Performance Analysis of Generalized Selection Combining of M-ary NCFSK Signals in Rayleigh Fading Channels

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Abstract— **In this paper, we propose and analyze the bit error performance of a generalized selection combining (GSC) receiver for M-ary noncoherent frequency shift keying (NCFSK) signals on i.i.d. Rayleigh fading channels with L antennas at the receiver. For each of the M hypotheses, the receiver combines the K largest outputs among the L available square-law detector outputs before proceeding to the bit detection process. We derive a closed-form expression for the bit error probability of the proposed (K, L) GSC receiver, and present numerical results to illustrate the bit error performance of this receiver for different values of M, K, and L. We also show that our generalized (K, L) GSC scheme and analysis encompass the previously reported schemes/analyses by Chyi *et al* and Hahn as special cases for $K = 1$ and $K = L$, respectively.**

Keywords: Generalized selection combining, M-ary noncoherent FSK, fading channels.

I. Introduction

The effects of multipath fading in a mobile radio environment can be alleviated by using diversity reception [1],[2]. Typical diversity combining schemes include maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC). Recently, there has been growing interest in the study of *generalized selection combining* (GSC) schemes where K out of L available diversity paths ($K \leq L$) are combined [5]-[16]. The structure of a GSC receiver depends on the type of modulation format used (M-PSK/M-FSK/other), type of detection (coherent/noncoherent), and whether the channel state information (CSI) is known at the receiver. Hybrid SC/MRC and hybrid SC/EGC schemes are possible depending on the knowledge of the CSI (fade amplitude and phase) at the receiver. In a hybrid SC/MRC scheme, the knowledge of fade amplitude and phase on each diversity branch is available at the receiver. The receiver first weights each branch with the complex conjugate of the fade and selects the best K out of L branch outputs. Hybrid SC/EGC schemes, on the other hand, typically employ noncoherent detection where the receiver does not have the knowledge of the channel phase.

In this paper, we are concerned with generalized selection combining of M-ary noncoherent FSK (NCFSK) signals on Rayleigh fading channels. The new contributions in this paper are a) we propose a novel hybrid SC/EGC scheme for M-ary NCFSK signals, where the receiver does not have knowledge of the fade amplitudes and phases, and b) derive a closed-form expression for the bit error probability of the proposed GSC receiver on i.i.d. Rayleigh fading channels. In the proposed GSC scheme, for each of the M hypotheses, the K largest outputs among the L available square-law detector outputs are combined before proceeding to the bit detection process. We present numerical results to illustrate the bit error performance of the proposed GSC receiver for different values of M, K, and L. Interestingly, we show that our generalized (K, L) GSC scheme and analysis encompass the previously reported schemes/analyses by Chyi *et al* in [3] and Hahn in [4] as special cases for $K = 1$ and $K = L$, respectively.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we present the proposed (K, L) GSC receiver for M-ary NCFSK signals and the bit error performance analysis. Numerical results and discussions are presented in Section IV, and conclusions are given in Section V.

II. System Model

We assume that the transmitted symbols are modulated by M-ary orthogonal FSK signals with $\underline{s}_i = [0, \ldots, 1, \ldots, 0]^T$ associated with the message m_i, where '1' is in the i^{th} position and $i = 1, 2, \ldots, M$. The complex orthogonal basis functions $\phi_l(t) = \exp(j2\pi f_l t)$, $l = 1, 2, \ldots, M$ are used to synthesize the transmitted information symbol $\underline{s}_m = [s_{m,1}, \ldots, s_{m,M}]$. That is, $s_m(t) = \sum_{j=1}^{M} s_{m,j}\phi_j(t)$, $m = 1, 2, \ldots, M$.

The modulated symbols are sent through the fading channel, and the signal is received through L antennas at the receiver. We assume that the fading process is frequency non-selective and remains constant over one symbol interval. Assuming perfect symbol timing at the receiver, the equivalent lowpass representation of the received symbols, after the noncoherent demodulation, when $s_1(t)$ is the transmitted signal, is given by [1]

$$\begin{aligned} r_{c,1}^{(l)} &= \sqrt{E_s}\alpha^{(l)}\cos\theta^{(l)} + n_{c,1}^{(l)}, \\ r_{s,1}^{(l)} &= \sqrt{E_s}\alpha^{(l)}\sin\theta^{(l)} + n_{s,1}^{(l)}, \quad l = 1, 2, \ldots, L, \end{aligned} \tag{1}$$

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and

$$\begin{aligned} r_{c,m}^{(l)} &= n_{c,m}^{(l)}, \\ r_{s,m}^{(l)} &= n_{s,m}^{(l)},\ l = 1, 2, \ldots, L \text{ and } m = 2, \ldots, M. \end{aligned} \tag{2}$$

In the above, E_s is the symbol energy per branch and is related to the total bit energy E_b as $E_s = \log_2(M)E_b/L$, thereby keeping the total SNR per bit, E_b/N_0, constant. Also, $\alpha^{(l)}$ and $\theta^{(l)}$ are the fade amplitude and random phase associated with the l^{th} antenna path, respectively. We assume that the $\alpha^{(l)}$'s are i.i.d. Rayleigh random variables with the density function $f_\alpha(x) = 2xe^{-x^2},\ x \geq 0$, where we have assumed that $E(\alpha^2) = 1$. It is also assumed that the random phases $\theta^{(l)}$ are uniformly distributed in $[0,\ 2\pi]$. Note that $n_{c,m}^{(l)}$ and $n_{s,m}^{(l)}$ represent real Gaussian random variables each with zero mean and variance σ^2. Here $\sigma^2 = N_0/2$, where N_0 is the one-sided power spectral density of the underlying Gaussian process.

III. Analysis

Let $X_m^{(l)} = [r_{c,m}^{(l)}]^2 + [r_{s,m}^{(l)}]^2$ denote the energy at the output of the m^{th} demodulator on the l^{th} diversity path, $m = 1, 2, \ldots, M$, $l = 1, 2, \ldots, L$. Assuming that $m = 1$ is the transmitted symbol, in the absence of knowledge of $\alpha^{(l)}$ and $\theta^{(l)}$, the pdf of $X_m^{(l)}$, after normalization by N_0, is given by [17]

$$\begin{aligned} f_{X_1^{(l)}}(x) &= \frac{1}{1+\overline{\gamma}} e^{-\frac{x}{1+\overline{\gamma}}},\ x \geq 0, \\ f_{X_m^{(l)}}(x) &= e^{-x},\ x \geq 0,\ m = 2, \ldots, M, \end{aligned} \tag{3}$$

where $\overline{\gamma} = E(\alpha^2)E_s/N_0 = E_s/N_0$ is the average SNR per symbol per branch. For each hypothesis m, $m = 1, 2, \ldots, M$, we combine the K largest outputs among the L available square-law detector outputs (i.e., the $X_m^{(l)}$'s) before proceeding to the bit detection process, as shown in Fig. 1. For each hypothesis m, the statistic Z_m at the output of (K, L) GSC combiner is given by

$$Z_m = \sum_{j=1}^{K} X_m^j,\ m = 1, 2, \ldots, M, \tag{4}$$

where $X_m^1, X^2, \ldots, X_m^K$ are the K largest among $X_m^{(1)}, X_m^{(2)}, \ldots, X_m^{(L)}$.

Since the random variables $X_m^{(l)}$ for $l = 1, 2, \ldots, L$ and $m = 2, \ldots, M$ are i.i.d., the random variables, $Z_2, Z_3, \ldots, Z_M$ are also i.i.d. With this, the symbol error probability (SEP), $P_s^{(K,L)GSC}$, is given by

$$\begin{aligned} P_s^{(K,L)GSC} &= \text{Prob}\left(Z_1 < \max(Z_2, \ldots, Z_M)\right) \\ &= 1 - \text{Prob}\left(\max(Z_2, \ldots, Z_M) < Z_1\right), \\ &= E_{Z_1}\left[1 - \prod_{m=2}^{M} \text{Prob}(Z_m < Z_1)\right] \\ &= 1 - \int_{z=0}^{\infty} \left[F_{Z_2}(z)\right]^{M-1} f_{Z_1}(z)dz, \end{aligned} \tag{5}$$

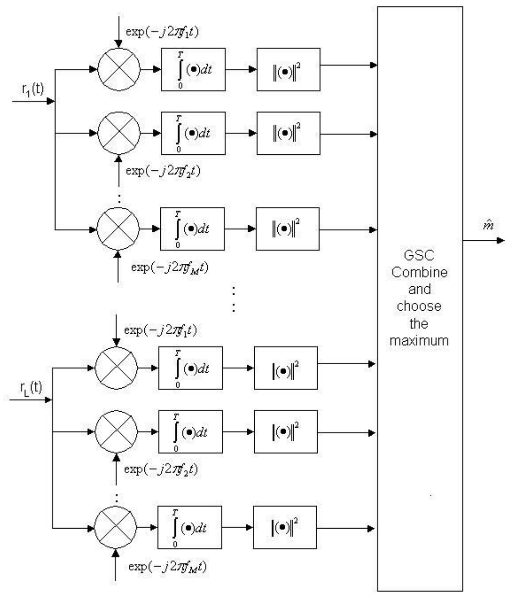

Fig. 1. Proposed (K, L)GSC receiver for M-ary NCFSK signals.

where $F_{Z_2}(z)$ is the cdf of the random variable Z_2 and $f_{Z_1}(z)$ is the pdf of the random variable Z_1. The bit error probability $P_b^{(K,L)GSC}$ is then given by [1]

$$P_b^{(K,L)GSC} = \frac{M}{2(M-1)} P_s^{(K,L)GSC}. \tag{6}$$

In order to obtain $F_{Z_2}(\cdot)$ and $f_{Z_1}(\cdot)$ we make use of the following result [14].

Let $U_1, U_2, \ldots, U_L$ be i.i.d. exponential random variables with mean μ_U. Let $U_{j:L}$, $j = 1, 2, \ldots, L$ be the order statistics of $U_1, U_2, \ldots, U_L$ such that $U_{1:L} \geq U_{2:L} \geq \cdots \geq U_{L:L}$. Then the cdf $F_V(x)$ and the pdf $f_V(x)$ of the random variable $V = \sum_{k=1}^{K} U_{k:L}$ are given by

$$\begin{aligned} F_V(x) &= \binom{L}{K}\left[1 - e^{-\frac{x}{\mu_U}} \sum_{l=0}^{K-1} \frac{(\frac{x}{\mu_U})^l}{l!} + \sum_{l=1}^{L-K} (-1)^{K+l-1} \binom{L-K}{l} \left(\frac{K}{l}\right)^{K-1} \right. \\ &\quad \left. \cdot \left(\frac{1 - e^{-(1+\frac{l}{K})\frac{x}{\mu_U}}}{1 + \frac{l}{K}} - \sum_{m=0}^{K-2} \left(\frac{-l}{K}\right)^m \left(1 - e^{-\frac{x}{\mu_U}} \sum_{k=0}^{m} \frac{(\frac{x}{\mu_U})^k}{k!}\right)\right)\right], \end{aligned} \tag{7}$$

and

$$\begin{aligned} f_V(x) &= \binom{L}{K}\left[\frac{x^{K-1} e^{-\frac{x}{\mu_U}}}{\mu_U^K (K-1)!} + \frac{1}{\mu_U} \sum_{l=1}^{L-K} (-1)^{K+l-1} \binom{L-K}{l} \left(\frac{K}{l}\right)^{K-1} \right. \\ &\quad \left. e^{-\frac{x}{\mu_U}} \cdot \left(e^{-\frac{lx}{K\mu_U}} - \sum_{m=0}^{K-2} \frac{1}{m!} \left(\frac{-lx}{K\mu_U}\right)^m\right)\right], \end{aligned} \tag{8}$$

respectively. Now, a) substituting $\mu_U = 1$ in Eqn. (7) and using the resulting $F_V(x)$ in place of $F_{Z_2}(z)$ in Eqn. (5), b) substituting $\mu_U = 1 + \overline{\gamma}$ in Eqn. (8) and using the resulting $f_V(x)$

in place of $f_{Z_1}(z)$ in Eqn. (5), and c) performing the integration, we obtain the following expression[1] for $P_s^{(K,L)GSC}$:

$$P_s^{(K,L)GSC} = 1 - \binom{L}{K}^M \sum_{i=0}^{M-1} \sum_{\underline{l}^i=\underline{1}^i}^{\overline{L-K}^i} \sum_{\underline{m}^i=\underline{0}^i}^{\overline{K-2}^i} \sum_{\underline{k}^i=\underline{0}^i}^{\underline{m}^i} \sum_{n=0}^{M-1-i} \sum_{s=0}^{n}$$

$$\sum_{\underline{r}^s=\underline{0}^s}^{\overline{K-1}^s} \sum_{\underline{q}^{n-s}=\underline{1}^{n-s}}^{\overline{L-K}^{n-s}} \frac{(-1)^n \binom{M-1}{i} \binom{M-1-i}{n} \binom{n}{s} \mathcal{T}_1^{M-1-i-n}}{\mu_2^{k_1+\cdots+k_i+r_1+\cdots+r_s}}$$

$$\left[\prod_{p=1}^{i} \frac{b(l_p)(-l_p/K)^{m_p}}{k_p!}\right] \frac{1}{\prod_{j=1}^{s} r_j!} \left[\prod_{j=1}^{n-s} \frac{b(q_j)}{1+\frac{q_j}{K}}\right] \cdot (\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_3), \quad (9)$$

where

$$\mathcal{R}_1 = \frac{1}{\mu_1^K (K-1)!} \frac{\Gamma(b+K)}{\left(\frac{1}{\mu_1}+a\right)^{b+K}}, \quad (10)$$

$$\mathcal{R}_2 = \frac{1}{\mu_1} \sum_{l=1}^{L-K} b(l) \frac{\Gamma(b+1)}{\left(a+\frac{1}{\mu_1}+\frac{l}{K\mu_1}\right)^{b+1}}, \quad (11)$$

and

$$\mathcal{R}_3 = \frac{1}{\mu_1} \sum_{l=1}^{L-K} \sum_{m=0}^{K-2} \frac{b(l)}{m!} \left(\frac{-l}{K\mu_1}\right)^m \frac{\Gamma(m+b+1)}{\left(a+\frac{1}{\mu_1}\right)^{m+b+1}}. \quad (12)$$

The other terms in Eqns. (9),(10),(11),(12) are defined as follows:

$$\begin{aligned}
\mu_1 &= 1+\overline{\gamma} \\
\mu_2 &= 1 \\
a &= \frac{i+n+q_1+\cdots+q_{n-s}}{\mu_2} \\
b &= k_1+\cdots+k_i+r_1+\cdots+r_s \\
b(l) &= (-1)^{K+l-1}\binom{L-K}{l}\left(\frac{K}{l}\right)^{K-1} \\
\mathcal{T}_1 &= 1+\sum_{l=1}^{L-K}\frac{b(l)}{1+\frac{l}{K}} - \sum_{l=1}^{L-K}\sum_{m=0}^{K-2} b(l)\left(\frac{-l}{K}\right)^m. \quad (13)
\end{aligned}$$

The summation $\sum_{\underline{\psi}^N=\underline{\xi}^N}^{\underline{\epsilon}^N}$, used in Eqn. (9), is defined as

$$\sum_{\underline{\psi}^N=\underline{\xi}^N}^{\underline{\epsilon}^N} = \sum_{\psi_1=\xi_1}^{\epsilon_1} \sum_{\psi_2=\xi_2}^{\epsilon_2} \cdots \sum_{\psi_N=\xi_N}^{\epsilon_N}. \quad (14)$$

Finally, $\underline{C}^N$ is a constant vector of dimension N with each element being C.

We note that, when $K = 1$, the proposed GSC scheme chooses the maximum of $Z_1, Z_2, \ldots, Z_M$ where $Z_m = \max(X_m^{(1)}, X_m^{(2)}, \ldots, X_m^{(L)})$, which is same as the scheme proposed by Chyi *el al* in [3]. By substituting $K = 1$ in Eqn. (9), we obtain the corresponding SEP as

$$P_s^{(1,L)GSC} = 1 - \sum_{n=0}^{L(M-1)} \sum_{l=0}^{L-1} (-1)^{l+n} \binom{L(M-1)}{n} \binom{L-1}{l} \times \frac{L}{1+l+n+n\overline{\gamma}}. \quad (15)$$

The above expression can be further simplified and is shown [19] to reduce to exactly the Eqn.(11) in [3].

On the other hand, when $K = L$, the proposed GSC scheme, for each hypothesis m, $m = 1, 2, \ldots, M$, combines the energies across all the L antenna paths before the bit detection process. Since we do not assume knowledge of random fade amplitudes and phases on each antenna path, this special case turns out to be the optimum square-law combiner with L-antenna diversity in [4]. By substituting $K = L$ in Eqn. (9), we obtain the corresponding SEP as

$$P_s^{(L,L)GSC} = 1 - \sum_{n=0}^{M-1} \sum_{\underline{l}^n=\underline{0}^n}^{\overline{L-1}^n} (-1)^n \binom{M-1}{n} \frac{\Gamma(l_1+\cdots+l_n+L)}{(L-1)!\prod_{p=1}^{n} l_p!} \times \frac{(1+\overline{\gamma})^{l_1+\cdots+l_n}}{(n+1+n\overline{\gamma})^{l_1+\cdots+l_n+L}}. \quad (16)$$

Here again, the above expression can be further simplified and is shown [19] to reduce to Eqn.(23) in [4]. Thus, our generalized (K, L)GSC scheme and analysis encompass the previously reported schemes by Chyi *et al* in [3] and Hahn in [4] as special cases for $K = 1$ and $K = L$, respectively.

IV. Results and Discussion

In this section, we present numerical results illustrating the bit error performance of the proposed (K, L) GSC receiver for different values of modulation alphabet size (M), number of receive antennas (L), and number of square-law detector outputs combined (K). Fig. 2 shows the bit error performance of the GSC receiver for 8-ary NCFSK with $L = 5$. The number of paths combined, K, is varied from 1 to 5. As expected, the performance improves as the number of paths combined is increased. It is noted that the $K = 1$, $L = 5$ curve corresponds to the performance of Chyi's maximum output selection combining scheme. It is observed that by increasing K from 1 to 2 (i.e., combining one more path compared to Chyi's SC scheme) the performance improves by about 1 dB at a BER of 10^{-5}. The $K = L = 5$ curve corresponds to the optimum square-law combining scheme performance, which represents the best possible performance for $M = 8$ and $L = 5$. It is observed that close to optimum square-law combiner ($K = L = 5$) performance can be achieved just by combining $K = 3$ out of $L = 5$ available paths (due to diminishing returns at increased values of K). This can result in reduced receiver complexity without incurring much loss compared to the optimum performance.

[1]The derivation of this expression is given in the Appendix.

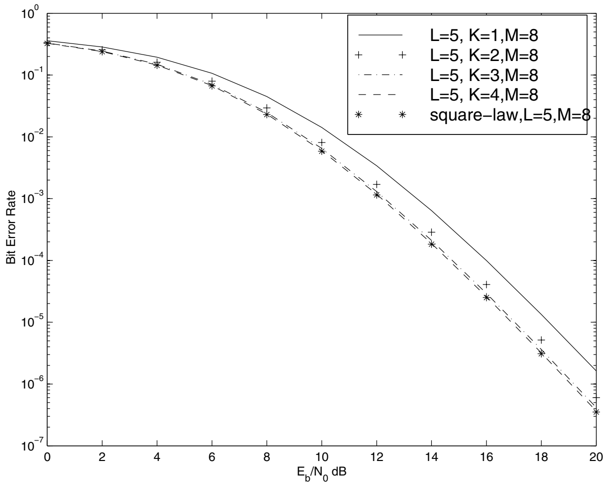

Fig. 2. Bit error performance of the GSC receiver for 8-ary NCFSK ($M = 8$) for different values of K ($= 1, 2, 3, 4, 5$) with $L = 5$.

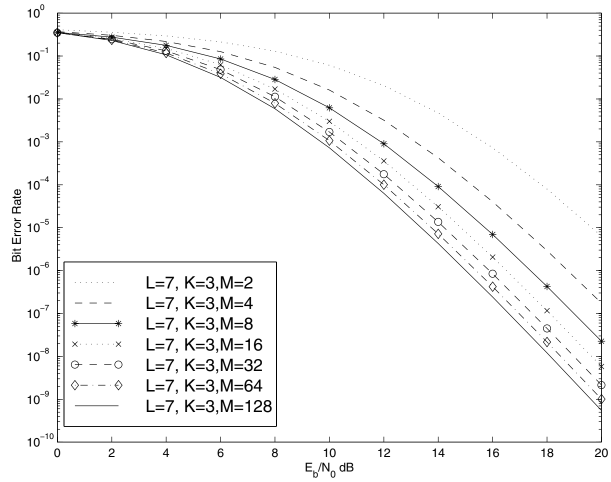

Fig. 4. Bit error performance of the GSC receiver for M-ary NCFSK with $K = 3$, $L = 7$, and $M = 2, 4, 8, 16, 32, 64, 128$.

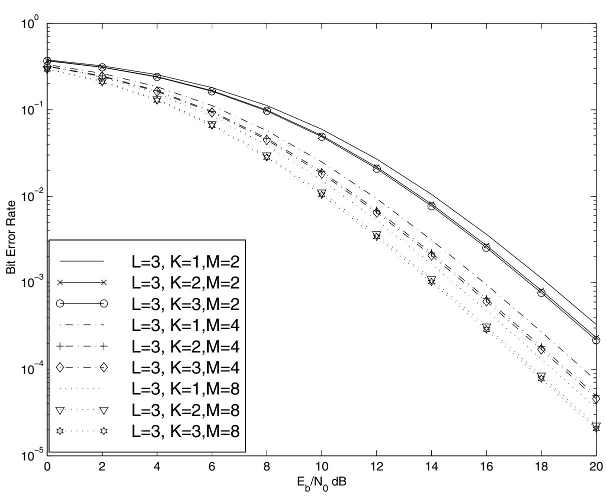

Fig. 3. Bit error performance of the GSC receiver for M-ary NCFSK for different values of M ($= 2, 4, 8$) with $L = 3$ and $K = 1, 2, 3$.

Fig. 3 gives the comparative performance of binary, 4-ary and 8-ary NCFSK schemes with $L = 3$ path diversity and $(K, 3)$ GSC reception, $K = 1, 2$ and 3. From Figure 3, it is observed that, for given K and L, bit error performance can be improved by increasing the modulation alphabet size, M. Fig. 4 also shows the performance of M-ary NCFSK schemes for various values of M ($= 2, 4, 8, 16, 32, 64$) using a $(3, 7)$ GSC receiver. From Fig. 4, it is observed that, by combining less than half the total number of available paths, significantly good error performance can be obtained by increasing the modulation alphabet size.

Fig. 5 illustrates the bit error performance as a function of the number of paths combined (K), for a given number of available paths ($L = 10$) at a given operating SNR per bit ($E_b/N_0 = 10$ dB), for various values of modulation alphabet size ($M = 2, 4, 8, 16, 32, 64, 128$). Fig. 5 illustrates the number of paths combined (K) beyond which the performance saturates (i.e., the value of K beyond which diminishing returns sets in). For example, when the modulation alphabet size M is small, the saturation occurs at lower values of K, and larger K becomes beneficial when M is made larger.

V. CONCLUSIONS

We proposed and analyzed the bit error performance of a GSC receiver for M-ary NCFSK signals on i.i.d. Rayleigh fading channels with L antennas at the receiver. For each of the M hypotheses, the receiver combines the K largest outputs among the L available square-law detector outputs before proceeding to the bit detection process. We derived a closed-form expression for the bit error probability of the proposed (K, L) GSC receiver, and presented numerical results illustrating the bit error performance of this receiver for different values of M, K, and L. Interestingly, we could show that our generalized (K, L) GSC scheme and analysis encompass the previously reported schemes/analyses by Chyi *et al* and Hahn as special cases for $K = 1$ and $K = L$, respectively.

APPENDIX

VI. DERIVATION OF EQN. (9)

In this Appendix, we present the derivation of Eqn. (9). We first arrive at a simplified expression for $[F_{Z_2}(z)]^{M-1}$ as follows. Define

$$\mathcal{C}_2(m, z) \quad = \quad e^{-\frac{z}{\mu_2}} \sum_{l=0}^{m} \frac{\left(\frac{z}{\mu_2}\right)^l}{l!}, \qquad (17)$$

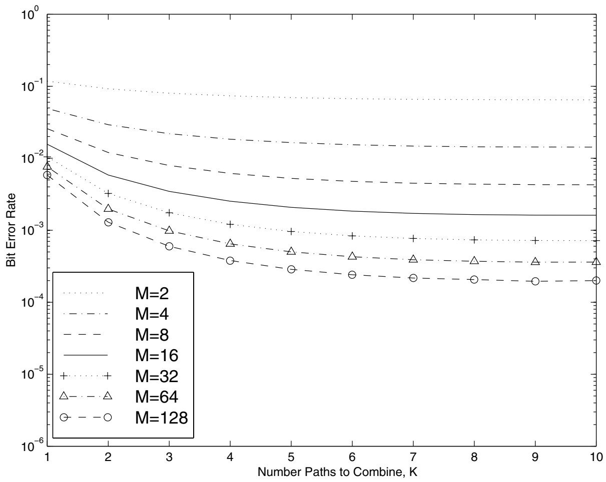

Fig. 5. Bit error performance of the GSC receiver for M-ary NCFSK as a function of K for $L = 10$, $E_b/N_0 = 10$ dB, and for various values of M $(= 2, 4, 8, 16, 32, 64, 128)$.

$$\mathcal{V}_2(z) = e^{-\frac{z}{\mu_2}} \sum_{l=0}^{K-1} \frac{\left(\frac{z}{\mu_2}\right)^l}{l!}, \tag{18}$$

and

$$\mathcal{W}_2(z) = \sum_{l=1}^{L-K} \frac{b(l)}{1+\frac{l}{K}} e^{-(1+\frac{l}{K})\frac{z}{\mu_2}}, \tag{19}$$

where $b(l)$ is defined as in Eqn. (13). With the above definitions, $F_{Z_2}(z)$ can be written as

$$F_{Z_2}(z) = \binom{L}{K}\left[\mathcal{H}_2(z) + \sum_{l=1}^{L-K}\sum_{m=0}^{K-2} b(l)(-\frac{l}{K})^m \mathcal{C}_2(m,z)\right], \tag{20}$$

where

$$H_2(z) = \mathcal{T}_1 - \mathcal{V}_2(z) - \mathcal{W}_2(z). \tag{21}$$

Upon expanding $[F_{Z_2}(z)]^{M-1}$, we obtain

$$\left[F_{Z_2}(z)\right]^{M-1} = \binom{L}{K}^{M-1} \sum_{i=0}^{M-1} \binom{M-1}{i} [\mathcal{H}_2(z)]^{M-1-i} \sum_{\underline{l}^i=\underline{1}^i}^{\underline{L-K}^i} \sum_{\underline{m}^i=\underline{0}^i}^{\underline{K-2}^i} \left\{\prod_{p=1}^{i} b(l_p)\left(-\frac{l_p}{K}\right)^{m_p} \mathcal{C}_2(m_p,z)\right\}. \tag{22}$$

In the above, l_p and m_p are integers which are implicitly defined as follows. For a given i, l and m are both vectors each of dimension i, i.e., $l = (l_1, l_2, ..., l_i)$ and $m = (m_1, m_2, ..., m_i)$. After the two summations in Eqn. (22), the product index p runs from 1 to i, as the number of product terms is the same as the dimension of l (or m). Upon expanding $\prod_{p=1}^{i} \mathcal{C}_2(m_p, z)$, Eqn. (22) can be simplified as

$$[F_{Z_2}(z)]^{M-1} = \binom{L}{K}^{M-1} \sum_{i=0}^{M-1} \binom{M-1}{i} \sum_{\underline{l}^i=\underline{1}^i}^{\underline{L-K}^i} \sum_{\underline{m}^i=\underline{0}^i}^{\underline{K-2}^i} \sum_{\underline{k}^i=\underline{0}^i}^{\underline{m}^i} \frac{1}{\prod_{p=1}^{i} k_p!\mu_2^{k_p}} \left\{\prod_{p=1}^{i} b(l_p)\left(-\frac{l_p}{K}\right)^{m_p}\right\} e^{-\frac{iz}{\mu_2}} z^{k_1+\cdots+k_i} [\mathcal{H}_2(z)]^{M-1-i} \tag{23}$$

Expanding $[\mathcal{H}_2(z)]^{M-1-i}$ using Eqns. (13), (18) and (19), we obtain

$$[\mathcal{H}_2(z)]^{M-1-i} = \sum_{n=0}^{M-1-i} (-1)^n \binom{M-1-i}{n} \mathcal{T}_1^{M-1-i-n} [\mathcal{V}_2(z) + \mathcal{W}_2(z)]^n$$
$$= \sum_{n=0}^{M-1-i} \sum_{s=0}^{n} (-1)^n \binom{M-1-i}{n} \binom{n}{s} \mathcal{T}_1^{M-1-i-n} [\mathcal{V}_2(z)]^s [\mathcal{W}_2(z)]^{n-s}. \tag{24}$$

From Eqn. (18), $[\mathcal{V}_2(z)]^s$ can be written as

$$[\mathcal{V}_2(z)]^s = e^{-\frac{sz}{\mu_2}} \sum_{\underline{r}^s=\underline{0}^s}^{\underline{K-1}^s} \prod_{j=1}^{s} \frac{z^{r_j}}{\mu_2^{r_j} r_j!}. \tag{25}$$

From Eqn. (19), $[\mathcal{W}_2(z)]^{n-s}$ can be written as

$$[\mathcal{W}_2(z)]^{n-s} = \sum_{\underline{q}^{n-s}=\underline{1}^{n-s}}^{\underline{L-K}^{n-s}} \left\{\prod_{j=1}^{n-s} \frac{b(q_j)}{(1+\frac{q_j}{K})}\right\} e^{-\frac{z(n-s+q_1+\cdots+q_{n-s})}{\mu_2}} \tag{26}$$

Substituting Eqns. (25), (26) in Eqn. (24), we obtain $[\mathcal{H}_2(z)]^{M-1-i}$, and the final expression for $[\mathcal{H}_2(z)]^{M-1-i}$ is given by

$$[\mathcal{H}_2(z)]^{M-1-i} = \sum_{n=0}^{M-1-i} \sum_{s=0}^{n} \sum_{\underline{r}^s=\underline{0}^s}^{\underline{K-1}^s} \sum_{\underline{q}^{n-s}=\underline{1}^{n-s}}^{\underline{L-K}^{n-s}} \frac{(-1)^n \binom{M-1-i}{n}\binom{n}{s} \mathcal{T}_1^{M-1-i-n}}{\mu_2^{r_1+\cdots+r_s} \prod_{j=1}^{s} r_j!} \prod_{j=1}^{n-s} \frac{b(q_j)}{(1+\frac{q_j}{K})} \cdot e^{-\frac{z(n+q_1+\cdots+q_{n-s})}{\mu_2}} z^{r_1+\cdots+r_s} \tag{27}$$

Substituting Eqn. (24) in Eqn. (23), we obtain $[F_{Z_2}(z)]^{M-1}$. The final expression for $[F_{Z_2}(z)]^{M-1}$ is then given by

$$[F_{Z_2}(z)]^{M-1} = \binom{L}{K}^{M-1} \sum_{i=0}^{M-1} \binom{M-1}{i} \sum_{\underline{l}^i=\underline{1}^i}^{\underline{L-K}^i} \sum_{\underline{m}^i=\underline{0}^i}^{\underline{K-2}^i} \sum_{\underline{k}^i=\underline{0}^i}^{\underline{m}^i} \frac{1}{\prod_{p=1}^{i} k_p!\mu_2^{k_p}} \sum_{n=0}^{M-1-i} \sum_{s=0}^{n} \sum_{\underline{r}^s=\underline{0}^s}^{\underline{K-1}^s} \sum_{\underline{q}^{n-s}=\underline{1}^{n-s}}^{\underline{L-K}^{n-s}} (-1)^n \binom{M-1-i}{n}\binom{n}{s} \frac{\mathcal{T}_1^{M-1-i-n}}{\mu_2^{r_1+\cdots+r_s}\prod_{j=1}^{s} r_j!} \left\{\prod_{j=1}^{n-s} \frac{b(q_j)}{(1+\frac{q_j}{K})}\right\} \left\{\prod_{p=1}^{i} b(l_p)\left(-\frac{l_p}{K}\right)^{m_p}\right\} e^{-\frac{z(i+n+q_1+\cdots+q_{n-s})}{\mu_2}} z^{k_1+\cdots+k_i+r_1+\cdots+r_s}. \tag{28}$$

In Eqn. (28) the term that is important to us in completing the error probability analysis is $e^{-\frac{z(i+n+q_1+\cdots+q_{n-s})}{\mu_2}} \times z^{k_1+\cdots+k_i+r_1+\cdots+r_s} \triangleq e^{-az}z^b$. When this term is integrated with $f_{Z_1}(z)$, we obtain [19]

$$\begin{aligned} N(a,b,\mu_1) &= \int_{z=0}^{\infty} e^{-az} z^b f_{Z_1}(z)dz \\ &= \binom{L}{K} \left[\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_3\right], \end{aligned} \tag{29}$$

where $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ are defined in Eqns. (10), (11) and (12), respectively. The second step in Eqn. (29) is obtained by substituting Eqn. (8) (with $\mu_U = \mu_1$) in the first step and carrying out the integration, where we have used the following result [18] to simplify the integral:

$$\int\limits_{x=0}^{\infty} e^{-ax} x^b dx = \frac{\Gamma(b+1)}{a^{b+1}}. \tag{30}$$

Combining Eqn. (29) with Eqn. (28), we obtain

$$\begin{aligned} &\int\limits_{z=0}^{\infty} [F_{Z_2}(z)]^{M-1} f_{Z_1}(z)dz = \binom{L}{K}^M \sum_{i=0}^{M-1} \binom{M-1}{i} \sum_{\underline{l}^i=\underline{1}^i}^{\overline{L-K}^i} \sum_{\underline{m}^i=\underline{0}^i}^{\overline{K-2}^i} \sum_{\underline{k}^i=\underline{0}^i}^{\underline{m}^i} \\ &\frac{1}{\prod_{p=1}^{i} k_p! \mu_2^{k_p}} \sum_{n=0}^{M-1-i} \sum_{s=0}^{n} \sum_{\underline{r}^s=\underline{0}^s}^{\overline{K-1}^s} \sum_{\underline{q}^{n-s}=\underline{1}^{n-s}}^{\overline{L-K}^{n-s}} \frac{(-1)^n \binom{M-1-i}{n} \binom{n}{s} \mathcal{T}_1^{M-1-i-n}}{\mu_2^{r_1+\cdots+r_s} \prod_{j=1}^{s} r_j!} \\ &\prod_{j=1}^{n-s} \frac{b(q_j)}{(1+\frac{q_j}{K})} \left\{ \prod_{p=1}^{i} b(l_p) \left(-\frac{l_p}{K}\right)^{m_p} \right\} \left[\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_3\right]. \end{aligned} \tag{31}$$

Finally, substituting Eqn. (31) in Eqn. (5), we obtain Eqn. (9).

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