Sum Secrecy Rate in MISO Full-Duplex Wiretap Channel with Imperfect CSI

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***Abstract*—In this paper, we consider the achievable sum secrecy rate in MISO (multiple-input-single-output) *full-duplex* wiretap channel in the presence of a passive eavesdropper and imperfect channel state information (CSI). We assume that the users participating in full-duplex communication have multiple transmit antennas, and that the users and the eavesdropper have single receive antenna each. The users have individual transmit power constraints. They also transmit jamming signals to improve the secrecy rates. We obtain the achievable perfect secrecy rate region by maximizing the worst case sum secrecy rate. We also obtain the corresponding transmit covariance matrices associated with the message signals and the jamming signals. Numerical results that show the impact of imperfect CSI on the achievable secrecy rate region are presented.**

keywords: MISO, full-duplex, physical layer security, secrecy rate, semidefinite programming.

I. Introduction

Transmitting messages with perfect secrecy using physical layer techniques was first studied in [1] on a physically degraded discrete memoryless wiretap channel model. Later, this work was extended to more general broadcast channel in [2] and Gaussian channel in [3], respectively. Wireless transmissions, being broadcast in nature, can be easily eavesdropped and hence require special attention to design modern secure wireless networks. Secrecy rate and capacity of point-to-point multi-antenna wiretap channels have been reported in the literature by several authors, e.g., [4]–[7]. In the above works, the transceiver operates in half-duplex mode, i.e., either it transmits or receives at any given time instant. On the other hand, full-duplex operation gives the advantage of simultaneous transmission and reception of messages [8]. But loopback self-interference and imperfect channel state information (CSI) are limitations. Full-duplex communication without secrecy constraint has been investigated by many authors, e.g., [9]–[12]. Full-duplex communication with secrecy constraint has been investigated in [13]–[15], where the achievable secrecy rate region of two-way (i.e., full-duplex) Gaussian and discrete memoryless wiretap channels have been characterized. In the above works, CSI on all the links are assumed to be perfect.

In this paper, we consider the achievable sum secrecy rate in MISO *full-duplex* wiretap channel in the presence of a passive eavesdropper and imperfect CSI. The users participating in full-duplex communication have multiple transmit antennas, and single receive antenna each. The eavesdropper is assumed to have single receive antenna. The norm of the CSI errors on all the links are assumed to be bounded in their respective absolute values. In addition to a message signal, each user transmits a jamming signal in order to improve the secrecy rates. The users operate under individual power constraints. For this scenario, we obtain the achievable perfect secrecy rate region by maximizing the worst case sum secrecy rate. We also obtain the corresponding transmit covariance matrices associated with the message signals and the jamming signals. Numerical results that illustrate the impact of imperfect CSI on the achievable secrecy rate region are presented. We also minimize the total transmit power (sum of the transmit powers of users 1 and 2) with imperfect CSI subject to receive signal-to-interference-plus-noise ratio (SINR) constraints at the users and eavesdropper, and individual transmit power constraints of the users.

The rest of the paper is organized as follows. The system model is given in Sec. II. Secrecy rate for perfect CSI is presented in Sec. III. Secrecy rate with imperfect CSI is studied in Sec. IV. Results and discussions are presented in Sec. V. Conclusions are presented in Sec. VI.

Notations : $\boldsymbol{A} \in \mathbb{C}^{N_1 \times N_2}$ implies that $\boldsymbol{A}$ is a complex matrix of dimension $N_1 \times N_2$. $\boldsymbol{A} \succeq \boldsymbol{0}$ and $\boldsymbol{A} \succ \boldsymbol{0}$ imply that $\boldsymbol{A}$ is a positive semidefinite matrix and positive definite matrix, respectively. Identity matrix is denoted by $\boldsymbol{I}$. $[.]^{*}$ denotes complex conjugate transpose operation. $\mathbb{E}[.]$ denotes expectation operator. $\|.\|$ denotes 2-norm operator. Trace of matrix $\boldsymbol{A} \in \mathbb{C}^{N \times N}$ is denoted by $\mathrm{Tr}(\boldsymbol{A})$.

II. System Model

We consider full-duplex communication between two users S_1 and S_2 in the presence of an eavesdropper E. S_1, S_2 are assumed to have M_1 and M_2 transmit antennas, respectively, and single receive antenna each. E is a passive eavesdropper and it has single receive antenna. The complex channel gains on various links are as shown in Fig. 1, where $\boldsymbol{h}_{11} \in \mathbb{C}^{1 \times M_1}$, $\boldsymbol{h}_{12} \in \mathbb{C}^{1 \times M_2}$, $\boldsymbol{h}_{21} \in \mathbb{C}^{1 \times M_1}$, $\boldsymbol{h}_{22} \in \mathbb{C}^{1 \times M_2}$, $\boldsymbol{z}_1 \in \mathbb{C}^{1 \times M_1}$, and $\boldsymbol{z}_2 \in \mathbb{C}^{1 \times M_2}$. S_1 and S_2 simultaneously transmit messages W_1 and W_2, respectively, in n channel uses. W_1 and W_2 are independent and equiprobable over $\{1, 2, \cdots, 2^{nR_1}\}$ and $\{1, 2, \cdots, 2^{nR_2}\}$, respectively. R_1 and R_2 are the information rates (bits per channel use) associated with W_1 and W_2, respectively, which need to be transmitted with perfect secrecy with respect to E [15]. S_1 and S_2 map W_1 and W_2 to codewords $\{\boldsymbol{x}_{1i}\}_{i=1}^{n}$ $\big(\boldsymbol{x}_{1i} \in \mathbb{C}^{M_1 \times 1}$, i.i.d. $\sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{\Phi}_1)$, $\boldsymbol{\Phi}_1 = \mathbb{E}[\boldsymbol{x}_{1i}\boldsymbol{x}_{1i}^{*}]\big)$ and $\{\boldsymbol{x}_{2i}\}_{i=1}^{n}$ $\big(\boldsymbol{x}_{2i} \in \mathbb{C}^{M_2 \times 1}$, i.i.d. $\sim$ $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{\Phi}_2)$, $\boldsymbol{\Phi}_2 = \mathbb{E}[\boldsymbol{x}_{2i}\boldsymbol{x}_{2i}^{*}]\big)$, respectively, of length n. In

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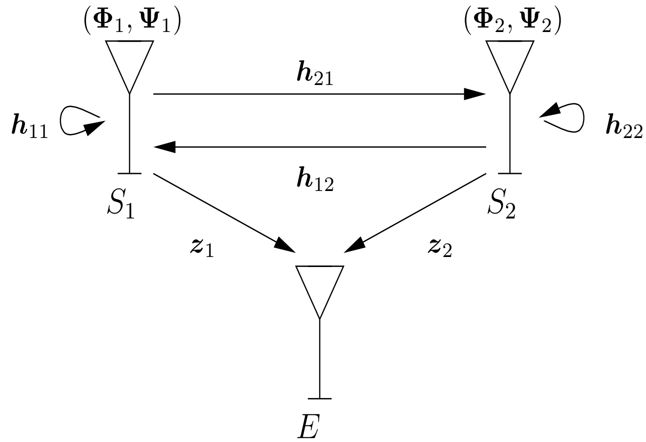

Fig. 1. System model for MISO full-duplex communication. S_1 has M_1 transmit antennas and single receive antenna. S_2 has M_2 transmit antennas and single receive antenna. E has single receive antenna.

order to degrade the eavesdropper channels and improve the secrecy rates, both S_1 and S_2 inject jamming signals $\{\boldsymbol{n}_{1i}\}_{i=1}^n$ $\big(\boldsymbol{n}_{1i} \in \mathbb{C}^{M_1\times 1}$, i.i.d. $\sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Psi}_1)$, $\boldsymbol{\Psi}_1 = \mathbb{E}[\boldsymbol{n}_{1i}\boldsymbol{n}_{1i}^*]\big)$ and $\{\boldsymbol{n}_{2i}\}_{i=1}^n$ $\big(\boldsymbol{n}_{2i} \in \mathbb{C}^{M_2\times 1}$, i.i.d. $\sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Psi}_2)$, $\boldsymbol{\Psi}_2 = \mathbb{E}[\boldsymbol{n}_{2i}\boldsymbol{n}_{2i}^*]\big)$, respectively, of length n. S_1 and S_2 transmit the symbols $\boldsymbol{x}_{1i} + \boldsymbol{n}_{1i}$ and $\boldsymbol{x}_{2i} + \boldsymbol{n}_{2i}$, respectively, during the ith channel use, $1 \leq i \leq n$. Hereafter, we will denote the symbols in $\{\boldsymbol{x}_{1i}\}_{i=1}^n$, $\{\boldsymbol{x}_{2i}\}_{i=1}^n$ $\{\boldsymbol{n}_{1i}\}_{i=1}^n$, and $\{\boldsymbol{n}_{2i}\}_{i=1}^n$ by $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, $\boldsymbol{n}_1$, and $\boldsymbol{n}_2$, respectively. We also assume that all the channel gains remain static over the codewords transmit duration. Let P_1 and P_2 be the transmit power budget for S_1 and S_2, respectively. This implies that

$$\text{Tr}(\boldsymbol{\Phi}_1 + \boldsymbol{\Psi}_1) \ \leq \ P_1, \quad \text{Tr}(\boldsymbol{\Phi}_2 + \boldsymbol{\Psi}_2) \ \leq \ P_2. \tag{1}$$

Let y_1, y_2, and y_E denote the received signals at S_1, S_2 and E, respectively. We have

$$y_1 = \boldsymbol{h}_{11}(\boldsymbol{x}_1 + \boldsymbol{n}_1) + \boldsymbol{h}_{12}(\boldsymbol{x}_2 + \boldsymbol{n}_2) + \eta_1, \tag{2}$$

$$y_2 = \boldsymbol{h}_{21}(\boldsymbol{x}_1 + \boldsymbol{n}_1) + \boldsymbol{h}_{22}(\boldsymbol{x}_2 + \boldsymbol{n}_2) + \eta_2, \tag{3}$$

$$y_E = \boldsymbol{z}_1(\boldsymbol{x}_1 + \boldsymbol{n}_1) + \boldsymbol{z}_2(\boldsymbol{x}_2 + \boldsymbol{n}_2) + \eta_E, \tag{4}$$

where η_1, η_2, and η_E are i.i.d. $(\sim \mathcal{CN}(0, N_0))$ receiver noise terms.

III. Sum secrecy rate - perfect CSI

In this section, we assume perfect CSI on all the links. Since S_1 knows the transmitted symbol $(\boldsymbol{x}_1 + \boldsymbol{n}_1)$, in order to detect $\boldsymbol{x}_2$, S_1 subtracts $\boldsymbol{h}_{11}(\boldsymbol{x}_1 + \boldsymbol{n}_1)$ from the received signal y_1, i.e.,

$$\begin{aligned} y_1^{'} &= y_1 - \boldsymbol{h}_{11}(\boldsymbol{x}_1 + \boldsymbol{n}_1) \\ &= \boldsymbol{h}_{12}(\boldsymbol{x}_2 + \boldsymbol{n}_2) + \eta_1. \end{aligned} \tag{5}$$

Similarly, since S_2 knows the transmitted symbol $(\boldsymbol{x}_2 + \boldsymbol{n}_2)$, to detect $\boldsymbol{x}_1$, S_2 subtracts $\boldsymbol{h}_{22}(\boldsymbol{x}_2 + \boldsymbol{n}_2)$ from the received signal y_2, i.e.,

$$\begin{aligned} y_2^{'} &= y_2 - \boldsymbol{h}_{22}(\boldsymbol{x}_2 + \boldsymbol{n}_2) \\ &= \boldsymbol{h}_{21}(\boldsymbol{x}_1 + \boldsymbol{n}_1) + \eta_2. \end{aligned} \tag{6}$$

Using (5) and (6), we get the following information rates for $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, respectively:

$$R_1^{'} \triangleq I\big(\boldsymbol{x}_1;\ y_2^{'}\big) = \log_2\Big(1 + \frac{\boldsymbol{h}_{21}\boldsymbol{\Phi}_1\boldsymbol{h}_{21}^*}{N_0 + \boldsymbol{h}_{21}\boldsymbol{\Psi}_1\boldsymbol{h}_{21}^*}\Big), \tag{7}$$

$$R_2^{'} \triangleq I\big(\boldsymbol{x}_2;\ y_1^{'}\big) = \log_2\Big(1 + \frac{\boldsymbol{h}_{12}\boldsymbol{\Phi}_2\boldsymbol{h}_{12}^*}{N_0 + \boldsymbol{h}_{12}\boldsymbol{\Psi}_2\boldsymbol{h}_{12}^*}\Big). \tag{8}$$

Using (4), we get the information leakage rate at E as

$$\begin{aligned} R_E^{'} &\triangleq I\big(\boldsymbol{x}_1, \boldsymbol{x}_2;\ y_E\big) \\ &= \log_2\Big(1 + \frac{\boldsymbol{z}_1\boldsymbol{\Phi}_1\boldsymbol{z}_1^* + \boldsymbol{z}_2\boldsymbol{\Phi}_2\boldsymbol{z}_2^*}{N_0 + \boldsymbol{z}_1\boldsymbol{\Psi}_1\boldsymbol{z}_1^* + \boldsymbol{z}_2\boldsymbol{\Psi}_2\boldsymbol{z}_2^*}\Big). \end{aligned} \tag{9}$$

Using (7), (8), and (9), we get the information capacities $C_1^{'}$, $C_2^{'}$, and $C_E^{'}$, respectively, as follows:

$$C_1^{'} = \log_2\Big(1 + \frac{\|\boldsymbol{h}_{21}\|^2\ P_1}{N_0}\Big), \tag{10}$$

$$C_2^{'} = \log_2\Big(1 + \frac{\|\boldsymbol{h}_{12}\|^2\ P_2}{N_0}\Big), \tag{11}$$

$$C_E^{'} = \log_2\Big(1 + \frac{\|\boldsymbol{z}_1\|^2\ P_1 + \|\boldsymbol{z}_2\|^2\ P_2}{N_0}\Big). \tag{12}$$

A secrecy rate pair (R_1, R_2) which falls in the following region is achievable [15]:

$$\begin{aligned} &0 \ \leq \ R_1 \ \leq \ R_1^{'}, \quad 0 \ \leq \ R_2 \ \leq \ R_2^{'}, \\ &0 \ \leq \ R_1 + R_2 \ \leq \ R_1^{'} + R_2^{'} - R_E^{'}, \\ &\boldsymbol{\Phi}_1 \succeq \mathbf{0}, \quad \boldsymbol{\Psi}_1 \succeq \mathbf{0}, \quad \text{Tr}(\boldsymbol{\Phi}_1 + \boldsymbol{\Psi}_1) \ \leq \ P_1, \\ &\boldsymbol{\Phi}_2 \succeq \mathbf{0}, \quad \boldsymbol{\Psi}_2 \succeq \mathbf{0}, \quad \text{Tr}(\boldsymbol{\Phi}_2 + \boldsymbol{\Psi}_2) \ \leq \ P_2. \end{aligned} \tag{13}$$

We intend to maximize the sum secrecy rate subject to the power constraint, i.e.,

$$\max_{\substack{\boldsymbol{\Phi}_1,\ \boldsymbol{\Psi}_1,\\ \boldsymbol{\Phi}_2,\ \boldsymbol{\Psi}_2}} \ R_1^{'} + R_2^{'} - R_E^{'} \tag{14}$$

$$\begin{aligned} = \max_{\substack{\boldsymbol{\Phi}_1,\ \boldsymbol{\Psi}_1,\\ \boldsymbol{\Phi}_2,\ \boldsymbol{\Psi}_2}} \ &\Big\{ \log_2\Big(1 + \frac{\boldsymbol{h}_{21}\boldsymbol{\Phi}_1\boldsymbol{h}_{21}^*}{N_0 + \boldsymbol{h}_{21}\boldsymbol{\Psi}_1\boldsymbol{h}_{21}^*}\Big) \\ &+ \log_2\Big(1 + \frac{\boldsymbol{h}_{12}\boldsymbol{\Phi}_2\boldsymbol{h}_{12}^*}{N_0 + \boldsymbol{h}_{12}\boldsymbol{\Psi}_2\boldsymbol{h}_{12}^*}\Big) \\ &- \log_2\Big(1 + \frac{\boldsymbol{z}_1\boldsymbol{\Phi}_1\boldsymbol{z}_1^* + \boldsymbol{z}_2\boldsymbol{\Phi}_2\boldsymbol{z}_2^*}{N_0 + \boldsymbol{z}_1\boldsymbol{\Psi}_1\boldsymbol{z}_1^* + \boldsymbol{z}_2\boldsymbol{\Psi}_2\boldsymbol{z}_2^*}\Big)\Big\} \end{aligned} \tag{15}$$

$$\begin{aligned} \text{s.t.} \quad &\boldsymbol{\Phi}_1 \succeq \mathbf{0}, \quad \boldsymbol{\Psi}_1 \succeq \mathbf{0}, \quad \text{Tr}(\boldsymbol{\Phi}_1 + \boldsymbol{\Psi}_1) \ \leq \ P_1, \\ &\boldsymbol{\Phi}_2 \succeq \mathbf{0}, \quad \boldsymbol{\Psi}_2 \succeq \mathbf{0}, \quad \text{Tr}(\boldsymbol{\Phi}_2 + \boldsymbol{\Psi}_2) \ \leq \ P_2. \end{aligned} \tag{16}$$

This is a non-convex optimization problem, and we solve it using two-dimensional search as follows.

Step1 : Divide the intervals $[0, C_1^{'}]$ and $[0, C_2^{'}]$ in K and L small intervals, respectively, of size $\triangle_1 = \frac{C_1^{'}}{K}$ and $\triangle_2 = \frac{C_2^{'}}{L}$ where K and L are large integers. Let $R_1^{'k} = k\triangle_1$ and $R_2^{'l} = l\triangle_2$, where $k = 0, 1, 2, \cdots, K$ and $l = 0, 1, 2, \cdots, L$.

Step2 : For a given $(R_1^{'k},\ R_2^{'l})$ pair, we minimize $R_E^{'}$ as follows:

$$R_E^{''kl} \triangleq \min_{\substack{\mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2}} \ \log_2\Big(1+\frac{\boldsymbol{z}_1\mathbf{\Phi}_1\boldsymbol{z}_1^{\ast}+\boldsymbol{z}_2\mathbf{\Phi}_2\boldsymbol{z}_2^{\ast}}{N_0+\boldsymbol{z}_1\mathbf{\Psi}_1\boldsymbol{z}_1^{\ast}+\boldsymbol{z}_2\mathbf{\Psi}_2\boldsymbol{z}_2^{\ast}}\Big) \quad (17)$$

$$\begin{aligned}
\text{s.t.}\quad R_1^{''k} &\triangleq \log_2\Big(1+\frac{\boldsymbol{h}_{21}\mathbf{\Phi}_1\boldsymbol{h}_{21}^{\ast}}{N_0+\boldsymbol{h}_{21}\mathbf{\Psi}_1\boldsymbol{h}_{21}^{\ast}}\Big) \ \geq \ R_1^{'k},\\
R_2^{''l} &\triangleq \log_2\Big(1+\frac{\boldsymbol{h}_{12}\mathbf{\Phi}_2\boldsymbol{h}_{12}^{\ast}}{N_0+\boldsymbol{h}_{12}\mathbf{\Psi}_2\boldsymbol{h}_{12}^{\ast}}\Big) \ \geq \ R_2^{'l},\\
&\mathbf{\Phi}_1 \succeq \mathbf{0}, \quad \mathbf{\Psi}_1 \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1) \ \leq \ P_1,\\
&\mathbf{\Phi}_2 \succeq \mathbf{0}, \quad \mathbf{\Psi}_2 \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \ \leq \ P_2. \quad (18)
\end{aligned}$$

The maximum sum secrecy rate is given by $\max_{\substack{k=0,1,2,\cdots,K,\\ l=0,1,2,\cdots,L}} \ (R_1^{''k}+R_2^{''l}-R_E^{''kl})$. We solve the optimization problem (17) as follows. Dropping the logarithm in the objective function in (17), we rewrite the optimization problem (17) in the following equivalent form:

$$\min_{t,\ \mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2} \quad t \quad (19)$$

s.t.

$$\begin{aligned}
(\boldsymbol{z}_1\mathbf{\Phi}_1\boldsymbol{z}_1^{\ast}+\boldsymbol{z}_2\mathbf{\Phi}_2\boldsymbol{z}_2^{\ast}) - t\big(N_0+\boldsymbol{z}_1\mathbf{\Psi}_1\boldsymbol{z}_1^{\ast}+\boldsymbol{z}_2\mathbf{\Psi}_2\boldsymbol{z}_2^{\ast}\big) \ &\leq \ 0,\\
\big(2^{R_1^{'k}}-1\big)\big(N_0+\boldsymbol{h}_{21}\mathbf{\Psi}_1\boldsymbol{h}_{21}^{\ast}\big)-\big(\boldsymbol{h}_{21}\mathbf{\Phi}_1\boldsymbol{h}_{21}^{\ast}\big) \ &\leq \ 0,\\
\big(2^{R_2^{'l}}-1\big)\big(N_0+\boldsymbol{h}_{12}\mathbf{\Psi}_2\boldsymbol{h}_{12}^{\ast}\big)-\big(\boldsymbol{h}_{12}\mathbf{\Phi}_2\boldsymbol{h}_{12}^{\ast}\big) \ &\leq \ 0,\\
\mathbf{\Phi}_1 \succeq \mathbf{0}, \quad \mathbf{\Psi}_1 \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1) \ &\leq \ P_1,\\
\mathbf{\Phi}_2 \succeq \mathbf{0}, \quad \mathbf{\Psi}_2 \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \ &\leq \ P_2. \quad (20)
\end{aligned}$$

Using the KKT conditions of the above optimization problem, the ranks of the optimum solutions $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ can be analyzed as in [20]. Further, for a given t, the above problem is formulated as the following semidefinite feasibility problem [19]:

$$\text{find} \quad \mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2 \quad (21)$$

subject to the constraints in (20). The minimum value of t, denoted by t_{min}^{kl}, can be obtained using bisection method [19] as follows. Let t_{min}^{kl} lie in the interval $[t_{lowerlimit},\ t_{upperlimit}]$. The value of $t_{lowerlimit}$ can be taken as 0 (corresponding to the minimum information rate of 0) and $t_{upperlimit}$ can be taken as $(2^{C_E^{'}}-1)$, which corresponds to the information capacity of the eavesdropper link. Check the feasibility of (21) at $t_{min}^{kl}=(t_{lowerlimit}+t_{upperlimit})/2$. If feasible, then $t_{upperlimit}=t_{min}^{kl}$, else $t_{lowerlimit}=t_{min}^{kl}$. Repeat this until $t_{upperlimit}-t_{lowerlimit} \ \leq \ \zeta$, where ζ is a small positive number. Using t_{min}^{kl} in (17), $R_E^{''kl}$ is given by

$$R_E^{''kl} \ = \ \log_2(1+t_{min}^{kl}). \quad (22)$$

IV. Sum secrecy rate - imperfect CSI

In this section, we assume that the available CSI on all the links are imperfect [16]–[18], i.e.,

$$\begin{aligned}
&\boldsymbol{h}_{11}=\boldsymbol{h}_{11}^0+\boldsymbol{e}_{11}, \quad \boldsymbol{h}_{12}=\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12}, \quad \boldsymbol{h}_{21}=\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21},\\
&\boldsymbol{h}_{22}=\boldsymbol{h}_{22}^0+\boldsymbol{e}_{22}, \quad \boldsymbol{z}_1=\boldsymbol{z}_1^0+\boldsymbol{e}_1, \quad \boldsymbol{z}_2=\boldsymbol{z}_2^0+\boldsymbol{e}_2,
\end{aligned}$$

where $\boldsymbol{h}_{11}^0$, $\boldsymbol{h}_{12}^0$, $\boldsymbol{h}_{21}^0$, $\boldsymbol{h}_{22}^0$, $\boldsymbol{z}_1^0$, and $\boldsymbol{z}_2^0$ are the estimates of $\boldsymbol{h}_{11}$, $\boldsymbol{h}_{12}$, $\boldsymbol{h}_{21}$, $\boldsymbol{h}_{22}$, $\boldsymbol{z}_1$, and $\boldsymbol{z}_2$, respectively, and $\boldsymbol{e}_{11}$, $\boldsymbol{e}_{12}$, $\boldsymbol{e}_{21}$, $\boldsymbol{e}_{22}$, $\boldsymbol{e}_1$, and $\boldsymbol{e}_2$ are the corresponding errors. We assume that the norm of the errors are bounded in their respective absolute values as:

$$\begin{aligned}
&\|\boldsymbol{e}_{11}\| \leq \epsilon_{11}, \quad \|\boldsymbol{e}_{12}\| \leq \epsilon_{12}, \quad \|\boldsymbol{e}_{21}\| \leq \epsilon_{21},\\
&\|\boldsymbol{e}_{22}\| \leq \epsilon_{22}, \quad \|\boldsymbol{e}_1\| \leq \epsilon_1, \quad \|\boldsymbol{e}_2\| \leq \epsilon_2.
\end{aligned}$$

We make the following assumptions with respect to the availability of the CSI at S_1, S_2, and E:

(a.) We assume that only the estimates $\boldsymbol{h}_{11}^0$, $\boldsymbol{h}_{21}^0$, $\boldsymbol{z}_1^0$, and $\boldsymbol{z}_2^0$ are available at S_1 while $\boldsymbol{h}_{12}$ is perfectly known at S_1 (coherent detection). Similarly, only the estimates $\boldsymbol{h}_{22}^0$, $\boldsymbol{h}_{12}^0$, $\boldsymbol{z}_1^0$, and $\boldsymbol{z}_2^0$ are available at S_2 while $\boldsymbol{h}_{21}$ is perfectly known at S_2 (coherent detection). We assume that E has perfect knowledge of $\boldsymbol{z}_1$, and $\boldsymbol{z}_2$. With the above error model, we rewrite (5), (6), and (4) as follows:

$$\begin{aligned}
y_1^{'} &= y_1-\boldsymbol{h}_{11}^0(\boldsymbol{x}_1+\boldsymbol{n}_1)\\
&= \boldsymbol{e}_{11}(\boldsymbol{x}_1+\boldsymbol{n}_1)+\boldsymbol{h}_{12}(\boldsymbol{x}_2+\boldsymbol{n}_2)+\eta_1, \quad (23)\\
y_2^{'} &= y_2-\boldsymbol{h}_{22}^0(\boldsymbol{x}_2+\boldsymbol{n}_2)\\
&= \boldsymbol{h}_{21}(\boldsymbol{x}_1+\boldsymbol{n}_1)+\boldsymbol{e}_{22}(\boldsymbol{x}_2+\boldsymbol{n}_2)+\eta_2, \quad (24)\\
y_E &= \boldsymbol{z}_1(\boldsymbol{x}_1+\boldsymbol{n}_1)+\boldsymbol{z}_2(\boldsymbol{x}_2+\boldsymbol{n}_2)+\eta_E. \quad (25)
\end{aligned}$$

(b.) We assume that while detecting $\boldsymbol{x}_2$, S_1 treats the residual term $\boldsymbol{e}_{11}(\boldsymbol{x}_1+\boldsymbol{n}_1)$ which appears in (23) as self-noise. Similarly, while detecting $\boldsymbol{x}_1$, S_2 treats the residual term $\boldsymbol{e}_{22}(\boldsymbol{x}_2+\boldsymbol{n}_2)$ which appears in (24) as self-noise.

Further, in order to compute $R_1^{'k}$, $R_2^{'l}$, and $R_E^{''kl}$, respectively, as described in **Step1** and **Step2** in Section III, we get the worst case capacities $C_1^{'}$, $C_2^{'}$ for S_1, S_2 links, and best case capacity $C_E^{'}$ for the eavesdropper link with imperfect CSI as follows:

$$C_1^{'}=\log_2\Big(1+\frac{\big|\|\boldsymbol{h}_{21}^0\|-\epsilon_{21}\big|^2P_1}{N_0}\Big) \ \text{ if } \ \big(\|\boldsymbol{h}_{21}^0\|>\ \epsilon_{21}\big), \quad 0 \ \text{ else.} \quad (26)$$

$$C_2^{'}=\log_2\Big(1+\frac{\big|\|\boldsymbol{h}_{12}^0\|-\epsilon_{12}\big|^2P_2}{N_0}\Big) \ \text{ if } \ \big(\|\boldsymbol{h}_{12}^0\|>\ \epsilon_{12}\big), \quad 0 \ \text{ else.} \quad (27)$$

$$C_E^{'}=\log_2\Big(1+\frac{\big|\|\boldsymbol{z}_1^0\|+\epsilon_1\big|^2P_1+\big|\|\boldsymbol{z}_2^0\|+\epsilon_2\big|^2P_2}{N_0}\Big). \quad (28)$$

Using (23), (24), and (25), we write the optimization problem (17) with imperfect CSI as follows:

$$R_E^{''kl} \ \triangleq \min_{\mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2} \ \max_{\boldsymbol{e}_1,\ \boldsymbol{e}_2} \ \log_2$$
$$\Big(1+\frac{(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\mathbf{\Phi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^{\ast}+(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\mathbf{\Phi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^{\ast}}{N_0+(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\mathbf{\Psi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^{\ast}+(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\mathbf{\Psi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^{\ast}}\Big) \quad (29)$$

$$\text{s.t.} \quad R_1^{''k} \ \triangleq \min_{\boldsymbol{e}_{21},\ \boldsymbol{e}_{22}} \ \log_2$$
$$\Big(1+\frac{(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\mathbf{\Phi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^{\ast}}{N_0+\boldsymbol{e}_{22}(\mathbf{\Phi}_2+\mathbf{\Psi}_2)\boldsymbol{e}_{22}^{\ast}+(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\mathbf{\Psi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^{\ast}}\Big) \ \geq \ R_1^{'k}, \quad (30)$$

$$R_2^{''l} \ \triangleq \min_{\boldsymbol{e}_{11},\ \boldsymbol{e}_{12}} \ \log_2$$
$$\Big(1+\frac{(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\mathbf{\Phi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^{\ast}}{N_0+\boldsymbol{e}_{11}(\mathbf{\Phi}_1+\mathbf{\Psi}_1)\boldsymbol{e}_{11}^{\ast}+(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\mathbf{\Psi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^{\ast}}\Big) \ \geq \ R_2^{'l}, \quad (31)$$

$$\|\boldsymbol{e}_{11}\|^2 \leq \epsilon_{11}^2, \quad \|\boldsymbol{e}_{12}\|^2 \leq \epsilon_{12}^2, \quad \|\boldsymbol{e}_{21}\|^2 \leq \epsilon_{21}^2,$$

$$\|\boldsymbol{e}_{22}\|^2 \le \epsilon_{22}^2, \quad \|\boldsymbol{e}_1\|^2 \le \epsilon_1^2, \quad \|\boldsymbol{e}_2\|^2 \le \epsilon_2^2, \tag{32}$$

$$\begin{aligned} &\boldsymbol{\Phi}_1 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_1 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1) \le P_1, \\ &\boldsymbol{\Phi}_2 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_2 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2) \le P_2. \end{aligned} \tag{33}$$

In the constraints (30) and (31), additional noise appear due the terms $\boldsymbol{e}_{22}(\boldsymbol{x}_2+\boldsymbol{n}_2)$ and $\boldsymbol{e}_{11}(\boldsymbol{x}_1+\boldsymbol{n}_1)$, respectively, which have been treated as self-noise.

We solve the optimization problem (29) as follows. Dropping the logarithm in the objective function in (29), we write the optimization problem (29) in the following equivalent form:

$$\min_{\boldsymbol{\Phi}_1,\, \boldsymbol{\Psi}_1,\, \boldsymbol{\Phi}_2,\, \boldsymbol{\Psi}_2} \ \max_{\boldsymbol{e}_1,\, \boldsymbol{e}_2} \left(\frac{(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\boldsymbol{\Phi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^* + (\boldsymbol{z}_2^0+\boldsymbol{e}_2)\boldsymbol{\Phi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^*}{N_0 + (\boldsymbol{z}_1^0+\boldsymbol{e}_1)\boldsymbol{\Psi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^* + (\boldsymbol{z}_2^0+\boldsymbol{e}_2)\boldsymbol{\Psi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^*} \right) \tag{34}$$

$$\text{s.t.} \quad \min_{\boldsymbol{e}_{21},\, \boldsymbol{e}_{22}} \left(\frac{(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\boldsymbol{\Phi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^*}{N_0 + \boldsymbol{e}_{22}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2)\boldsymbol{e}_{22}^* + (\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\boldsymbol{\Psi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^*} \right) \ge (2^{R_1'^k}-1), \tag{35}$$

$$\min_{\boldsymbol{e}_{11},\, \boldsymbol{e}_{12}} \left(\frac{(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\boldsymbol{\Phi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^*}{N_0 + \boldsymbol{e}_{11}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1)\boldsymbol{e}_{11}^* + (\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\boldsymbol{\Psi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^*} \right) \ge (2^{R_2'^l}-1), \tag{36}$$

$$\begin{aligned} &\|\boldsymbol{e}_{11}\|^2 \le \epsilon_{11}^2, \quad \|\boldsymbol{e}_{12}\|^2 \le \epsilon_{12}^2, \quad \|\boldsymbol{e}_{21}\|^2 \le \epsilon_{21}^2, \\ &\|\boldsymbol{e}_{22}\|^2 \le \epsilon_{22}^2, \quad \|\boldsymbol{e}_1\|^2 \le \epsilon_1^2, \quad \|\boldsymbol{e}_2\|^2 \le \epsilon_2^2, \end{aligned} \tag{37}$$

$$\begin{aligned} &\boldsymbol{\Phi}_1 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_1 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1) \le P_1, \\ &\boldsymbol{\Phi}_2 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_2 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2) \le P_2. \end{aligned} \tag{38}$$

Solving the above optimization problem is hard due to the presence of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ in both the numerator and denominator of the objective function in (34). Similarly, $\boldsymbol{e}_{21}$ and $\boldsymbol{e}_{12}$ appear in both the numerator and denominator of the constraints in (35) and (36), respectively. By independently constraining the various quadratic terms appearing in the objective function in (34) and the constraints (35) and (36), we get the following upper bound for the above optimization problem:

$$\min_{\boldsymbol{\Phi}_1,\, \boldsymbol{\Psi}_1,\, \boldsymbol{\Phi}_2,\, \boldsymbol{\Psi}_2} \ \min_{t_1,\, t_2, \cdots,\, t_{10}} \left(\frac{t_1+t_2}{N_0+t_3+t_4} \right) \tag{39}$$

$$\text{s.t.} \quad t_3 \ge 0, \ t_4 \ge 0, \ t_5 \ge 0, \ t_8 \ge 0, \tag{40}$$

$$\forall \boldsymbol{e}_1 \quad \text{s.t.} \quad \|\boldsymbol{e}_1\|^2 \le \epsilon_1^2 \implies$$

$$(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\boldsymbol{\Phi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^* - t_1 \le 0, \tag{41}$$

$$\forall \boldsymbol{e}_1 \quad \text{s.t.} \quad \|\boldsymbol{e}_1\|^2 \le \epsilon_1^2 \implies$$

$$-(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\boldsymbol{\Psi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^* + t_3 \le 0, \tag{42}$$

$$\forall \boldsymbol{e}_2 \quad \text{s.t.} \quad \|\boldsymbol{e}_2\|^2 \le \epsilon_2^2 \implies$$

$$(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\boldsymbol{\Phi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^* - t_2 \le 0, \tag{43}$$

$$\forall \boldsymbol{e}_2 \quad \text{s.t.} \quad \|\boldsymbol{e}_2\|^2 \le \epsilon_2^2 \implies$$

$$-(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\boldsymbol{\Psi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^* + t_4 \le 0, \tag{44}$$

$$\left(\frac{t_5}{N_0+t_6+t_7} \right) \ge (2^{R_1'^k}-1), \tag{45}$$

$$\forall \boldsymbol{e}_{21} \quad \text{s.t.} \quad \|\boldsymbol{e}_{21}\|^2 \le \epsilon_{21}^2 \implies$$

$$-(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\boldsymbol{\Phi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^* + t_5 \le 0, \tag{46}$$

$$\forall \boldsymbol{e}_{21} \quad \text{s.t.} \quad \|\boldsymbol{e}_{21}\|^2 \le \epsilon_{21}^2 \implies$$

$$(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\boldsymbol{\Psi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^* - t_7 \le 0, \tag{47}$$

$$\forall \boldsymbol{e}_{22} \quad \text{s.t.} \quad \|\boldsymbol{e}_{22}\|^2 \le \epsilon_{22}^2 \implies$$

$$\boldsymbol{e}_{22}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2)\boldsymbol{e}_{22}^* - t_6 \le 0, \tag{48}$$

$$\left(\frac{t_8}{N_0+t_9+t_{10}} \right) \ge (2^{R_2'^l}-1), \tag{49}$$

$$\forall \boldsymbol{e}_{12} \quad \text{s.t.} \quad \|\boldsymbol{e}_{12}\|^2 \le \epsilon_{12}^2 \implies$$

$$-(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\boldsymbol{\Phi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^* + t_8 \le 0, \tag{50}$$

$$\forall \boldsymbol{e}_{12} \quad \text{s.t.} \quad \|\boldsymbol{e}_{12}\|^2 \le \epsilon_{12}^2 \implies$$

$$(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\boldsymbol{\Psi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^* - t_{10} \le 0, \tag{51}$$

$$\forall \boldsymbol{e}_{11} \quad \text{s.t.} \quad \|\boldsymbol{e}_{11}\|^2 \le \epsilon_{11}^2 \implies$$

$$\boldsymbol{e}_{11}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1)\boldsymbol{e}_{11}^* - t_9 \le 0, \tag{52}$$

$$\begin{aligned} &\boldsymbol{\Phi}_1 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_1 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1) \le P_1, \\ &\boldsymbol{\Phi}_2 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_2 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2) \le P_2. \end{aligned} \tag{53}$$

We use the S-procedure to transform the pairs of quadratic inequalities in (41), (42), (43), (44), (46), (47), (48), (50), (51), and (52) to equivalent linear matrix inequalities (LMI) [19]. With this, we get the following single minimization form for the above optimization problem:

$$\min_{\substack{\boldsymbol{\Phi}_1,\, \boldsymbol{\Psi}_1,\, \boldsymbol{\Phi}_2,\, \boldsymbol{\Psi}_2, \\ t_1,\, t_2, \cdots,\, t_{10}, \\ \lambda_1,\, \lambda_2, \cdots,\, \lambda_{10}, \\ t}} \ t \tag{54}$$

$$\begin{aligned}
\text{s.t.} \quad & t_3 \ge 0, \ t_4 \ge 0, \ t_5 \ge 0, \ t_8 \ge 0, \\
& (t_1+t_2) - t(N_0+t_3+t_4) \le 0, \\
& (2^{R_1'^k}-1)(N_0+t_6+t_7) - t_5 \le 0, \\
& (2^{R_2'^l}-1)(N_0+t_9+t_{10}) - t_8 \le 0, \\
& \begin{bmatrix} -\boldsymbol{\Phi}_1+\lambda_1\boldsymbol{I} & -\boldsymbol{\Phi}_1\boldsymbol{z}_1^{0*} \\ -\boldsymbol{z}_1^0\boldsymbol{\Phi}_1^* & -\boldsymbol{z}_1^0\boldsymbol{\Phi}_1\boldsymbol{z}_1^{0*}+t_1-\lambda_1\epsilon_1^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_1 \ge 0, \\
& \begin{bmatrix} \boldsymbol{\Psi}_1+\lambda_2\boldsymbol{I} & \boldsymbol{\Psi}_1\boldsymbol{z}_1^{0*} \\ \boldsymbol{z}_1^0\boldsymbol{\Psi}_1^* & \boldsymbol{z}_1^0\boldsymbol{\Psi}_1\boldsymbol{z}_1^{0*}-t_3-\lambda_2\epsilon_1^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_2 \ge 0, \\
& \begin{bmatrix} -\boldsymbol{\Phi}_2+\lambda_3\boldsymbol{I} & -\boldsymbol{\Phi}_2\boldsymbol{z}_2^{0*} \\ -\boldsymbol{z}_2^0\boldsymbol{\Phi}_2^* & -\boldsymbol{z}_2^0\boldsymbol{\Phi}_2\boldsymbol{z}_2^{0*}+t_2-\lambda_3\epsilon_2^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_3 \ge 0, \\
& \begin{bmatrix} \boldsymbol{\Psi}_2+\lambda_4\boldsymbol{I} & \boldsymbol{\Psi}_2\boldsymbol{z}_2^{0*} \\ \boldsymbol{z}_2^0\boldsymbol{\Psi}_2^* & \boldsymbol{z}_2^0\boldsymbol{\Psi}_2\boldsymbol{z}_2^{0*}-t_4-\lambda_4\epsilon_2^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_4 \ge 0, \\
& \begin{bmatrix} \boldsymbol{\Phi}_1+\lambda_5\boldsymbol{I} & \boldsymbol{\Phi}_1\boldsymbol{h}_{21}^{0*} \\ \boldsymbol{h}_{21}^0\boldsymbol{\Phi}_1^* & \boldsymbol{h}_{21}^0\boldsymbol{\Phi}_1\boldsymbol{h}_{21}^{0*}-t_5-\lambda_5\epsilon_{21}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_5 \ge 0, \\
& \begin{bmatrix} -\boldsymbol{\Psi}_1+\lambda_6\boldsymbol{I} & -\boldsymbol{\Psi}_1\boldsymbol{h}_{21}^{0*} \\ -\boldsymbol{h}_{21}^0\boldsymbol{\Psi}_1^* & -\boldsymbol{h}_{21}^0\boldsymbol{\Psi}_1\boldsymbol{h}_{21}^{0*}+t_7-\lambda_6\epsilon_{21}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_6 \ge 0, \\
& \begin{bmatrix} -(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2)+\lambda_7\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & t_6-\lambda_7\epsilon_{22}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_7 \ge 0, \\
& \begin{bmatrix} \boldsymbol{\Phi}_2+\lambda_8\boldsymbol{I} & \boldsymbol{\Phi}_2\boldsymbol{h}_{12}^{0*} \\ \boldsymbol{h}_{12}^0\boldsymbol{\Phi}_2^* & \boldsymbol{h}_{12}^0\boldsymbol{\Phi}_2\boldsymbol{h}_{12}^{0*}-t_8-\lambda_8\epsilon_{12}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_8 \ge 0, \\
& \begin{bmatrix} -\boldsymbol{\Psi}_2+\lambda_9\boldsymbol{I} & -\boldsymbol{\Psi}_2\boldsymbol{h}_{12}^{0*} \\ -\boldsymbol{h}_{12}^0\boldsymbol{\Psi}_2^* & -\boldsymbol{h}_{12}^0\boldsymbol{\Psi}_2\boldsymbol{h}_{12}^{0*}+t_{10}-\lambda_9\epsilon_{12}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_9 \ge 0, \\
& \begin{bmatrix} -(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1)+\lambda_{10}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & t_9-\lambda_{10}\epsilon_{11}^2 \end{bmatrix} \succeq \boldsymbol{0}, \quad \lambda_{10} \ge 0, \\
& \boldsymbol{\Phi}_1 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_1 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_1+\boldsymbol{\Psi}_1) \le P_1, \\
& \boldsymbol{\Phi}_2 \succeq \boldsymbol{0}, \quad \boldsymbol{\Psi}_2 \succeq \boldsymbol{0}, \quad \mathrm{Tr}(\boldsymbol{\Phi}_2+\boldsymbol{\Psi}_2) \le P_2.
\end{aligned} \tag{55}$$

For a given t, the above problem is formulated as the following semidefinite feasibility problem [19]:

$$\text{find} \quad \mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2,\ t_1,\cdots,t_{10},\ \lambda_1,\cdots,\lambda_{10}, \quad (56)$$

subject to the constraints in (55). The minimum value of t, denoted by t_{min}^{kl}, can be obtained using bisection method [19] as described in Section III. The value of $t_{lowerlimit}$ can be taken as 0 (corresponding to the minimum information rate of 0). The value of $t_{upperlimit}$ can be taken as $(2^{C_E^{'}}-1)$, which corresponds to the best case information capacity of the eavesdropper link. Using t_{min}^{kl} in (29), the upper bound on $R_E^{''kl}$ is given by

$$R_E^{''kl} \ \leq \ \log_2\big(1+t_{min}^{kl}\big). \quad (57)$$

Similarly, denoting the optimal values of $t_5,\cdots,t_{10}$ by $t_5^{kl},\cdots,t_{10}^{kl}$, we obtain the lower bounds on $R_1^{''k}$ and $R_2^{''l}$ as

$$R_1^{''k} \ \geq \ \log_2\Big(1+\frac{t_5^{kl}}{N_0+t_6^{kl}+t_7^{kl}}\Big), \quad (58)$$

$$R_2^{''l} \ \geq \ \log_2\Big(1+\frac{t_8^{kl}}{N_0+t_9^{kl}+t_{10}^{kl}}\Big). \quad (59)$$

Using the upper bound from (57) and lower bounds from (58) and (59), the lower bound on the worst case sum secrecy rate is given by $\max_{\substack{k=0,1,2,\cdots,K,\\ l=0,1,2,\cdots,L}} \ (R_1^{''k}+R_2^{''l}-R_E^{''kl})$.

Remark: We note that when S_1 and S_2 do not transmit jamming signals, the optimization problems (34) and (39) will be equivalent, and the sum secrecy rate will be exact. However, the lower bound on the sum secrecy rate as obtained above with jamming strategies will always be greater than or equal to the (exact) sum secrecy rate with no jamming strategies.

A. Transmit Power Minimization with SINR Constraints

In this subsection, we minimize the total transmit power (i.e., S_1 transmit power plus S_2 transmit power) with imperfect CSI subject to receive SINR constraints at S_1, S_2, E, and individual transmit power constraints. The optimization problem to minimize the total transmit power is as follows:

$$\min_{\mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2} \quad \text{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1)+\text{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \quad (60)$$

$$\text{s.t.} \quad \max_{\boldsymbol{e}_1,\ \boldsymbol{e}_2} \Bigg(\frac{(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\mathbf{\Phi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^*+(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\mathbf{\Phi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^*}{N_0+(\boldsymbol{z}_1^0+\boldsymbol{e}_1)\mathbf{\Psi}_1(\boldsymbol{z}_1^0+\boldsymbol{e}_1)^*+(\boldsymbol{z}_2^0+\boldsymbol{e}_2)\mathbf{\Psi}_2(\boldsymbol{z}_2^0+\boldsymbol{e}_2)^*}\Bigg) \leq \ \gamma_E, \quad (61)$$

$$\min_{\boldsymbol{e}_{21},\ \boldsymbol{e}_{22}} \Bigg(\frac{(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\mathbf{\Phi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^*}{N_0+\boldsymbol{e}_{22}(\mathbf{\Phi}_2+\mathbf{\Psi}_2)\boldsymbol{e}_{22}^*+(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})\mathbf{\Psi}_1(\boldsymbol{h}_{21}^0+\boldsymbol{e}_{21})^*}\Bigg) \geq \ \gamma_{S_2}, \quad (62)$$

$$\min_{\boldsymbol{e}_{11},\ \boldsymbol{e}_{12}} \Bigg(\frac{(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\mathbf{\Phi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^*}{N_0+\boldsymbol{e}_{11}(\mathbf{\Phi}_1+\mathbf{\Psi}_1)\boldsymbol{e}_{11}^*+(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})\mathbf{\Psi}_2(\boldsymbol{h}_{12}^0+\boldsymbol{e}_{12})^*}\Bigg) \geq \ \gamma_{S_1}, \quad (63)$$

$$\|\boldsymbol{e}_{11}\|^2 \leq \epsilon_{11}^2, \quad \|\boldsymbol{e}_{12}\|^2 \leq \epsilon_{12}^2, \quad \|\boldsymbol{e}_{21}\|^2 \leq \epsilon_{21}^2,$$
$$\|\boldsymbol{e}_{22}\|^2 \leq \epsilon_{22}^2, \quad \|\boldsymbol{e}_1\|^2 \leq \epsilon_1^2, \quad \|\boldsymbol{e}_2\|^2 \leq \epsilon_2^2, \quad (64)$$

$$\mathbf{\Phi}_1 \succeq \mathbf{0}, \quad \mathbf{\Psi}_1 \succeq \mathbf{0}, \quad \text{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1) \ \leq \ P_1,$$
$$\mathbf{\Phi}_2 \succeq \mathbf{0}, \quad \mathbf{\Psi}_2 \succeq \mathbf{0}, \quad \text{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \ \leq \ P_2. \quad (65)$$

The left hand side of the inequality in the constraint (61) corresponds to the best case received SINR at the eavesdropper over the region of CSI error uncertainty. Similarly, the left hand side of the inequality in the constraints (62) and (63) correspond to the worst case received SINR at S_2, and S_1, respectively. γ_E, γ_{S_2}, and γ_{S_1} are known SINR thresholds at E, S_2, and S_1, respectively. Solving the above optimization problem is hard due to the presence of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ in both the numerator and denominator of the SINR expression of the eavesdropper in (61). Similarly, $\boldsymbol{e}_{21}$ and $\boldsymbol{e}_{12}$ appear in both the numerator and denominator of the SINR expressions of S_2 and S_1 in the constraints (62) and (63), respectively. By independently constraining the various quadratic terms appearing in the constraints (61), (62), and (63), and further using the S-procedure, we get the following upper bound for the above optimization problem:

$$\min_{\substack{\mathbf{\Phi}_1,\ \mathbf{\Psi}_1,\ \mathbf{\Phi}_2,\ \mathbf{\Psi}_2,\\ t_1,\ t_2,\cdots,\ t_{10},\\ \lambda_1,\ \lambda_2,\cdots,\ \lambda_{10}}} \quad \text{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1)+\text{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \quad (66)$$

$$\text{s.t.} \quad t_3 \ \geq \ 0,\ t_4 \ \geq \ 0,\ t_5 \ \geq \ 0,\ t_8 \ \geq \ 0,$$
$$\big(t_1+t_2\big)-\gamma_E\big(N_0+t_3+t_4\big) \ \leq \ 0,$$
$$\gamma_{S_2}\big(N_0+t_6+t_7\big)-t_5 \ \leq \ 0,$$
$$\gamma_{S_1}\big(N_0+t_9+t_{10}\big)-t_8 \ \leq \ 0,$$
$$\begin{bmatrix} -\mathbf{\Phi}_1+\lambda_1\boldsymbol{I} & -\mathbf{\Phi}_1\boldsymbol{z}_1^{0*} \\ -\boldsymbol{z}_1^0\mathbf{\Phi}_1^* & -\boldsymbol{z}_1^0\mathbf{\Phi}_1\boldsymbol{z}_1^{0*}+t_1-\lambda_1\epsilon_1^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_1 \geq 0,$$
$$\begin{bmatrix} \mathbf{\Psi}_1+\lambda_2\boldsymbol{I} & \mathbf{\Psi}_1\boldsymbol{z}_1^{0*} \\ \boldsymbol{z}_1^0\mathbf{\Psi}_1^* & \boldsymbol{z}_1^0\mathbf{\Psi}_1\boldsymbol{z}_1^{0*}-t_3-\lambda_2\epsilon_1^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_2 \geq 0,$$
$$\begin{bmatrix} -\mathbf{\Phi}_2+\lambda_3\boldsymbol{I} & -\mathbf{\Phi}_2\boldsymbol{z}_2^{0*} \\ -\boldsymbol{z}_2^0\mathbf{\Phi}_2^* & -\boldsymbol{z}_2^0\mathbf{\Phi}_2\boldsymbol{z}_2^{0*}+t_2-\lambda_3\epsilon_2^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_3 \geq 0,$$
$$\begin{bmatrix} \mathbf{\Psi}_2+\lambda_4\boldsymbol{I} & \mathbf{\Psi}_2\boldsymbol{z}_2^{0*} \\ \boldsymbol{z}_2^0\mathbf{\Psi}_2^* & \boldsymbol{z}_2^0\mathbf{\Psi}_2\boldsymbol{z}_2^{0*}-t_4-\lambda_4\epsilon_2^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_4 \geq 0,$$
$$\begin{bmatrix} \mathbf{\Phi}_1+\lambda_5\boldsymbol{I} & \mathbf{\Phi}_1\boldsymbol{h}_{21}^{0*} \\ \boldsymbol{h}_{21}^0\mathbf{\Phi}_1^* & \boldsymbol{h}_{21}^0\mathbf{\Phi}_1\boldsymbol{h}_{21}^{0*}-t_5-\lambda_5\epsilon_{21}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_5 \geq 0,$$
$$\begin{bmatrix} -\mathbf{\Psi}_1+\lambda_6\boldsymbol{I} & -\mathbf{\Psi}_1\boldsymbol{h}_{21}^{0*} \\ -\boldsymbol{h}_{21}^0\mathbf{\Psi}_1^* & -\boldsymbol{h}_{21}^0\mathbf{\Psi}_1\boldsymbol{h}_{21}^{0*}+t_7-\lambda_6\epsilon_{21}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_6 \geq 0,$$
$$\begin{bmatrix} -(\mathbf{\Phi}_2+\mathbf{\Psi}_2)+\lambda_7\boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & t_6-\lambda_7\epsilon_{22}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_7 \geq 0,$$
$$\begin{bmatrix} \mathbf{\Phi}_2+\lambda_8\boldsymbol{I} & \mathbf{\Phi}_2\boldsymbol{h}_{12}^{0*} \\ \boldsymbol{h}_{12}^0\mathbf{\Phi}_2^* & \boldsymbol{h}_{12}^0\mathbf{\Phi}_2\boldsymbol{h}_{12}^{0*}-t_8-\lambda_8\epsilon_{12}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_8 \geq 0,$$
$$\begin{bmatrix} -\mathbf{\Psi}_2+\lambda_9\boldsymbol{I} & -\mathbf{\Psi}_2\boldsymbol{h}_{12}^{0*} \\ -\boldsymbol{h}_{12}^0\mathbf{\Psi}_2^* & -\boldsymbol{h}_{12}^0\mathbf{\Psi}_2\boldsymbol{h}_{12}^{0*}+t_{10}-\lambda_9\epsilon_{12}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_9 \geq 0,$$
$$\begin{bmatrix} -(\mathbf{\Phi}_1+\mathbf{\Psi}_1)+\lambda_{10}\boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & t_9-\lambda_{10}\epsilon_{11}^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_{10} \geq 0,$$
$$\mathbf{\Phi}_1 \succeq \mathbf{0}, \quad \mathbf{\Psi}_1 \succeq \mathbf{0}, \quad \text{Tr}(\mathbf{\Phi}_1+\mathbf{\Psi}_1) \ \leq \ P_1,$$
$$\mathbf{\Phi}_2 \succeq \mathbf{0}, \quad \mathbf{\Psi}_2 \succeq \mathbf{0}, \quad \text{Tr}(\mathbf{\Phi}_2+\mathbf{\Psi}_2) \leq \ P_2, \quad (67)$$

where $t_1,t_2,\cdots,t_{10}$ are as defined in the optimization problem (39). The above problem can be easily solved using semidefinite programming techniques.

V. Results and Discussions

In this section, we present numerical results on the secrecy rate under perfect and imperfect CSI conditions. We assume that $M_1 = M_2 = 2$. We have used the following channel gains as the estimates: $\boldsymbol{h}_{12}^0 = [0.0838+0.5207i,\ 0.2226-$

$0.2482i]$, $\boldsymbol{h}_{21}^0 = [0.4407 + 0.6653i,\ 0.5650 - 0.0015i]$, $\boldsymbol{z}_1^0 = [0.0765 + 0.0276i,\ -0.0093 + 0.0062i]$, $\boldsymbol{z}_2^0 = [-0.0449 + 0.0314i,\ -0.0396 - 0.0672i]$. We assume that the magnitudes of the CSI errors on all the links are equal, i.e., $\epsilon_{11} = \epsilon_{12} = \epsilon_{21} = \epsilon_{22} = \epsilon_1 = \epsilon_2 = \epsilon$. We also assume that $N_0 = 1$. In Fig. 2 and Fig. 3, we plot the (R_1, R_2) region obtained

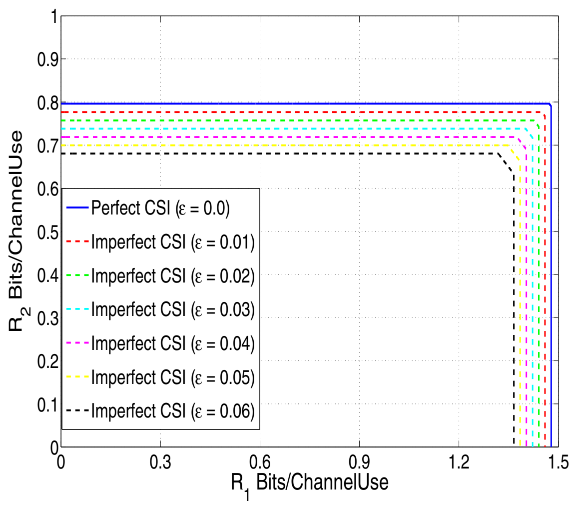

Fig. 2. Achievable (R_1, R_2) region in full-duplex communication. $P_1 = P_2 = 3$ dB, $M_1 = M_2 = 2$, $N_0 = 1$, $\epsilon = 0.0,\ 0.01,\ 0.02,\ 0.03,\ 0.04,\ 0.05,\ 0.06$.

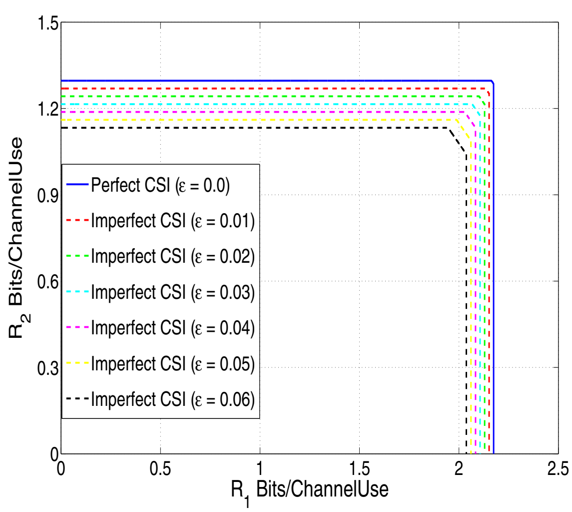

Fig. 3. Achievable (R_1, R_2) region in full-duplex communication. $P_1 = P_2 = 6$ dB, $M_1 = M_2 = 2$, $N_0 = 1$, $\epsilon = 0.0,\ 0.01,\ 0.02,\ 0.03,\ 0.04,\ 0.05,\ 0.06$.

by maximizing the sum secrecy rate for various values of $\epsilon = 0.0,\ 0.01,\ 0.02,\ 0.03,\ 0.04,\ 0.05, 0.06$. Results in Fig. 2 and Fig. 3 are generated for fixed powers $P_1 = P_2 = 3$ dB and $P_1 = P_2 = 6$ dB, respectively. We observe that as the magnitude of the CSI error increases the corresponding sum secrecy rate decreases which results in the shrinking of the achievable rate region. Also, as the power is increased from 3 dB to 6 dB, the achievable secrecy rate region increases.

VI. Conclusions

We investigated the sum secrecy rate and the corresponding achievable secrecy rate region in MISO full-duplex wiretap channel when the CSI on all the links were assumed to be imperfect. We obtained the transmit covariance matrices associated with the message signals and the jamming signals which maximized the worst case sum secrecy rate. Numerical results illustrated the impact of imperfect CSI on the achievable secrecy rate region.

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