots, 0, \underbrace{1}_{i\text{th coordinate}}, 0, \cdots, 0]^T.$$

In the first phase of transmission, S transmits SSK signal to the relays and destinations. Let $\mathcal{D}$ denote the *decoding set*, which consists of the indices of the relay nodes which decode the signal correctly. $\mathcal{D}$ can be any one of $\{\emptyset\}, \{1\}, \{2\}, \{1, 2\}$. Forwarding happens in the second phase only when $\mathcal{D}$ is non-empty. If none of the relays decode the signal correctly (i.e., $\mathcal{D} = \{\emptyset\}$), then there is no forwarding and D_k processes only the signal it directly received from S (denoted by $\mathbf{y}_{sd_k}$) for detection. If any one of the relays decodes the signal correctly $\big($i.e., $\mathcal{D} = \{1\}$ or $\{2\}\big)$, the node D_k combines $\mathbf{y}_{sd_k}$ and the signal vector received from R_m ($m = 1$ or 2), $\mathbf{y}_{r_m d_k}$ and performs optimal detection. The detected signal at D_k is then given by

$$\tilde{\mathbf{x}}_k = \underset{\mathbf{s} \in \mathbb{S}_{n_s}}{\arg\min} \parallel \mathbf{y}_k - \mathbf{H}_k \mathbf{s} \parallel^2, \tag{2}$$

where $\mathbf{y}_k = \left[\mathbf{y}_{sd_k}^T \ \mathbf{y}_{r_m d_k}^T\right]^T$, and $\mathbf{H}_k = \left[\mathbf{H}_{sd_k}^T \ \mathbf{H}_{r_m d_k}^T\right]^T$.

A. Relay selection

When both the relays decode correctly, i.e., $\mathcal{D} = \{1, 2\}$, one of the relays is selected and the selected relay will forward the decoded signal. This relay selection is done as follows. Each destination node determines which relay in the decoding set is the best for itself and feeds back the information about the index of this relay to S. To do this, node D_k selects the best relay based on R_m-to-D_k channel metric

$$\mu_{r_m d_k} \triangleq \underset{p, q \in \mathbb{N}; q > p}{\min} ||\mathbf{h}_{r_m d_k}^q - \mathbf{h}_{r_m d_k}^p||^2, \tag{3}$$

where $\mathbf{h}_{r_m d_k}^p$ is the pth column $\mathbf{H}_{r_m d_k}$ and $\mathbb{N} \triangleq \{1, 2, \cdots, n_s\}$. If $\eta_{r_1 d_k} > \eta_{r_2 d_k}$, then R_1 is selected as the best relay by D_k, and R_2 is selected otherwise, i.e., the index of the selected best relay at D_k is given by

$$i_{d_k} = \underset{m \in \mathcal{D}}{\arg\max} \ \mu_{r_m d_k}. \tag{4}$$

The metric $\eta_{r_m d_k}$ is defined as in (3) because the pairwise error probability (PEP) of SSK in point-to-point channel is dependent on the euclidean distance between the columns of the channel matrix [5].

Let L_m denote the number of destination nodes that selected relay R_m as the best relay. Then we can write

$$L_m = \sum_{k=1}^{K} I_{\{i_{d_k} = m\}}, \tag{5}$$

where $I_{i_{d_k} = m}$ is the indicator function. The relay which is chosen by most of the destination nodes is the selected as the best relay, i.e., the index of the selected relay is given by

$$i_s = \underset{m \in \{1, 2\}}{\text{argmax}} \ L_m. \tag{6}$$

The source informs the relays and destination nodes about the index of the best relay, and the best relay forwards the decoded signal. In case both the relays are selected by equal number of destination nodes, the best relay is selected by S randomly and the relays and the destination nodes are informed about the index of the best relay by S. The selected relay R_{i_s} forwards the decoded signal. At destination D_k, the received signal vectors from the source and the selected relay ($\mathbf{y}_{sd_k}$ and $\mathbf{y}_{r_{i_s} d_k}$) are combined and optimal detection is performed as in (2).

In this selection scheme, the source S does not require any channel knowledge. The relay R_m needs only the knowledge of the S-to-R_m channel and does not need the knowledge of the R_m-to-D_k channels. The destination node D_k needs the knowledge of S-to-D_k channel (i.e., direct link from S) and R_m-to-D_k channels for all m.

III. ABEP Analysis

A. Exact analysis for $n_s = 2$

We denote the end-to-end bit error event at D_k as E_k. The probability of end-to-end bit error is given by

$$P(E_k) \quad = \sum_{\mathcal{D} \in \mathcal{P}(\mathbb{I}_R)} P(E_k | \mathcal{D}) P(\mathcal{D}), \tag{7}$$

where $\mathbb{I}_R \triangleq \{1, 2\}$ is the set of indices of the relays, $\mathcal{P}(\mathbb{I}_R)$ is the power set of $\mathbb{I}_R$. Consider an arbitrary set $\mathcal{A} \in \mathcal{P}(\mathbb{I}_R)$ of cardinality T. T can be any integer between 0 (corresponding to null set) and 2 (corresponding to the set with indices of the two relays). The probability that the decoding set $\mathcal{D} = \mathcal{A}$ can be written as

$$P(\mathcal{D} = \mathcal{A}) = \prod_{m \in \mathcal{A}} (1 - P(E_{sr_m})) \prod_{n \in \mathcal{A}^c} P(E_{sr_n}), \tag{8}$$

where E_{sr_m} is the error event in S-to-R_m link, and $\mathcal{A}^c \triangleq \mathbb{I}_R \backslash \mathcal{A}$. The probability of the event E_{sr_m} is given by [5]

$$P(E_{sr_m}) = \gamma_{sr_m}^{n_{r_m}} \sum_{t=0}^{n_{r_m}-1} \binom{n_{r_m} + t - 1}{t} \left(1 - \gamma_{sr_m}\right)^t, \tag{9}$$

where $\gamma_{sr_m} = \frac{1}{2}\left(1 - \sqrt{\frac{\Omega_{sr_m}}{\Omega_{sr_m} + 2}}\right)$, and $\Omega_{sr_m} = \frac{\sigma_{sr_m}^2}{\sigma^2}$.

Next, consider the probability $P(E_k | \mathcal{D} = \mathcal{A})$, i.e., the probability of error at D_k given $\mathcal{D} = \mathcal{A}$. When $T = 0$, i.e., $\mathcal{D} = \{\emptyset\}$, the error is due error event in S-to-D_k link (E_{sd_k}) and the error probability $P(E_k|\mathcal{D}=\{\emptyset\})$ is given by

$$P(E_k|\mathcal{D}=\{\emptyset\}) = P(E_{sd_k}) = \gamma_{sd_k}^{n_{d_k}}\sum_{t=0}^{n_{d_k}-1}\binom{n_{d_k}+t-1}{t}\left(1-\gamma_{sd_k}\right)^t \quad (10)$$

where $\gamma_{sd_k} = \frac{1}{2}\left(1-\sqrt{\frac{\Omega_{sd_k}}{\Omega_{sd_k}+2}}\right)$, and $\Omega_{sd_k} = \frac{\sigma^2_{sd_k}}{\sigma^2}$. When $T=1$, no relay selection is required, since the only relay in $\mathcal{A}$ acts as the best relay. Suppose $m\in\mathcal{A}$. We denote, $\eta_{r_md_k} = \frac{||\mathbf{h}^2_{r_md_k}-\mathbf{h}^1_{r_md_k}||^2}{2\sigma^2}$ and $\eta_{sd_k} = \frac{||\mathbf{h}^2_{sd_k}-\mathbf{h}^1_{sd_k}||^2}{2\sigma^2}$, where $\mathbf{h}^q_{sd_k}$ is the qth column of $\mathbf{H}_{sd_k}$. The conditional probability of error at D_k for the given channel gain can be found out from (2) as $Q(\sqrt{\eta_{sd_k}+\eta_{r_md_k}})$. Here, η_{sd_k} and $\eta_{r_md_k}$ are distributed as $\Gamma(n_{d_k},\Omega_{sd_k})$ and $\Gamma(n_{d_k},\Omega_{r_md_k})$, respectively, where $\Gamma(a,b)$ denotes gamma distribution with shape parameter a and scale parameter b, and $\Omega_{sd_k} = \frac{\sigma^2_{sd_k}}{\sigma^2}$, $\Omega_{r_md_k} = \frac{\sigma^2_{r_md_k}}{\sigma^2}$. On averaging $Q(\sqrt{\eta_{sd_k}+\eta_{r_md_k}})$, we get $P(E_k|\mathcal{D}=\{m\})$ as follows:

$$P(E_k|\mathcal{D}=\{m\}) = \int_0^\infty Q\left(\sqrt{\alpha}\right)f_{\eta_{sd_k}+\eta_{r_md_k}}(\alpha)d\alpha$$
$$= \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(\frac{-\alpha}{2\sin^2\theta}\right)d\theta f_{\eta_{sd_k}+\eta_{r_md_k}}(\alpha)d\alpha \quad (11)$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}G_{\eta_{sd_k}}\left(\frac{-1}{2\sin^2\theta}\right)G_{\eta_{r_md_k}}\left(\frac{-1}{2\sin^2\theta}\right)d\theta \quad (12)$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{\Omega_{sd_k}}{2\sin^2\theta}\right)^{-n_{d_k}}\left(1+\frac{\Omega_{r_md_k}}{2\sin^2\theta}\right)^{-n_{d_k}}d\theta. \quad (13)$$

Eqn. (11) follows from Craig's formula [23]. In (12), $G_{\eta_{sd_k}}$ and $G_{\eta_{r_md_k}}$ denote moment generating functions (MGF) of η_{sd_k} and $\eta_{r_md_k}$, respectively. The integral in (12) follows from (11), through few steps involving change in the order of integral. The integral in (13) follows from (12) since $G_{\eta_{sd_k}}(\frac{-1}{2\sin^2\theta}) = \left(1+\frac{\Omega_{sd_k}}{2\sin^2\theta}\right)^{-n_{d_k}}$ and $G_{\eta_{r_md_k}}(\frac{-1}{2\sin^2\theta}) = \left(1+\frac{\Omega_{r_md_k}}{2\sin^2\theta}\right)^{-n_{d_k}}$. An exact closed-form expression of the integral of the form in (13) is available in [24, appendix 5A].

For $T>1$, i.e., in the case of $\mathcal{A}=\{1,2\}$,

$$P(E_k|\mathcal{D}=\mathcal{A}) = \sum_{l_1\in\mathcal{A}}\cdots\sum_{l_K\in\mathcal{A}}P(\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K})$$
$$= \sum_{l_1\in\mathcal{A}}\cdots\sum_{l_K\in\mathcal{A}}P(\varphi_1=R_{l_1})\cdots P(\varphi_K=R_{l_K}) P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}), \quad (14)$$

where φ_k denote the selected relay by D_k. In (14), the probability $P(\varphi_k=R_{l_k})$ can be written as

$$P(\varphi_k=R_{l_k}) = P(\mu_{r_{l_k}d_k}>\mu_{r_td_k}, : t\in\mathcal{A}, t\neq l_k) \quad (15)$$

For binary SSK, $\mu_{r_td_k} = ||\mathbf{h}^2_{r_td_k}-\mathbf{h}^1_{r_td_k}||^2$ and is distributed as $\Gamma(n_{d_k},2\sigma^2_{r_td_k})$. Hence from (15), we can write

$$P(\varphi_k=R_{l_k}) = \int_0^\infty\int_0^\beta f_{\mu_{r_td_k}}(\beta_t)d\beta_t f_{\mu_{r_{l_k}d_k}}(\beta)d\beta$$
$$= 1-\sum_{q=0}^{n_{d_k}-1}\frac{(q+n_{d_k}-1)!}{(n_{d_k}-1)!q!}\frac{\left(\sigma^{-2}_{r_td_k}+\sigma^{-2}_{r_{l_k}d_k}\right)^{-(q+n_{d_k})}}{\sigma^{2q}_{r_td_k}\sigma^{2n_{d_k}}_{r_{l_k}d_k}}. \quad (16)$$

Now consider $P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K})$. Denote the best relay as B_R. For any realization $\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}$ in (14), we can write

$$P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) = \sum_{m=1}^{2}P(B_R=R_m|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) P(E_k|B_R=R_m,\varphi_k=R_{l_k}). \quad (17)$$

In (17), the following cases can happen.

Case 1: For any $m\in\mathcal{A}$, $L_m>L_n, n\neq m, n\in\mathcal{A}$. In this case, $P(B_R=R_m|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K})=1$. Hence we can write from (17)

$$P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) = P(E_k|B_R=R_m,\varphi_k=R_{l_k}). \quad (18)$$

Case 2: $L_1=L_2$. In this case, any one among R_1,R_2 is selected as the best relay with equal probability, i.e., $P(B_R=R_1|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) = P(B_R=R_2|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) = \frac{1}{2}$. Hence, from (17), we can write

$$P(E_k|\varphi_1=R_{l_1},\cdots,\varphi_K=R_{l_K}) = \sum_{m=1}^{2}\frac{1}{2}P(E_k|B_R=R_m,\varphi_k=R_{l_k}). \quad (19)$$

In (18) and (19), $P(E_k|B_R=R_m,\varphi_k=R_{l_k})$ can be derived by averaging the corresponding conditional probability for the given channel, $Q(\sqrt{\eta_{sd_k}+\eta_{r_md_k}})$, as

$$P(E_k|B_R=R_m,\varphi_k=R_{l_k}) = \int_0^\infty Q\left(\sqrt{\alpha}\right)f_{\eta_{sd_k}+\eta_{r_md_k}|\varphi_k=R_{l_k}}(\alpha)d\alpha$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}G_{\eta_{sd_k}}\left(-\frac{1}{2\sin^2\theta}\right)G_{\eta_{r_md_k}|\varphi_k=R_{l_k}}\left(-\frac{1}{2\sin^2\theta}\right)d\theta \quad (20)$$

In (20), $G_{\eta_{r_md_k}|\varphi_k=R_{l_k}}$ is given by

$$G_{\eta_{r_md_k}|\varphi_k=R_{l_k}}\left(\frac{-1}{2\sin^2\theta}\right) = \int_0^\infty \exp\left(\frac{-\alpha}{2\sin^2\theta}\right)f_{\eta_{r_md_k}|\varphi_k=R_{l_k}}(\alpha)d\alpha. \quad (21)$$

The density function $f_{\eta_{r_md_k}|\varphi_k=R_{l_k}}(\alpha)$ can be written as

$$f_{\eta_{r_md_k}|\varphi_k=R_{l_k}}(\alpha) = \frac{d}{d\alpha}\frac{P(\eta_{r_md_k}\leqslant\alpha,\varphi_k=R_{l_k})}{P(\varphi_k=R_{l_k})}. \quad (22)$$

For $m=l_k$,

$$\frac{d}{d\alpha}P(\eta_{r_md_k}\leqslant\alpha,\varphi_k=R_m) = \frac{d}{d\alpha}\int_0^\alpha\int_0^\gamma f_{\eta_{r_qd_k}}(\gamma_q)d\gamma_q f_{\eta_{r_md_k}}(\gamma)d\gamma$$
$$= f_{\eta_{r_md_k}}(\alpha) - \underbrace{\sum_{t=0}^{n_{d_k}-1}\frac{(t+n_{d_k}-1)!(\Omega^{-1}_{r_qd_k}+\Omega^{-1}_{r_md_k})^{-\tau}}{(n_{d_k}-1)!t!\,\Omega^{n_{d_k}}_{r_md_k}\Omega^t_{r_qd_k}}}_{\xi}f_{\Lambda_t}(\alpha), \quad (23)$$

where $q \in \mathcal{A}, q \neq m$; $\tau = t + n_{d_k}$; $f_{\Lambda_t}(\alpha)$ denotes the probability density function of $\Lambda_t \sim \Gamma(t + n_{d_k}, \Omega_\Lambda)$, where $\Omega_\Lambda = (\Omega_{r_q d_k}^{-1} + \Omega_{r_m d_k}^{-1})^{-1} = (\Omega_{r_1 d_k}^{-1} + \Omega_{r_2 d_k}^{-1})^{-1}$. Hence, from (21), (22), and (23), we can write

$$G_{\eta_{r_m d_k}|\varphi_k = R_{l_k}}\Big(\frac{-1}{2\sin^2\theta}\Big) = \frac{1}{P(\varphi_k = R_m)} \Big[G_{\eta_{r_m d_k}}\Big(\frac{-1}{2\sin^2\theta}\Big) - \sum_{t=0}^{n_{d_k}-1} \xi\, G_{\Lambda_t}\Big(\frac{-1}{2\sin^2\theta}\Big)\Big]. \quad (24)$$

where G_{Λ_t} denotes the MGF of Λ_t. When $m \neq l_k$

$$\begin{aligned} &\frac{d}{d\alpha}\frac{P(\eta_{r_m d_k} \leqslant \alpha, \varphi_k = R_{l_k})}{P(\varphi_k = R_{l_k})} \\ &= \frac{1}{P(\varphi_k = R_{l_k})}\frac{d}{d\alpha}\int_0^\alpha \int_\gamma^\infty f_{\eta_{r_{l_k} d_k}}(\gamma_{l_k}) d\gamma_{l_k} f_{\eta_{r_m d_k}}(\gamma) d\gamma \\ &= \sum_{t=0}^{n_{d_k}-1} \underbrace{\frac{(t + n_{d_k} - 1)!(\Omega_{r_{l_k} d_k}^{-1} + \Omega_{r_m d_k}^{-1})^{-(t+n_{d_k})}}{t!(n_{d_k} - 1)!\Omega_{r_{l_k} d_k}^{t}\Omega_{r_m d_k}^{n_{d_k}} P(\varphi_k = R_{l_k})}}_{\Upsilon} f_{\Lambda_t}(\alpha). \quad (25) \end{aligned}$$

Hence, from (21), (25),

$$G_{\eta_{r_m d_k}|\varphi_k = R_{l_k}}\Big(\frac{-1}{2\sin^2\theta}\Big) = \sum_{t=0}^{n_{d_k}-1} \Upsilon\, G_{\Lambda_t}\Big(\frac{-1}{2\sin^2\theta}\Big). \quad (26)$$

For $m = l_k$, from (20), (21), (24), we can write

$$\begin{aligned} &P(E_k|B_R = R_m, \varphi_k = R_m) \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} G_{\eta_{sd_k}}\Big(-\frac{1}{2\sin^2\theta}\Big)\Big[G_{\eta_{r_m d_k}}\Big(-\frac{1}{2\sin^2\theta}\Big) \\ &\quad - \sum_{t=0}^{n_{d_k}-1} \xi\, G_{\Lambda_t}\Big(-\frac{1}{2\sin^2\theta}\Big)\Big]\frac{d\theta}{P(\varphi_k = R_m)} \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Big(1 + \frac{\Omega_{sd_k}}{2\sin^2\theta}\Big)^{-n_{d_k}} \Big[\Big(1 + \frac{\Omega_{r_m d_k}}{2\sin^2\theta}\Big)^{-n_{d_k}} \\ &\quad - \sum_{t=0}^{n_{d_k}-1} \xi\, \Big(1 + \frac{\Omega_\Lambda}{2\sin^2\theta}\Big)^{-(t+n_{d_k})}\Big]\frac{d\theta}{P(\varphi_k = R_m)}. \quad (27) \end{aligned}$$

For the case of $m \neq l_k$, we can write from (20), (21), (26)

$$\begin{aligned} &P(E_k|B_R = R_m, \varphi_k = R_{l_k}) \\ &= \sum_{t=0}^{n_{d_k}-1} \frac{\Upsilon}{\pi}\int_0^{\frac{\pi}{2}} G_{\eta_{sd_k}}\Big(\frac{-1}{2\sin^2\theta}\Big) G_{\Lambda_t}\Big(\frac{-1}{2\sin^2\theta}\Big) d\theta \\ &= \sum_{t=0}^{n_{d_k}-1} \frac{\Upsilon}{\pi}\int_0^{\frac{\pi}{2}} \Big(1 + \frac{\Omega_{sd_k}}{2\sin^2\theta}\Big)^{-n_{d_k}} \Big(1 + \frac{\Omega_\Lambda}{2\sin^2\theta}\Big)^{-(t+n_{d_k})} d\theta. \quad (28) \end{aligned}$$

Using the closed-form expressions of the integrals in (27) and (28) in (18) and (19), and using the expression of $P(\varphi_k = R_{l_k})$ from (16), we get the expression of the probability $P(E_k |\varphi_1 = R_{l_1}, \cdots, \varphi_K = R_{l_K})$. Then , using the expression of $P(\varphi_k = R_{l_k})$ from (16) and the expression of $P(E_k|\varphi_1 = R_{l_1}, \cdots, \varphi_K = R_{l_K})$ thus obtained, in (14), we get the expression of $P(E_k|\mathcal{D} = \mathcal{A})$ for $\mathcal{A} = \{1, 2\}$. Using the closed-form expression of the integral in (13), we get the expression of the probability $P(E_k|\mathcal{D} = \mathcal{A})$ for $\mathcal{A} = \{1\}, \{2\}$. Using the expression derived in (9), in (8), we get the expression of the probability $P(\mathcal{D} = \mathcal{A})$ for all the possibilities of $\mathcal{A}$. Substituting the expressions of $P(\mathcal{D} = \mathcal{A})$ and $P(E_k|\mathcal{D} = \mathcal{A})$ for possibilities of $\mathcal{A}$, in (7), we get the ABEP expression.

B. Approximate analysis for $n_s > 2$

For the case of $n_s > 2$, an exact ABEP analysis turns out to be rather difficult. Therefore, we adopt the union bound approach which uses the ABEP expression for the case of $n_s = 2$ in Section III-A. We propose the approximate ABEP for the non-binary SSK case as

$$\begin{aligned} P(E_k) &\approx \sum_{i_1=1}^{n_s-1}\sum_{i_2=i_1+1}^{n_s} \frac{N(i_1, i_2)}{n_s} P(E_{i_1 \to i_2, k}) \\ &= P_{o_k}\sum_{i_1=1}^{n_s-1}\sum_{i_2=i_1+1}^{n_s} \frac{N(i_1, i_2)}{n_s}, \quad (29) \end{aligned}$$

where $N(i_1, i_2)$ is the number of bit errors at destination when the source transmits $\mathbf{s}_{i_1}$ and destination decodes it as $\mathbf{s}_{i_2}$. $P(E_{i_1 \to i_2, k})$ is the average pairwise error probability (APEP) of incorrectly decoding $\mathbf{s}_{i_2}$ at destination when $\mathbf{s}_{i_1}$ is transmitted. For any pair of $\mathbf{s}_{i_2}$ and $\mathbf{s}_{i_1}$, the APEP at D_k is same and is denoted by P_{o_k}. Since P_{o_k} involves any two SSK symbols at a time, its analytical expression is same as the expression for the case of $n_s = 2$, i.e., the analytical expression of $P(E_k)$ in (7) derived in Section III-A.

C. Diversity analysis

The end-to-end ABEP is a function of different constituent probabilities. We analyze the asymptotic expression of each of these probabilities in order to find out the diversity gain of the system for SSK. We define SNR as $\beta = \frac{1}{\sigma^2}$. Hence in Section III-A, $\Omega_{sr_m} = \sigma_{sr_m}^2\beta$, $\Omega_{sd_k} = \sigma_{sd_k}^2\beta$, and $\Omega_{r_m d_k} = \sigma_{r_m d_k}^2\beta$. First consider $P(E_{sr_m})$. From (9), we can write at high SNR [10, Eq. (18)]

$$P(E_{sr_m}) = C_1\beta^{-n_{r_m}} + o\big(\beta^{-n_{r_m}}\big), \quad (30)$$

where C_1 is independent of β. Putting this asymptotic expression (30) in (8), we can get the asymptotic expression of $P(\mathcal{D} = \mathcal{A})$. The asymptotic expression of the probability $P(E_k|\mathcal{D} = \{\emptyset\})$ can be obtained similarly from (10) as

$$P(E_k|\mathcal{D} = \{\emptyset\}) = C_2\beta^{-n_{d_k}} + o\big(\beta^{-n_{d_k}}\big), \quad (31)$$

where C_2 is independent of β. From (13), we can write for high SNR [25, Proposition 3]

$$\begin{aligned} P(E_k|\mathcal{D} = \{m\}) &\approxeq \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Big(\frac{\Omega_{sd_k}}{2\sin^2\theta}\Big)^{-n_{d_k}} \Big(\frac{\Omega_{r_m d_k}}{2\sin^2\theta}\Big)^{-n_{d_k}} d\theta \\ &= \frac{\beta^{-2n_{d_k}}}{\pi}\int_0^{\frac{\pi}{2}} \Big(\frac{\sigma_{sd_k}^2}{2\sin^2\theta}\Big)^{-n_{d_k}} \Big(\frac{\sigma_{r_m d_k}^2}{2\sin^2\theta}\Big)^{-n_{d_k}} d\theta \\ &= C_3\beta^{-2n_{d_k}} + o\big(\beta^{-2n_{d_k}}\big), \quad (32) \end{aligned}$$

where C_3 is independent of β. The probability $P(\varphi_k = R_{l_k})$ in (14) is independent of β. Next consider the probability $P(E_k|B_R = R_m, \varphi_k = R_{l_k})$ in (17). From (27) and (28), we can derive the following for the high SNR scenario using the same approach as in (32)

$$P(E_k|B_R = R_m, \varphi_k = R_{l_k}) = C_4\beta^{-2n_{d_k}} + o\big(\beta^{-2n_{d_k}}\big), \quad (33)$$

where C_4 is independent of β. Hence from (17), (18), (19), (33), we can write the high SNR asymptotic expression of $P(E_k|\mathcal{D} = \{1, 2\})$ as

$$P(E_k|\mathcal{D} = \{1, 2\}) = C_5\beta^{-2n_{d_k}} + o\big(\beta^{-2n_{d_k}}\big), \quad (34)$$

where C_5 is independent of β. Hence using the asymptotic expressions in (30), (31), (32), (34) in the end-to-end ABEP expression of (7), we can write the diversity order offered by the system as the minimum exponent of β in the asymptotic ABEP expression. So the diversity order can be written as

$$\lambda_k = \min\{2n_{d_k}; n_{d_k} + 2n_{r_m}, m \in \mathbb{I}_R\}. \quad (35)$$

For $n_s > 2$, each of the constituent probabilities considered in (30), (31), (32), (34) show similar asymptotic behavior since each of these probabilities are upper bounded by the corresponding union bound which is the linear combination of APEPs. For SSK the expression of each of the APEP corresponding to any particular constituent probability is identical and its asymptotic expression is same as that of the ABEP of $n_s = 2$ case. So the diversity order for non-binary SSK remains the same as the binary SSK and is given by (35).

IV. Numerical Results

In this section, we present the numerical plots of ABEP of SSK under the relay selection scheme obtained through the analytical expressions derived in the previous section. For the purpose of verification, the ABEP obtained through simulation is also presented. In all the numerical results, we keep the channel parameters $\sigma^2_{sr_m} = \sigma^2_{r_m d_k} = \sigma^2_{sd_k} = 0$ dB.

A. Comparison with single-antenna QAM and SM

In Fig. 2, we show an instance where SSK outperforms other single-RF-chain transmission schemes in cooperative relaying. We compare three systems with same bpcu (4 bpcu): (i) SSK with $n_s = 16$, (ii) SM with $n_s = 2$ and 8-PSK, and (iii) single-antenna system with 16-QAM on each link with one relay and one destination. The number of receive antennas at the relay and destination are kept at 4. It can be seen that at 10^{-3} ABEP, SSK has nearly 2 dB and 7 dB SNR advantage over SM and single-antenna 16-QAM transmission, respectively.

B. Validation of exact analysis for $n_s = 2$

In Fig. 3, we plot SNR vs ABEP curves for binary SSK with $K = 2, 3$ destinations. The ABEP curves obtained through analysis derived in Section III-A as well as through simulation are shown. The ABEP vs SNR curves corresponding to single-relay and no-relay scenarios are also plotted for comparison. From Fig. 3, we can make the following observations: (i) the simulated and analytical ABEP curves show exact match, thus validating the analysis, and (ii) the system with two relays under the relay selection scheme in this paper outperforms the system with single relay and the system of direct communication without relay.

C. Validation of approximate analysis for $n_s > 2$

In Fig. 4, we plot the ABEP of SSK at 3 and 4 bpcu, as a function of SNR, for $K = 2$, $n_{d_k} = n_{r_m} = 4$; $m, k = 1, 2$, obtained from the approximate analytical derivation in Section III-B as well as through simulation. At 4 bpcu, the simulated ABEP points fall almost exactly on the analytical ABEP curves. At 3 bpcu, the analytical ABEP curve closely follows the simulated ABEP points.

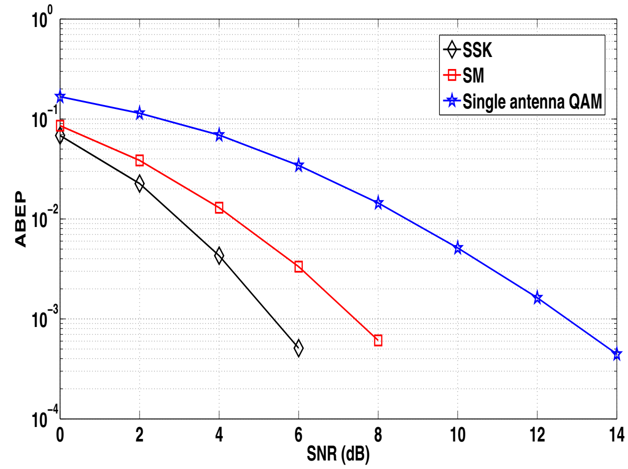

Fig. 2. SNR versus ABEP comparison at 4 bpcu between (i) SSK ($n_s = 16$), (ii) SM ($n_s = 2$, 8-PSK), and (iii) single-antenna 16-QAM. $n_{d_1} = n_{r_1} = 4$.

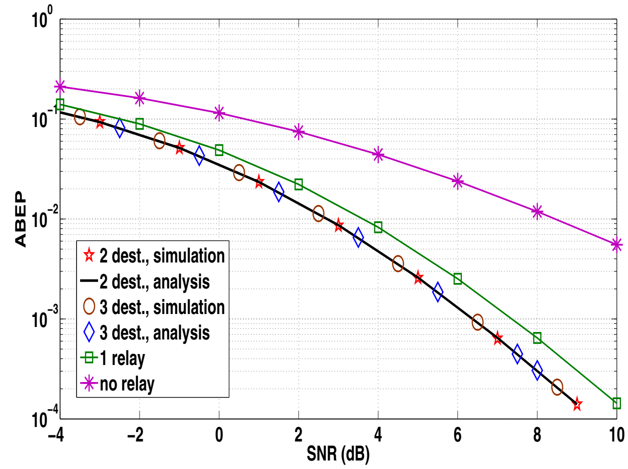

Fig. 3. SNR versus ABEP for binary SSK with $K = 2, 3$. $n_{d_k} = n_{r_m} = 4$; $m = 1, 2$; $k = 1, 2, 3$. Simulation and analysis. ABEP of systems with single relay and no relay are also given.

D. Validation of diversity analysis

The diversity gain of the system at D_k is determined by the slope in $\log_{10}$SNR vs $\log_{10}P(E_k)$ plot [26]. In Fig. 5, we plot $\log_{10}P(E_k)$ as a function of $\log_{10}$SNR for $n_s = 2, K = 2$, and for two sets of the number of receive antennas $n_{d_1} = n_{d_2} = n_{r_1} = n_{r_2} = 1, 2$, in 0 to 32 dB SNR range. For $n_{d_1} = n_{d_2} = n_{r_1} = n_{r_2} = 2$ and $n_{d_1} = n_{d_2} = n_{r_1} = n_{r_2} = 1$, the plots are parallel to the lines of slope 4 and 2, respectively. These diversity orders of 4 and 2 evident in the figure are consistent with those obtained analytically from (35) in Section III-C.

V. Conclusion

We studied SSK in dual-hop decode-and-forward cooperative multicast networks, which has not been reported before. We proposed and analyzed a relay selection strategy for SSK in cooperative multicast system with two relays. We analyzed the ABEP of the system in exact closed-form for binary SSK. For non-binary SSK we derived an approximate ABEP expression.

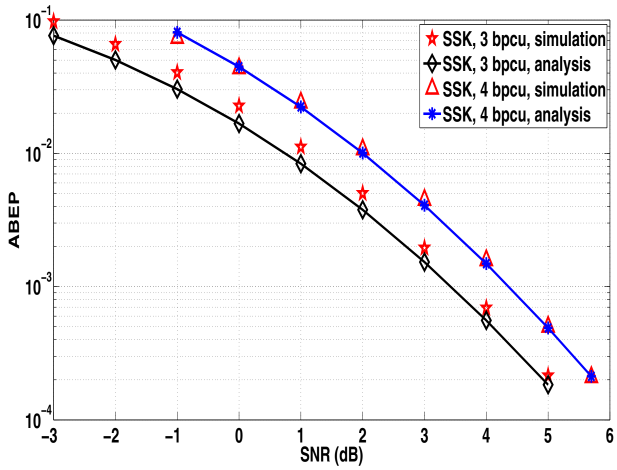

Fig. 4. SNR versus ABEP for SSK with $K = 2$ at 3,4 bpcu. $n_{d_k} = n_{r_m} = 4$; $m, k = 1, 2$. Simulation and analysis.

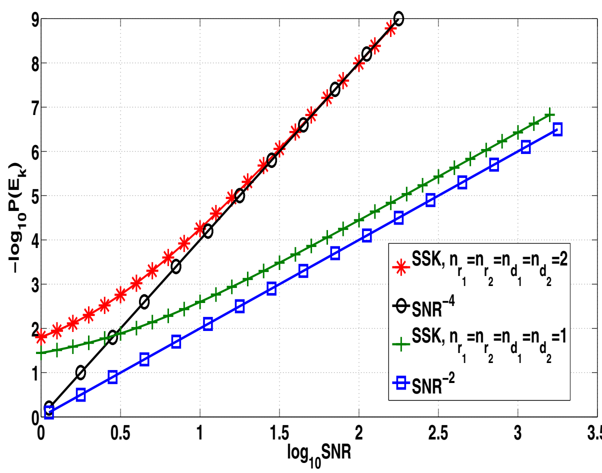

Fig. 5. $\log_{10}$SNR versus $\log_{10} P(E_k)$ for SSK at $n_s = 2$, $K = 2$. $n_{d_k} = n_{r_m} = 1, 2$; $m, k = 1, 2$.

We also derived the diversity order through asymptotic ABEP analysis. Analytical and simulation results matched very well, thus validating the analysis. In future, we propose to extend the relay selection scheme for any number of relays. Another logical extension of this work can be the consideration of SM in cooperative multicast systems and devising and analyzing similar relaying schemes.

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