Performance of Noncoherent Turbo Detection on Rayleigh Fading Channels

A. Ramesh*, **A. Chockalingam**† and **L. B. Milstein**‡

∗ Wireless and Broadband Communications
Synopsys (INDIA) Pvt. Ltd, Bangalore 560095, INDIA

† Department of Electrical Communication Engineering
Indian Institute of Science, Bangalore 560012, INDIA

‡ Department of Electrical and Computer Engineering
University of California, San Diego, La Jolla, CA 92093, U.S.A

Abstract—**In this paper, we are concerned with the performance of turbo codes on Rayleigh fading channels when noncoherent detection is employed. Performance of turbo codes with noncoherent detection on AWGN channels has been analyzed by Hall and Wilson. They compared the performance of turbo codes with the achievable information theoretic channel capacity for noncoherent detection on AWGN channels. We, in this paper, derive the information theoretic channel capacity for noncoherent detection on Rayleigh fading channels, and show that noncoherent turbo decoder achieves performance close to this theoretical capacity (within 1.5 dB). We further show that the performance of noncoherent turbo decoder without the knowledge of channel fades is quite close (within 0.5 dB at 10^{-5} BER) to that with perfect knowledge of the channel fades. Optimum decoding of turbo codes requires the knowledge of channel SNR. With this requirement in mind, we propose online SNR estimation schemes, based on the ratio of certain observables at the output of the noncoherent detector, which when used in the turbo decoder gives performance very close to that with perfect knowledge of channel SNR.**

Keywords – Turbo codes, fading channels, MAP decoding, noncoherent detection.

I. Introduction

Turbo codes have been shown to offer near-capacity performance on AWGN channels and significantly good performance on fully-interleaved flat Rayleigh fading channels [1]–[4]. The performance of turbo codes on *noncoherent* AWGN channels has been examined by Hall and Wilson in [5], where they considered DPSK (differential phase shift keying) and BFSK (binary frequency shift keying) modulation schemes. In this paper, we analyze the performance of turbo codes on Rayleigh fading channels with noncoherent detection using BFSK modulation. We compute the information theoretic channel capacity for noncoherent detection on Rayleigh fading channels, and show that the noncoherent turbo decoder achieves performance within 1.5 dB of capacity.

This work was supported in part by the Office of Naval Research under Grant N00014-98-1-0875, by the TRW foundation, and by Nokia Mobile Phones.

Optimum decoding of turbo codes requires the knowledge of channel SNR [6],[7]. With this requirement in mind, we propose an online SNR estimation scheme, based on certain observables at the output of the noncoherent detector, for both the AWGN channel and the Rayleigh fading channel. Our approach in this paper is similar to that in [7], where we derived SNR estimation schemes for coherent detection. The proposed SNR estimation scheme, when used in the noncoherent turbo decoder, gives performance very close to that with perfect knowledge of channel SNR.

The rest of the paper is organized as follows. In Section II, we introduce the system model and derive the modified transition metric for noncoherent turbo detection of BFSK symbols on Rayleigh fading channels without channel state information (CSI). In Section III, we calculate the capacity of Rayleigh fading channels using noncoherent BFSK with/without CSI. Section IV gives the modified log-MAP algorithm. In Section V, we derive SNR estimates on both AWGN and Rayleigh fading channels with noncoherent detection. Results and discussion are given in Section VI. Conclusions are provided in Section VII.

II. System Model

Following the notation in [8], we assume that the transmitted symbols are BFSK modulated with $\mathbf{s}_0 = [1,0]^T$ and $\mathbf{s}_1 = [0,1]^T$ denoting the vector representation of BFSK symbols associated with the messages m_0 and m_1, respectively. The transmitted symbols are passed through a fading channel and noise is added to them at the receiver front end. The equivalent low pass model of the received symbols is given by [8]

$$\underline{r}_c = \alpha \mathbf{s}_i \cos\theta + \underline{n}_c \quad (1)$$

and

$$\underline{r}_s = \alpha \mathbf{s}_i \sin\theta + \underline{n}_s. \quad (2)$$

Here, $\mathbf{s}_i$ is the transmitted BFSK signal point corresponding to the message m_i, $i \in \{0,1\}$. The received vectors $\underline{r}_c$ and $\underline{r}_s$ denote the outputs of the quadrature demodulators, θ is a random phase which is distributed uniformly over $[0, 2\pi]$, and α is the random fade amplitude experienced by the transmitted symbol $\mathbf{s}_i$. We assume that α's are i.i.d Rayleigh random variables with the density function given by

$$f_\alpha(a) = 2ae^{-a^2}, \quad a \geq 0, \tag{3}$$

we normalized the second moment of α to unity (i.e., $E[\alpha^2] = 1$). The noise vectors $\underline{n}_c$ and $\underline{n}_s$ denote the in phase and quadrature phase noise whose components have zero mean and variance σ^2, where, $\sigma^2 = N_0/2E_s$. Given $\underline{r}_c = \underline{x}$ and $\underline{r}_s = \underline{y}$, the optimum receiver sets $\widehat{m}$ equal to that m_i for which $\mathrm{Prob}(m_i|\underline{r}_c = \underline{x}, \underline{r}_s = \underline{y})$ is maximum [8]. With the signals m_i, $i \in \{0,1\}$ being equally probable, equivalently, we have to maximize

$$p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i) = E_\alpha\left\{E_\theta\left[p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i,\alpha,\theta)\right]\right\}, \tag{4}$$

where $E_\alpha[\cdot$ and $E_\theta[\cdot]$ denote the expectation operations with respect to α and θ, respectively. The quantity $p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i,\alpha,\theta)$ can be calculated as

$$\begin{aligned} p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i,\alpha,\theta) &= p_{\underline{n}_c}(\underline{x} - \alpha\underline{s}_i\cos\theta)p_{\underline{n}_s}(\underline{y} - \alpha\underline{s}_i\sin\theta) \\ &\sim e^{-\alpha^2\frac{E_s}{N_0}}e^{\frac{2E_s}{N_0}(\underline{x}\cdot\underline{s}_i\alpha\cos\theta + \underline{y}\cdot\underline{s}_i\alpha\sin\theta)} \\ &\sim e^{-\alpha^2\frac{E_s}{N_0}}e^{\frac{2E_sX_i}{N_0}\alpha\cos(\theta-\phi_i)}, \end{aligned} \tag{5}$$

where $X_i = \sqrt{(\underline{x}.\underline{s}_i)^2 + (\underline{y}.\underline{s}_i)^2}$ and $\phi_i = \tan^{-1}\left(\frac{\underline{y}\cdot\underline{s}_i}{\underline{x}\cdot\underline{s}_i}\right)$. From Eqn. (5)

$$\begin{aligned} p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i,\alpha) &\sim E_\theta\left(\frac{E_s}{\pi N_0}e^{-(2+\alpha^2)\frac{E_s}{N_0}}e^{\frac{2X_iE_s}{N_0}\alpha\cos(\theta-\phi_i)}\right) \\ &\sim e^{-\alpha^2\frac{E_s}{N_0}}I_0\left(2\alpha\frac{X_iE_s}{N_0}\right), \end{aligned} \tag{6}$$

where $I_0(\cdot)$ is the modified Bessel function of the zeroth order and first kind [8]. We can obtain $p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i)$, from Eqn. (6), as

$$\begin{aligned} p_{\underline{r}_c,\underline{r}_s}(\underline{x},\underline{y}|m_i) &\sim E_\alpha\left[e^{-\alpha^2\frac{E_s}{N_0}}I_0\left(2\alpha\frac{X_iE_s}{N_0}\right)\right] \\ &\sim \frac{2\gamma e^{-2\gamma}}{1+\gamma}e^{\frac{X_i^2\gamma^2}{1+\gamma}}, \end{aligned} \tag{7}$$

where[1] $\gamma = E_s/N_0$. Therefore, the optimum receiver has to maximize the above quantity over i.

III. Channel Capacity

In [5], Hall and Wilson obtained the capacity limits for noncoherent detection on AWGN channel. In this section, we obtain the capacity limits for noncoherent detection on a Rayleigh fading channel using BFSK signaling. Channel capacity is defined as the maximum over the input distribution, $P_X(x)$, of the mutual information between the channel output (Y) and the input (X), $I(X,Y)$. For the fading channel, if the fading amplitude is known, the mutual information is conditioned on this knowledge. In the following, we consider a) Rayleigh fading with channel state information (CSI), i.e., perfect knowledge of the fade amplitudes is available at the receiver, and b) Rayleigh fading without CSI (NCSI).

[1] In deriving Eqn. (7) we have used the result $\int\limits_{t=0}^{\infty} te^{-at^2}J_0(bt)dt = \frac{1}{2a}e^{-\frac{b^2}{4a}}$, with $J_0(it) = I_0(t)$, $i = \sqrt{-1}$.

A. Rayleigh fading with CSI

With perfect CSI, the conditional p.d.fs $p(X_i|\underline{s}_j,\alpha)$, $i,j \in \{0,1\}$ can be computed in a straightforward way, and are given by

$$\begin{aligned} p(X_0|\underline{s}_0,\alpha) &= 2\gamma X_0 e^{-(X_0^2+\alpha^2)\gamma}I_0(2X_0\alpha\gamma), \\ p(X_1|\underline{s}_0,\alpha) &= 2\gamma X_1 e^{-X_1^2\gamma}, \\ p(X_0|\underline{s}_1,\alpha) &= 2\gamma X_0 e^{-X_0^2\gamma}, \quad \text{and} \\ p(X_1|\underline{s}_1,\alpha) &= 2\gamma X_1 e^{-(X_1^2+\alpha^2)\gamma}I_0(2X_1\alpha\gamma). \end{aligned} \tag{8}$$

The channel capacity is given by

$$\begin{aligned} C_{BFSK}^{Ray,CSI} = &\int\limits_{a=0}^{\infty} f_\alpha(a)\int\limits_{X_0=0}^{\infty}\int\limits_{X_1=0}^{\infty} p(X_0|\underline{s}_0,a)p(X_1|\underline{s}_0,a)\cdot \\ &\log\left(\frac{p(X_0|\underline{s}_0,a)p(X_1|\underline{s}_0,a)}{\sum_{i=0}^{1}P(\underline{s}_i)p(X_0|\underline{s}_i,a)p(X_1|\underline{s}_i,a)}\right) \\ &da\,dX_0\,dX_1. \end{aligned} \tag{9}$$

Upon substituting the p.d.f's of Eqn. (8) in Eqn. (9), we obtain

$$\begin{aligned} C_{BFSK}^{Ray,CSI} = &\int\limits_{a=0}^{\infty} f_\alpha(a)\int\limits_{X_0=0}^{\infty}\int\limits_{X_1=0}^{\infty} 4\gamma^2X_0X_1e^{-(X_0^2+X_1^2+\alpha^2)\gamma}\cdot \\ &I_0(2X_0\alpha\gamma)\log\left(\frac{2I_0(2X_0\alpha\gamma)}{I_0(2X_0\alpha\gamma)+I_0(2X_1\alpha\gamma)}\right) \\ &da\,dX_0\,dX_1. \end{aligned} \tag{10}$$

B. Rayleigh fading without CSI (NCSI)

With no channel state information (NCSI), the conditional p.d.fs $p(X_i|\underline{s}_j)$, $i,j \in \{0,1\}$ are given by

$$\begin{aligned} p(X_0|\underline{s}_0) &= \frac{2X_0\gamma}{1+\gamma}e^{\frac{-X_0^2\gamma}{1+\gamma}}, \\ p(X_1|\underline{s}_0) &= 2\gamma X_1 e^{-X_1^2\gamma}, \\ p(X_0|\underline{s}_1) &= 2\gamma X_0 e^{-X_0^2\gamma}, \quad \text{and} \\ p(X_1|\underline{s}_1) &= \frac{2X_1\gamma}{1+\gamma}e^{\frac{-X_1^2\gamma}{1+\gamma}}. \end{aligned} \tag{11}$$

The channel capacity is given by

$$C_{BFSK}^{Ray,NCSI} = \int\limits_{X_0=0}^{\infty}\int\limits_{X_1=0}^{\infty} p(X_0|\underline{s}_0)p(X_1|\underline{s}_0)\cdot$$

Code Rate	E_b/N_0 (min) AWGN	E_b/N_0 (min) Rayleigh, CSI	E_b/N_0 (min) Rayleigh, NCSI
1/4	6.72 dB	7.22 dB	8.02 dB
1/3	6.87 dB	7.47 dB	8.07 dB
1/2	6.91 dB	8.31 dB	8.51 dB
2/3	6.96 dB	9.96 dB	10.07 dB
4/5	7.67 dB	11.07 dB	11.57 dB

TABLE I

CHANNEL CAPACITY LIMITS FOR NONCOHERENT BFSK

$$\log\left(\frac{p(X_0|\underline{s}_0)p(X_1|\underline{s}_0)}{\sum_{i=0}^{1} P(\underline{s}_i)p(X_0|\underline{s}_i)p(X_1|\underline{s}_i)}\right) dX_0\, dX_1. \quad (12)$$

Upon substituting the p.d.fs of Eqn. (11) in Eqn. (12), we obtain

$$C_{BFSK}^{Ray,NCSI} = \int_{X_0=0}^{\infty}\int_{X_1=0}^{\infty} 4\frac{X_0X_1\gamma^2}{1+\gamma} e^{-\left(\frac{X_0^2\gamma}{1+\gamma}+X_1^2\gamma\right)} \cdot \log\left(\frac{2}{1+e^{\frac{(X_1^2-X_0^2)\gamma^2}{1+\gamma}}}\right) dX_0\, dX_1. \quad (13)$$

By evaluating the capacity expressions in Eqns. (10) and (13) via numerical integration, we obtain the minimum E_b/N_0 required to achieve an arbitrarily small probability of error for different code rates. We tabulated these values in Table I for code rates of interest. Also, the capacity limits for the AWGN channel are given for comparison purposes. The results in Table I show that noncoherent BFSK does not benefit from letting the code rate diminish indefinitely. In Fig. 1, we have plotted the capacity versus E_s/N_0 on Rayleigh fading channels with and without CSI, with noncoherent detection. The capacity for the AWGN channel is also shown. From Fig. 1, we observe that the loss in channel capacity with no CSI is quite small compared to the capacity with CSI. The same observation can be made from Table I as well.

IV. LOG-MAP DECODER WITH NONCOHERENT DETECTION

In this section, we modify the log-MAP decoder [10],[12], for the case of noncoherent BFSK signaling. To do so, we need to calculate the transition metric defined by $\gamma_k(s,t) =$ Prob$(\mathbf{X}_k, S_k = t|S_{k-1} = s)$, where $\mathbf{X}_k = (X_{k,0}^s, X_{k,1}^s, X_{k,0}^p, X_{k,1}^p)$ for a rate-1/3 turbo code. Here $X_{k,0}^s$, $X_{k,1}^s$ are the envelope outputs corresponding to the transmitted information symbol x_k^s. $X_{k,0}^p$, $X_{k,1}^p$ are the envelope outputs corresponding to the transmitted parity symbol x_k^p. Here $p \in \{p_1, p_2\}$, where p_1 signifies the first parity and p_2 signifies the second parity. It is to be noted that, for the first decoder the received amplitudes due to transmitted symbol and parity (i.e., first parity) have same time alignment, whereas for the second decoder the received amplitudes are due to the interleaved version of the transmitted symbols and again have

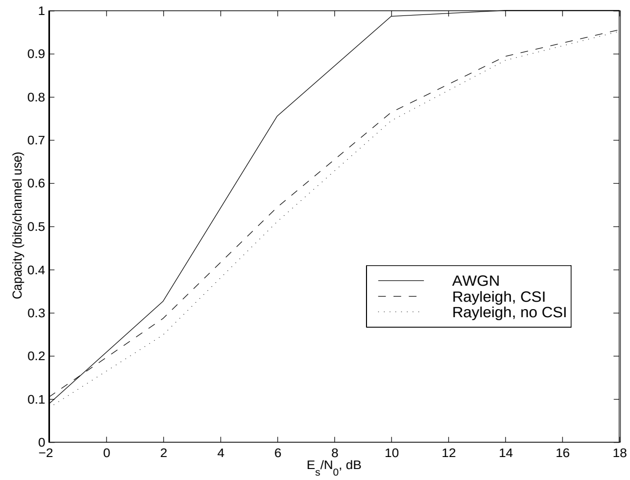

Fig. 1. Channel capacity on the fully interleaved Rayleigh fading channel with noncoherent BFSK signaling. Also shown is the capacity on the AWGN channel.

the same time alignment with the amplitudes due to the second parity symbol. Also, S_k, S_{k-1} are the encoder states at time instances k, $k-1$, respectively. Applying Baye's theorem, we can write $\gamma_k(s,t)$ as

$$\begin{aligned}\gamma_k(s,t) &= \text{Prob}(\mathbf{X}_k, S_k = t|S_{k-1} = s)\\ &= \text{Prob}(\mathbf{X}_k|S_k = t, S_{k-1} = s)\text{Prob}(S_k = t|S_{k-1} = s)\\ &= p(\mathbf{X}_k|\mathbf{x}_k)\text{Prob}(S_k = t|S_{k-1} = s)\\ &= p(\mathbf{X}_k|\mathbf{x}_k)\text{Prob}(x_k^s). \quad (14)\end{aligned}$$

The last step in the above equation is due to the fact that the state transition between any given pair of states s and t uniquely determines the information bit x_k^s. Define

$$\begin{aligned}c_k(s,t) &= \log(\gamma_k(s,t))\\ &= \log(p(\mathbf{X}_k|\mathbf{x}_k)) + \log(\text{Prob}(x_k^s)). \quad (15)\end{aligned}$$

The first term in the above equation is derived as follows. With perfect channel interleaving, we get

$$\begin{aligned}\log(p(\mathbf{X}_k|\mathbf{x}_k)) &= \log(p(X_{k,0}^s, X_{k,1}^s|x_k^s)) +\\ &\quad \log(p(X_{k,0}^p, X_{k,1}^p|x_k^p)). \quad (16)\end{aligned}$$

Using Eqn. (7) in Eqn. (16), and discarding all the constant terms and terms which do not depend on the code symbols $\{\mathbf{x}_k\}$, we obtain

$$\begin{aligned}\log(p(\mathbf{X}_k|\mathbf{x}_k)) &= \frac{\gamma^2}{1+\gamma}(X_{k,0}^s(1-x_k^s) + X_{k,1}^s x_k^s +\\ &\quad X_{k,0}^p(1-x_k^p) + X_{k,1}^p x_k^p. \quad (17)\end{aligned}$$

The second term in Eqn. (15) can be calculated as follows. Define the quantity $\widehat{L}_k$ as

$$\widehat{L}_k = \log\left(\frac{\text{Prob}(x_k^s = 1)}{\text{Prob}(x_k^s = 0)}\right). \quad (18)$$

This gives us Prob$(x_k^s = 1) = \frac{e^{\widehat{L}_k}}{1+e^{\widehat{L}_k}}$ and Prob$(x_k^s = 0) = \frac{1}{1+e^{\widehat{L}_k}}$. With these, we have,

$$\log(\text{Prob}(x_k^s)) = x_k^s \widehat{L}_k. \tag{19}$$

Combining the results of Eqns. (17) and (19) and substituting in Eqn. (15), we obtain

$$\begin{aligned} c_k(s,t) &= \widehat{L}_k x_k^s + \frac{\gamma^2}{1+\gamma}(X_{k,0}^s(1-x_k^s) + X_{k,1}^s x_k^s + \\ & X_{k,0}^p(1-x_k^p) + X_{k,1}^p x_k^p). \end{aligned} \tag{20}$$

V. SNR Estimation

Optimum decoding of turbo codes require the knowledge of channel SNR [6],[7]. In [7], we proposed an SNR estimation scheme over generalized fading channels and applied it in decoding turbo codes with coherent BPSK signaling. From Eqn. (20), we observe that noncoherent turbo detection also requires the knowledge of channel SNR, γ. In Sections V-A and V-B, we derive the SNR estimation schemes for non-coherent detection on an AWGN channel and for a Rayleigh fading channel, respectively.

A. SNR Estimation on AWGN Channel

For the case of an AWGN channel, the inphase and quadrature phase demodulator's outputs are given by

$$\begin{aligned} \underline{r}_c &= \underline{s}_i \cos\theta + \underline{n}_c \\ \underline{r}_s &= \underline{s}_i \sin\theta + \underline{n}_s. \end{aligned} \tag{21}$$

We want to estimate the received SNR, $\gamma = \frac{E_s}{2\sigma^2} = \frac{E_s}{N_0}$. In Eqn. (21), the actual data polarity is unknown. Our interest is to devise a blind algorithm which does not require the transmission of known training symbols to estimate the SNR. Accordingly, we formulate an estimator for the SNR based on a block observation of $\underline{r}_c$ and/or $\underline{r}_s$ as

$$z_{AWGN} = \frac{E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)\right]^2}{E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)^2\right]}. \tag{22}$$

In the above formulation, we have used the observables at the output of the inphase demodulator. The same result can also be obtained with the observable, $\underline{r}_s$, at the output of quadrature phase demodulator. The numerator in the Eqn. (22) can be computed as

$$\begin{aligned} E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)\right] &= E(\cos^2\theta)E(\underline{s}_i \cdot \underline{s}_i) + E(\underline{n}_c \cdot \underline{n}_c) + \\ & 2E(\cos\theta)E(\underline{s}_i \cdot \underline{n}_c) \\ &= \frac{1}{2} + 2\sigma^2, \end{aligned} \tag{23}$$

since $E(|\underline{s}_i|^2) = 1$, $E(\cos\theta) = 0$, and $E(\underline{s}_i \cdot \underline{n}_c) = 0$. The denominator in the Eqn. (22) can be computed as

$$\begin{aligned} E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)^2\right] &= E\left[\left(|\underline{s}_i|^2\cos^2\theta + |\underline{n}_c|^2 + 2\cos\theta\,\underline{s}_i \cdot \underline{n}_c\right)^2\right] \\ &= E(\cos^4\theta)E(|\underline{s}_i|^4) + E(|\underline{n}_c|^4) + \\ & 4E(\cos^2\theta)E\left[\left(\underline{s}_i \cdot \underline{n}_c\right)^2\right] + \\ & 2E(\cos^2\theta)E(|\underline{s}_i|^2)E(|\underline{n}_c|^2) \\ &= \frac{3}{8} + 8\sigma^4 + 4\sigma^2. \end{aligned} \tag{24}$$

From Eqns. (23) and (24), z_{AWGN} can be obtained as

$$z_{AWGN} = \frac{2(2+\gamma)^2}{16 + 16\gamma + 3\gamma^2}. \tag{25}$$

For a given value of z_{AWGN} (computed from a block observation of the $\underline{r}'_c s$), the corresponding estimate of γ can be computed from Eqn. (25). For easy implementation, an approximate relation between z_{AWGN} and γ can be obtained through an exponential curve fitting for Eqn. (25). The exponential fit is given by

$$\gamma = a_3 e^{\left(a_0 e^{(a_1 z_{AWGN})} + a_2 z_{AWGN}\right)}, \tag{26}$$

where $a_0 = -5.86 \cdot 10^4$, $a_1 = -18.99$, $a_2 = -13.37$, and $a_3 = 2.66 \cdot 10^4$.

B. SNR Estimation on Rayleigh Fading Channel

For the case of a Rayleigh fading channel, the inphase and quadrature phase demodulator's outputs are given by the Eqns. (1) and (2). We want to estimate the average received SNR, $\gamma = \frac{E_s}{2\sigma^2}E(\alpha^2) = \frac{E_s}{N_0}$. Similar to the approach we have in Section V-A, we formulate an estimator for the SNR, based on a block observation of the $\underline{r}_c$'s, as

$$z_{Ray} = \frac{E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)\right]^2}{E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)^2\right]}. \tag{27}$$

The numerator in Eqn. (27) can be computed as

$$\begin{aligned} E\left[\left(\underline{r}_c \cdot \underline{r}_c\right)\right] &= E(\cos^2\theta)E(\alpha^2)E(\underline{s}_i \cdot \underline{s}_i) + E(\underline{n}_c \cdot \underline{n}_c) + \\ & 2E(\cos\theta)E(\alpha)E(\underline{s}_i \cdot \underline{n}_c) \\ &= \frac{1}{2} + 2\sigma^2. \end{aligned} \tag{28}$$

The denominator in the Eqn. (27) can be computed as

$$\begin{aligned} E[(\underline{r}_c \cdot \underline{r}_c)^2] &= E[(\alpha^2|\underline{s}_i|^2\cos^2\theta + |\underline{n}_c|^2 + 2\alpha\cos\theta\,\underline{s}_i \cdot \underline{n}_c)^2] \\ &= E(\alpha^4)E(\cos^4\theta)E(|\underline{s}_i|^4) + E(|\underline{n}_c|^4) + \\ & 4E(\alpha^2)E(\cos^2\theta)E[\left(\underline{s}_i \cdot \underline{n}_c\right)^2] + \\ & 2E(\alpha^2)E(\cos^2\theta)E(|\underline{s}_i|^2)E(|\underline{n}_c|^2) \\ &= \frac{3}{4} + 8\sigma^4 + 4\sigma^2. \end{aligned} \tag{29}$$

From Eqns. (28) and (29), z_{Ray} can be obtained as

$$z_{Ray} = \frac{(2+\gamma)^2}{8 + 8\gamma + 3\gamma^2}. \tag{30}$$

For a given value of z_{Ray} (computed from a block observation of the $\underline{r}'_c s$), the corresponding estimate of γ can

True SNR, γ (dB)	Block size=1000 bits		Block Size=5000 bits	
	E[$\widehat{\gamma}$], dB	SD [$\widehat{\gamma}$], dB	E[$\widehat{\gamma}$], dB	SD[$\widehat{\gamma}$], dB
1.23	0.78	2.688	0.845	0.522
2.23	1.97	1.814	2.03	0.351
3.23	3.15	1.117	3.19	0.214
4.23	4.21	0.618	4.25	0.117
5.23	5.10	0.302	5.13	0.056

TABLE II

MEAN AND STANDARD DEVIATION OF THE SNR ESTIMATE, $\widehat{\gamma}$, FOR DIFFERENT VALUES OF THE TRUE SNR, γ, FOR NONCOHERENT AWGN CHANNEL.

True SNR, γ (dB)	Block size=1000 bits		Block Size=5000 bits	
	E[$\widehat{\gamma}$], dB	SD [$\widehat{\gamma}$], dB	E[$\widehat{\gamma}$], dB	SD[$\widehat{\gamma}$], dB
1.23	1.11	1.570	1.20	0.299
2.23	2.21	1.176	2.28	0.225
3.23	3.26	0.882	3.31	0.170
4.23	4.23	0.678	4.27	0.132
5.23	5.10	0.540	5.13	0.106

TABLE III

MEAN AND STANDARD DEVIATION OF THE SNR ESTIMATE, $\widehat{\gamma}$, FOR DIFFERENT VALUES OF THE TRUE SNR, γ, FOR RAYLEIGH FADING CHANNEL WITH NONCOHERENT BFSK.

be computed from Eqn. (30). For easy implementation, an approximate relation between z_{Ray} and γ can be obtained through an exponential curve fitting for Eqn. (30). The exponential fit is given by

$$\gamma \quad = \quad b_3 e^{\left(b_0 e^{(b_1 z_{Ray})} + b_2 z_{Ray}\right)}, \qquad (31)$$

where $b_0 = -7.63 \cdot 10^{-7}$, $b_1 = 29.03$, $b_2 = -11.75$, and $b_3 = 514.15$.

Figure 2 shows the γ versus z plots corresponding to exponential fits for the AWGN and the Rayleigh fading channels as per Eqns. (26) and (31), along with the true value plots as per Eqns. (25) and (30), respectively. It is seen that the fit is very accurate over the SNR values of interest. In order to obtain an estimate for z_{AWGN} (z_{Ray}), we replace the expectations in Eqn. (22) for AWGN (and Eqn. (27) for Rayleigh fading) with the corresponding block averages, yielding

$$\widehat{z} \quad = \quad \frac{[\overline{\underline{r}_c \cdot \underline{r}_c}]^2}{(\underline{r}_c \cdot \underline{r}_c)^2}. \qquad (32)$$

Substituting Eqn. (32) into Eqn. (25) for AWGN (and Eqn. (30) for Rayleigh fading) we get the SNR estimates, $\widehat{\gamma}$, for the AWGN channel (and the Rayleigh fading channel). We tested the accuracy of the fit by evaluating the mean and standard deviation of the SNR estimates, $\widehat{\gamma}$, determined by over 20000 trials. The block sizes considered are 1000 and 5000 bits (3000 and 15000 code symbols). Tables II and III give these results. Note that the true SNR value ($\gamma = E_s/N_0$) ranges from 1.23 dB to 5.23 dB in Tables II and III, and corresponds to E_b/N_0 values in the range 6 to 10 dB for a rate-1/3 turbo code. From Tables II and III, it can be seen that the mean SNR estimates $\widehat{\gamma}$ through the exponential curve fit are quite close to the true value of SNR γ and the standard deviation of the estimate is reduced as the block size is increased.

VI. RESULTS AND DISCUSSION

Simulations were performed using the proposed online estimator to provide $\widehat{\gamma}$ for the iterative decoding of turbo codes on flat Rayleigh fading channels. For details regarding turbo coding and decoding, the reader is referred to [9]. In this paper, we consider a rate-1/3 turbo code using two 16-state (constraint length = 5) recursive systematic convolutional (RSC) encoders with generator $(21/37)_8$. A random turbo interleaver is employed. The number of information bits per frame is 5000. The transmitted symbols are corrupted by flat i.i.d Rayleigh fading and AWGN.

We used the log-MAP algorithm in the iterative decoder [11],[12]. The number of decoding iterations is eight. The decoder performance is evaluated for seven different cases: a) coherent AWGN channel with BPSK signaling and E_s/N_0 known to the receiver (coherent AWGN (Ideal)), b) noncoherent AWGN channel with BFSK signaling and E_s/N_0 known to the receiver (noncoherent AWGN (Ideal)), c) noncoherent AWGN channel with BFSK signaling and E_s/N_0 estimated at the receiver using the *exponential* fit of Eqn. (26) (noncoherent AWGN (Est)), d) Noncoherent Rayleigh fading channel with CSI and E_s/N_o known to the receiver (Rayleigh CSI (Ideal)), e) Noncoherent Rayleigh fading channel with CSI and estimating the SNR using the *exponential* fit of Eqn. (31) (Rayleigh CSI (Est)), f) Noncoherent Rayleigh fading channel without CSI and E_s/N_o known to the receiver (Rayleigh NCSI (Ideal)), and g) noncoherent Rayleigh fading channel without CSI and estimating the SNR from the *exponential* fit in (31) (Rayleigh NCSI (Est)). In cases c), e), and g), the *average* SNR estimate is computed from the $\underline{r}_c$'s over each frame (i.e., 15000 symbol observations) according to Eqns. (26) and (31). This estimated average SNR is then given as the channel information to the turbo decoder.

Figure 3 shows the simulated performance of the turbo decoder when the proposed SNR estimates derived in Section V are used, relative to the performance when perfect CSI is used. For coherent BPSK, the performance is within 0.7 dB of the channel capacity as previously noted in the literature for BER=10^{-5} [1]. From Figure 3, we observe that, for BFSK on an AWGN channel, this capacity relationship is preserved, as a bit error rate of 10^{-5} is achieved at 8.2 dB, which is about 1.3 dB away from the limit of 6.87 dB (refer $E_b/N_o(min)$ required for rate 1/3 on AWGN in Table I). Similarly, for the case of Rayleigh fading with CSI, bit error rate of 10^{-5}

is achieved at $E_b/N_0 = 9.0$ dB, whereas, the capacity limit is 7.47 dB (ref. Table I). For the case of Rayleigh fading without CSI, a bit error rate of 10^{-5} is achieved at $E_b/N_0 = 9.6$ dB which is 1.6 dB away from the limit of 8.07 dB (ref. Table I). Figure 3 also shows that the performance loss in turbo decoding without CSI is within 0.5 dB compared with the case of ideal CSI. Finally, we observe that the bit error performance of turbo codes on noncoherent channels with estimated SNR is quite close to the ideal SNR case (to within 0.5 dB).

VII. CONCLUSIONS

In this paper, we analyzed the performance of turbo codes on Rayleigh fading channels with noncoherent detection and without channel state information (CSI). We obtained the modified turbo decoder transition metric for the case of no CSI. We proposed an online SNR estimation technique, for decoding turbo codes, based on the ratio of certain observables at the output of the noncoherent detector, for both the AWGN channel and the Rayleigh fading channel. With information theoretic capacity calculations, we showed that, for Rayleigh fading channels, the channel capacity with non-coherent detection without CSI is quite close to the case with CSI. Finally, we showed, through simulations, that the performance of a noncoherent turbo decoder on Rayleigh fading channels without CSI is quite close to the performance with CSI, for both the cases of true SNR and estimated SNR, and is just about 1.5 dB away from the performance on a noncoherent AWGN channel.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes," *Proc. IEEE ICC'93*, pp. 1064-1070, 1993.

[2] D. Divsalar, S. Dolinar, R. J. McEliece, and F. Pollara, "Transfer Function Bounds on the Performance of Turbo Codes," TDA Progress Report 42-122, JPL, Caltech, August 1995.

[3] S. Benedetto and G. Montorsi, "Unveiling Turbo Codes: Some Results on Parallel Concatenated Coding Schemes," *IEEE Trans. Info. Theory,* vol. 42, pp. 409-429, March 1996.

[4] E. K. Hall and S. G. Wilson, "Design and Analysis of Turbo Codes on Rayleigh Fading Channels," *IEEE Jl. Sel. Areas Commun.,* vol. 16, no. 2, pp. 160-174, February 1998.

[5] E. K. Hall and S. G. Wilson, "Turbo Codes for Noncoherent Channels," *Proc. IEEE Commun. Theory Mini-Conf.,* Phoenix, AZ, pp. 66-70, 1997.

[6] T. A. Summers and S. G. Wilson, "SNR Mismatch and Online Estimation in Turbo Decoding," *IEEE Trans. Commun.,* vol. 46, no. 4, pp. 421-423, April 1998.

[7] A. Ramesh, A. Chockalingam, and L. B. Milstein, "SNR Estimation over Generalized Fading Channels and its Application to Turbo Decoding," *IEEE ICC'2001,* Helsinki, , June 2001.

[8] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering,* New York: Wiley, 1965.

[9] W. E. Ryan, "A Turbo Code Tutorial," *Proc. IEEE Globecom'98,* 1998.

[10] http://www.ee.vt.edu/valenti/turbo.html

[11] S. Benedetto, G. Montorsi, D. Divsalar, and F. Pollara, "Soft-Output Decoding Algorithms in Iterative Decoding of Turbo Codes," *JPL TDA Progress Report 42-124,* pp. 63-87, February 1996.

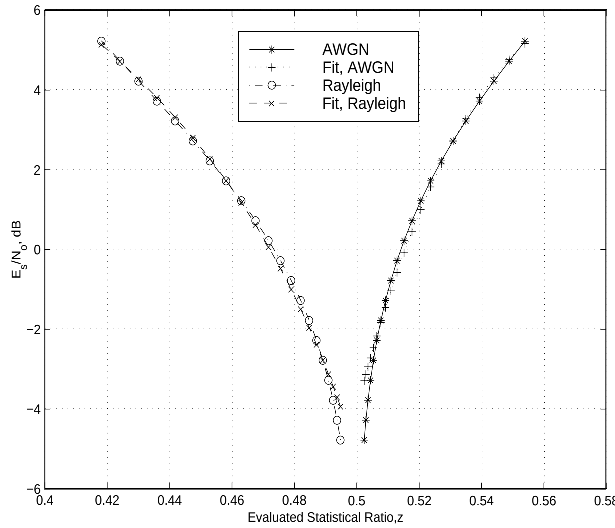

Fig. 2. Received SNR, γ, versus estimated parameter, z.

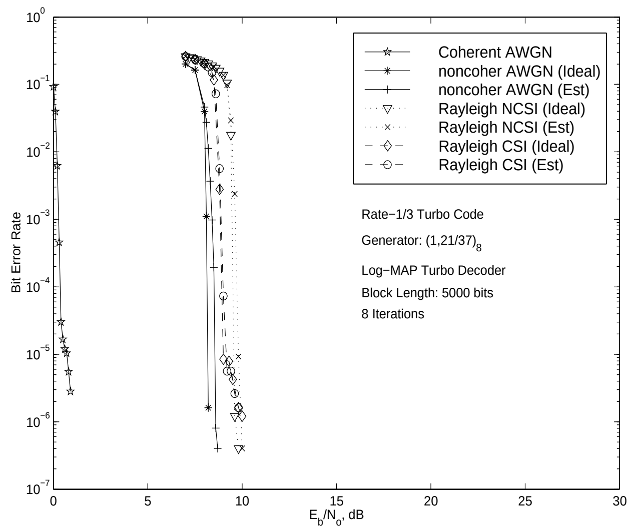

Fig. 3. Comparison of turbo decoder performance on Rayleigh fading channel with/without CSI and estimated channel SNR. Also shown are the plots for coherent AWGN and noncoherent AWGN channel. NCSI indicates "no channel state information".

[12] A. J. Viterbi, "An Intuitive Justification and a Simplified Implementation of the MAP Decoder for Convolutional Codes," *IEEE Jl. Sel. Areas Commun.,* vol. 16, pp. 260-264, February 1998.