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Non-linear Transceiver Designs with Imperfect CSIT Using Convex Optimization

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***Abstract*—In this paper, we consider non-linear transceiver designs for multiuser multi-input multi-output (MIMO) downlink in the presence of imperfections in the channel state information at the transmitter (CSIT). The base station (BS) is equipped with multiple transmit antennas and each user terminal is equipped with multiple receive antennas. The BS employs Tomlinson-Harashima precoding (THP) for inter-user interference pre-cancellation at the transmitter. We investigate robust THP transceiver designs based on the minimization of BS transmit power with mean square error (MSE) constraints, and balancing of MSE among users with a constraint on the total BS transmit power. We show that these design problems can be solved by iterative algorithms, wherein each iteration involves a pair of convex optimization problems. The robustness of the proposed algorithms to imperfections in CSIT is illustrated through simulations.**

I. INTRODUCTION

Multiuser multiple-input multiple-output (MIMO) wireless communication systems have potential to offer the benefits of transmit diversity and increased channel capacity. Multiuser interference is a major factor that limits the performance of such systems. By employing appropriately designed precoder at the base station (BS) and receive filter at the receiver terminals, this interference can be considerably reduced. Joint precoder/receive filter design require the knowledge of the channel sate information (CSI). Linear and non-linear transceiver designs have been widely investigated [1]–[6]. Non-linear transceivers improve the performance of the system compared to linear transceivers [7].

Non-linear transceiver designs employing Tomlinson-Harashima precoding (THP) have been widely studied [6], [8]–[10]. Non-linear precoder design based on minimization of SMSE under a BS transmit power constraint for MISO downlink is considered in [8]. THP design based power minimization under SINR constraints for MISO downlink is reported in [10]. Iterative THP transceiver designs for multiuser MIMO systems are considered in [6], [9].

All the studies on transceiver designs mentioned above assume the availability of perfect CSI at the transmitter (CSIT). However, in practice, the CSIT is usually imperfect due to different factors like estimation error, feedback delay, quantization etc. Hence, it is of interest to develop transceiver designs that are robust to errors in CSIT. Robust linear and non-linear transceiver designs for multiuser multi-input single-output (MISO) downlink have been widely studied [11]–[14]. Linear and non-linear zero-forcing (ZF) precoders for MISO systems are reported in [11]. Linear robust precoders with SINR and MSE constraints for MISO downlink is studied in [12]–[14].

In this paper, we consider robust THP transceiver designs for the downlink of a multiuser *MIMO* system in the presence of imperfect CSIT which has not been reported so far. In this regard, in [15], we have formulated a THP transceiver optimization problem that minimizes sum MSE (SMSE) with a transmit power constraint and proposed an iterative algorithm to solve this problem. An alternate optimization problem is to minimize the total BS transmit power with constraints on the MSE at the user terminals. A linear transceiver design based on this criterion has been reported in [16]. To our knowledge, non-linear transceiver design based on this criterion has not been reported so far. Accordingly, in this paper, we investigate robust THP transceiver design that minimizes total BS transmit power with constraints on MSE at user terminals. In addition to this, we also consider the related problem of balancing of user MSEs under a constraint on the total BS transmit power in the presence of CSIT imperfections. We assume that the CSIT error can be characterized by an uncertainty set. We adopt a minimax approach to the robust design. This results in a conservative design, but ensures that the constraints are satisfied for all members of uncertainty set.

The rest of the paper is organized as follows. The system model and the CSIT error model are presented in Section II. The proposed robust THP transceiver designs are presented in Section III. Simulation results and comparisons are presented in Section IV. Conclusions are presented in Section V.

II. SYSTEM MODEL

We consider a multiuser MIMO downlink, where a base station (BS) communicates with M users on the downlink. A block diagram of the system considered is shown in Fig. 1. The BS employs Tomlinson-Harashima precoding (THP) for inter-user interference pre-cancellation. The BS employs N_t transmit antennas and the kth user is equipped with N_{r_k} receive antennas, $1 \leq k \leq M$. Let $\mathbf{u}_k$ denote[1] the $L_k \times 1$ data symbol vector for the kth user, where L_k, $k = 1, 2, \cdots, M$, is the number of data streams for the kth user. Stacking the data vectors for all the users, we get the global data vector

[1]Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec(.) operator stacks the columns of the input matrix into one column-vector. $\|\cdot\|_F$ denotes the Frobenius norm, and $\mathbb{E}\{\cdot\}$ denotes the expectation operator. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A} - \mathbf{B}$ is positive semi-definite.

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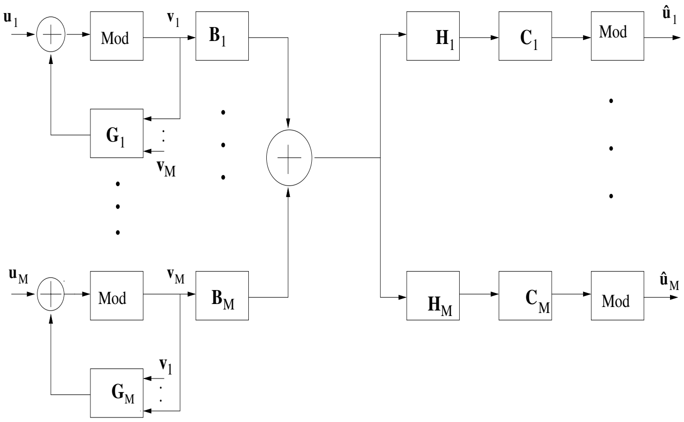

Fig. 1. Multiuser MIMO downlink system model with Tomlinson-Harashima Precoding.

$\mathbf{u} = [\mathbf{u}_1^T, \cdots, \mathbf{u}_M^T]^T$. The output of the k modulo operator is denoted by $\mathbf{v}_k$. Let $\mathbf{B}_k \in \mathbb{C}^{N_t \times L_k}$ represent the precoding matrix for the kth user. The global precoding matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_M]$. The transmit vector is given by

$$\mathbf{x} \;=\; \mathbf{B}\mathbf{v}, \tag{1}$$

where $\mathbf{v} = [\mathbf{v}_1^T, \cdots, \mathbf{v}_M^T]^T$. The feedback filters

$$\mathbf{G}_k = \left[\underline{\mathbf{G}}_{k,1} \cdots \underline{\mathbf{G}}_{k,k-1} \quad \mathbf{0}_{L_k \times \sum_{j=k}^{M} L_j} \right], \quad 1 \le k \le M, \tag{2}$$

where $\underline{\mathbf{G}}_{kj} \in \mathbb{C}^{L_k \times L_j}$, perform the interference pre-subtraction. We consider only inter-user interference pre-subtraction. When THP is used, both the transmitter and receivers employ modulo operator, $\mathrm{Mod}(\cdot)$. For a complex number x, the modulo operator performs the following operation

$$\mathrm{Mod}(x) \;=\; x - a\left\lfloor \frac{\Re(x)}{a} + \frac{1}{2} \right\rfloor - \mathbf{j}\, a \left\lfloor \frac{\Im(x)}{a} + \frac{1}{2} \right\rfloor, \tag{3}$$

where $\mathbf{j} = \sqrt{-1}$, and a depends on the constellation [17]. For a vector argument $\mathbf{x} = [x_1 \; x_2 \; \cdots \; x_N]^T$,

$$\mathrm{Mod}(\mathbf{x}) \;=\; [\mathrm{Mod}(x_1) \; \mathrm{Mod}(x_2) \cdots \; \mathrm{Mod}(x_N)]^T. \tag{4}$$

The vectors $\mathbf{u}_k$ and$\mathbf{v}_k$ are related as

$$\mathbf{v}_k \;=\; \mathrm{Mod}\Big(\mathbf{u}_k - \sum_{j=1}^{k-1} \underline{\mathbf{G}}_{k,j} \mathbf{v}_j \Big). \tag{5}$$

The kth component of the transmit vector $\mathbf{x}$ is transmitted from the kth transmit antenna. Let $\mathbf{H}_k$ denote the $N_{r_k} \times N_t$ channel matrix of the kth user. The overall channel matrix is given by

$$\mathbf{H} \;=\; \left[\mathbf{H}_1^T \quad \mathbf{H}_2^T \cdots \mathbf{H}_M^T \right]^T. \tag{6}$$

The entries of the channel matrices are assumed to be zero-mean, unit-variance complex Gaussian random variables. The received signal vectors are given by

$$\mathbf{y}_k \;=\; \mathbf{H}_k \mathbf{B}\mathbf{v} + \mathbf{n}_k, \quad 1 \le k \le M. \tag{7}$$

The kth user estimates its data vector as

$$\begin{aligned} \widehat{\mathbf{u}}_k &= (\mathbf{C}_k \mathbf{y}_k) \mod a \\ &= (\mathbf{C}_k \mathbf{H}_k \mathbf{B}\mathbf{v} + \mathbf{C}_k \mathbf{n}_k) \mod a, \quad 1 \le k \le M, \end{aligned} \tag{8}$$

where $\mathbf{C}_k$ is the $L_k \times N_{r_k}$ dimensional receive filter of the kth user, and $\mathbf{n}_k$ is the zero-mean noise vector with $\mathbb{E}\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_n^2 \mathbf{I}$. Stacking the estimated vectors of all users, the global estimate vector can be written as

$$\widehat{\mathbf{u}} \;=\; \mathbf{C}\mathbf{H}\mathbf{B}\mathbf{v} + \mathbf{C}\mathbf{n} \mod a, \tag{9}$$

where $\mathbf{C}$ is a block diagonal matrix with $\mathbf{C}_k$, $1 \le k \le M$ on the diagonal, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_M^T]^T$. The global receive matrix $\mathbf{C}$ has block diagonal structure as the receivers are non-cooperative. Neglecting the modulo loss, and assuming $\mathbb{E}\{\mathbf{v}_k \mathbf{v}_k^H\} = \mathbf{I}$, we can write MSE between the symbol vector $\mathbf{u}_k$ and the estimate $\widehat{\mathbf{u}}_k$ at the kth user as [6]

$$\begin{aligned} \epsilon_k &= \mathbb{E}\{\|\widehat{\mathbf{u}}_k - \mathbf{u}_k\|^2\} \\ &= \mathrm{tr}\Big[(\mathbf{C}_k \mathbf{H}_k \mathbf{B} - \bar{\mathbf{G}}_k)(\mathbf{C}_k \mathbf{H}_k \mathbf{B} - \bar{\mathbf{G}}_k)^H + \sigma_n^2 \mathbf{C}_k \mathbf{C}_k^H \Big], \\ & \quad 1 \le k \le M, \end{aligned} \tag{10}$$

where $\bar{\mathbf{G}}_k = \left[\underline{\mathbf{G}}_{k,1} \cdots \underline{\mathbf{G}}_{k,k-1} \quad \mathbf{I}_{L_k, L_k} \quad \mathbf{0}_{L_k \times \sum_{j=k+1}^{M} L_j} \right]$.

A. CSIT Error Model

In this paper, we consider CSIT error that can be characterized by an uncertainty region. The true channel matrix of the kth user, $\mathbf{H}_k$, is represented as

$$\mathbf{H}_k \;=\; \widehat{\mathbf{H}}_k + \mathbf{E}_k, \quad 1 \le k \le M, \tag{11}$$

where $\widehat{\mathbf{H}}_k$ is the CSIT of the kth user, and $\mathbf{E}_k$ is the CSIT error matrix. The overall channel matrix can be written as

$$\mathbf{H} \;=\; \widehat{\mathbf{H}} + \mathbf{E}, \tag{12}$$

where $\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \quad \widehat{\mathbf{H}}_2^T \cdots \widehat{\mathbf{H}}_M^T]^T$, and $\mathbf{E} = [\mathbf{E}_1^T \quad \mathbf{E}_2^T \cdots \mathbf{E}_M^T]^T$. The true channel $\mathbf{H}_k$ belongs to the uncertainty set $\mathcal{R}_k$ given by

$$\mathcal{R}_k \;=\; \{\zeta \,|\, \zeta = \widehat{\mathbf{H}}_k + \mathbf{E}_k, \|\mathbf{E}_k\|_F \le \delta_k\}, \; 1 \le k \le M, \tag{13}$$

where δ_k is the CSIT *uncertainty size*. Equivalently,

$$\|\mathbf{E}_k\|_F \;\le\; \delta_k, \quad 1 \le k \le M. \tag{14}$$

This model is suitable for systems where $\mathbf{E}$ is dominated by quantization errors [14].

III. Robust Transceiver Designs

In this section, we consider THP transceiver designs based on two different optimality criteria. The first design seeks to minimize the total BS transmit power required to ensure that the MSE at the users does not exceed the upper limit. The second design is based on balancing the MSE at the user terminals under a constraint on the total BS transmit power.

A. Robust MSE-constrained Transceiver Design

Transceiver designs that satisfy QoS constraints are of practical interest. Such designs in the context of multiuser MISO downlink with perfect CSIT have been reported in the literature [18]–[20]. Robust linear precoder designs for MISO downlink with SINR constraints are described in [12]. Here, we address the problem of robust THP transceiver design for multiuser MIMO with MSE constraints in the presence of CSIT imperfections. THP designs are of interest because of their better performance compared to the linear designs.

When the CSIT is perfect, the transceiver design under MSE constraints can be written as

$$\min_{\mathbf{B},\mathbf{G},\mathbf{C}} \quad \text{tr}(\mathbf{B}\mathbf{B}^H) \tag{15}$$

$$\text{Subject to} \quad \epsilon_k \leq \eta_k,\ 1 \leq k \leq M, \tag{16}$$

where η_k is the maximum allowed MSE at kth user terminal. This problem can be written as the following convex optimization problem:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{G},\mathbf{C}} \quad & r \qquad (17)\\ \text{Subject to} \quad & \|\mathbf{D}_k\mathbf{h}_k - \bar{\mathbf{g}}_k\|^2 + \sigma_n^2\|\mathbf{c}_k\|^2 \leq \eta_k,\ 1 \leq k \leq M,\\ & \|\mathbf{b}\|^2 \leq r, \end{aligned}$$

where $\mathbf{D}_k = (\mathbf{B}^T \otimes \mathbf{C}_k)$, $\widehat{\mathbf{h}}_k = \text{vec}(\widehat{\mathbf{H}}_k)$, $\mathbf{b} = \text{vec}(\mathbf{B})$, $\mathbf{c}_k = \text{vec}(\mathbf{C}_k)$, $\bar{\mathbf{g}}_k = \text{vec}(\bar{\mathbf{G}}_k)$, $\mathbf{h}_k = \text{vec}(\mathbf{H}_k)$, and r is a slack variable. We can express this optimization problem in which the data and optimization variables are real. In order to do this, we use the following notation where for any complex matrix $\mathbf{Q}$, the corresponding real matrix

$$\widetilde{\mathbf{Q}} = \begin{bmatrix} \Re(\mathbf{Q}) & \Im(\mathbf{Q}) \\ -\Im(\mathbf{Q}) & \Re(\mathbf{Q}) \end{bmatrix}, \tag{18}$$

and for any complex vector $\mathbf{q}$, the corresponding real vector

$$\widetilde{\mathbf{q}} = \begin{bmatrix} \Re(\mathbf{q}) \\ -\Im(\mathbf{q}) \end{bmatrix}. \tag{19}$$

Using this notation, we rewrite the problem in (17) as

$$\begin{aligned} \min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},\widetilde{\mathbf{c}},r} \quad & r \qquad (20)\\ \text{Subject to} \quad & \|\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k - \widetilde{\bar{\mathbf{g}}}_k\|^2 + \sigma_n^2\|\widetilde{\mathbf{c}}_k\|^2 \leq \eta_k,\ 1 \leq k \leq M,\\ & \|\widetilde{\mathbf{b}}\|^2 \leq r. \end{aligned}$$

1) Transceiver design in the presence of CSIT errors: When the CSIT is imperfect, the robust transceiver design with MSE constraints can be written as

$$\begin{aligned} \min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},\widetilde{\mathbf{c}},r} \quad & r \qquad (21)\\ \text{Subject to} \quad & \|\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k - \widetilde{\bar{\mathbf{g}}}_k\|^2 + \sigma_n^2\|\widetilde{\mathbf{c}}_k\|^2 \leq \eta_k,\\ & \forall\widetilde{\mathbf{h}}_k \in \mathcal{R}_k,\ 1 \leq k \leq M,\\ & \|\widetilde{\mathbf{b}}\|^2 \leq r. \end{aligned}$$

In the above problem, the true channel, unknown to the transmitter, may lie anywhere in the uncertainty region. In order to ensure, a priori, that the constraints are met for the actual channel, the transceiver should be so designed that the constraints are met for all members of the uncertainty set. This, in effect, is a semi-infinite optimization problem, which in general is intractable. We show, in the following, that an appropriate transformation makes the problem in (21) tractable.

The optimization problem in (21) is not jointly convex in $\mathbf{b}$, $\mathbf{g}$, and $\mathbf{c}$. But, for fixed $\mathbf{c}$, it is convex in $\mathbf{b}$ and $\mathbf{c}$, and vice versa. So, in order to solve this optimization problem, we propose an iterative algorithm, wherein each iteration solves two problems $\mathcal{S}1$ and $\mathcal{S}2$. Problem $\mathcal{S}1$, which is given below, involves the optimization over $\{\mathbf{b}, \mathbf{g}\}$ for fixed $\mathbf{c}$.

$$\begin{aligned} \min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},r} \quad & r \qquad (22)\\ \text{Subject to} \quad & \|\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k - \widetilde{\bar{\mathbf{g}}}_k\|^2 + \sigma_n^2\|\widetilde{\mathbf{c}}_k\|^2 \leq \eta_k,\\ & \forall\widetilde{\mathbf{h}}_k \in \mathcal{R}_k,\ 1 \leq k \leq M\\ & \|\widetilde{\mathbf{b}}\|^2 \leq r. \end{aligned}$$

Problem $\mathcal{S}2$, given below, involves the optimization over $\{\mathbf{c}\}$ for fixed $\{\mathbf{b}, \mathbf{g}\}$

$$\begin{aligned} \min_{\widetilde{\mathbf{c}},s_1,\cdots,s_M} \quad & s_k \qquad (23)\\ \text{Subject to} \quad & \|\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k - \widetilde{\bar{\mathbf{g}}}_k\|^2 + \sigma_n^2\|\widetilde{\mathbf{c}}_k\|^2 \leq s_k,\ \forall\widetilde{\mathbf{h}}_k \in \mathcal{R}_k,\\ & 1 \leq k \leq M, \end{aligned}$$

where $s_1, \cdots, s_M$ are slack variables. The convex optimization problem $\mathcal{S}1$ can be expressed as a semi-definite program (SDP) which can be solved efficiently. Towards this end, we rewrite the problem in (22) as the following SOCP:

$$\begin{aligned} \min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},t} \quad & t \qquad (24)\\ \text{Subject to} \quad & \left\| \begin{bmatrix} \widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k - \widetilde{\bar{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k \end{bmatrix} \right\| \leq \sqrt{\eta_k},\\ & \forall\widetilde{\mathbf{h}}_k \in \mathcal{R}_k,\ 1 \leq k \leq M\\ & \|\widetilde{\mathbf{b}}\| < t, \end{aligned}$$

where t is a slack variable. We invoke the following Lemma to recast the above SOCP with infinite constraints in a tractable form.

Lemma 1 [21] The following robust SOCP

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \qquad (25)\\ \text{Subject to} \quad & \|\mathbf{A}\mathbf{x} + \mathbf{b}\|_2 \leq \mathbf{d}^T\mathbf{x} + \gamma,\ \forall\mathbf{A} \in \mathcal{V},\ \mathbf{d} \in \mathcal{W}, \end{aligned}$$

where

$$\mathcal{V} = \{[\mathbf{A}; \mathbf{b}] = [\mathbf{A}^0; \mathbf{b}^0] + \sum_{k=1}^{N} \alpha_k[\mathbf{A}_k; \mathbf{b}_k] | \boldsymbol{\alpha}^T\boldsymbol{\alpha} \leq 1\}, \tag{26}$$

$$\mathcal{W} = \{(\mathbf{d}, \lambda) = (\mathbf{d}^0, \lambda^0) + \sum_{k=1}^{M} \alpha_k(\mathbf{d}_k, \gamma_k) | \boldsymbol{\alpha}^T\boldsymbol{\alpha} \leq 1\} \tag{27}$$

is equivalent to the following SDP:

$$\min_{\mathbf{x},\mu,\lambda} \quad \mathbf{c}^T\mathbf{x} \tag{28}$$

$$\text{Subject to} \quad \mathbf{\Gamma}_1 \succeq \mathbf{0}, \tag{29}$$

$$\mathbf{\Gamma}_2 \succeq \mathbf{0}, \tag{30}$$

where

$$\mathbf{\Gamma}_1 = \begin{bmatrix} \lambda-\mu & \mathbf{0} & (\mathbf{A}\mathbf{x}+\mathbf{b})^T \\ \mathbf{0} & \mu\mathbf{I} & [\mathbf{A}^1\mathbf{x}\cdots\mathbf{A}^N\mathbf{x}]^T \\ \mathbf{A}^0\mathbf{x} & [\mathbf{A}^1\mathbf{x}\cdots\mathbf{A}^N\mathbf{x}] & \lambda\mathbf{I} \end{bmatrix}, \tag{31}$$

and

$$\mathbf{\Gamma}_2 = \begin{bmatrix} \mathbf{d}^0\mathbf{x}+\gamma-\lambda & [\mathbf{d}^1\mathbf{x}\cdots\mathbf{f}^M\mathbf{x}] \\ [\mathbf{d}^1\mathbf{x}\cdots\mathbf{f}^M\mathbf{x}] & (\mathbf{d}^0\mathbf{x}+\gamma-\lambda)\mathbf{I} \end{bmatrix}. \tag{32}$$

In the present problem, only $\widetilde{\mathbf{h}}_k,\ 1 \le k \le M$ is uncertain and it can be expressed as

$$\mathcal{R}_k = \{\widetilde{\mathbf{h}}_k = \widetilde{\widetilde{\mathbf{h}}}_k + \delta_k\boldsymbol{\alpha}\big|\|\boldsymbol{\alpha}\| \le 1\}. \tag{33}$$

Moreover, as the uncertainty is limited to $\widetilde{\mathbf{h}}_k,\ 1 \le k \le M$, the constraint $\mathbf{\Gamma}_2 \succeq \mathbf{0}$ reduces to the condition that $\lambda \le \gamma$. Considering these facts and applying Lemma 1, we can express the problem in (22) as the following SDP:

$$\min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},\mu,t} \quad t \tag{34}$$

$$\text{Subject to} \quad \begin{bmatrix} \sqrt{\eta_k}-\mu_k & \mathbf{0} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k\end{bmatrix}^T \\ \mathbf{0} & \mu_k\mathbf{I} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\delta_k \\ \mathbf{0}\end{bmatrix}^T \\ \begin{bmatrix}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k\end{bmatrix} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\delta_k \\ \mathbf{0}\end{bmatrix} & \sqrt{\eta_k}\mathbf{I} \end{bmatrix} \succeq \mathbf{0}$$

$$1 \le k \le M,$$

$$\|\widetilde{\mathbf{b}}\| \le t. \tag{35}$$

Similarly, by applying Lemma 1, we can show that the problem $\mathcal{S}2$ in (23) can be formulated as the following optimization problem:

$$\min_{\mathbf{c},\tau,\mu} \quad \tau_k^2 \tag{36}$$

$$\text{Subject to} \quad \begin{bmatrix} \tau_k-\mu_k & \mathbf{0} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k\end{bmatrix}^T \\ \mathbf{0} & \mu_k\mathbf{I} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\delta_k \\ \mathbf{0}\end{bmatrix}^T \\ \begin{bmatrix}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k\end{bmatrix} & \begin{bmatrix}\widetilde{\mathbf{D}}_k\delta_k \\ \mathbf{0}\end{bmatrix} & \tau_k\mathbf{I} \end{bmatrix} \succeq \mathbf{0},$$

$$1 \le k \le M, \tag{37}$$

where $\tau_1, \cdots, \tau_M$ are slack variables. The proposed robust MSE-constrained transceiver design algorithm iterates over problem $\mathcal{S}1$ and $\mathcal{S}2$. In the next subsection, we show that this algorithm converges to a limit.

2) Convergence of the proposed algorithm: At the $(n+1)$th iteration, we compute $\widetilde{\mathbf{b}}^{n+1}$ and $\widetilde{\mathbf{g}}^{n+1}$ by solving the optimization problem $\mathcal{S}1$ with fixed $\widetilde{\mathbf{c}}^n$. We assume that the problem $\mathcal{S}1$ is feasible, otherwise the iteration terminates. The solution of $\mathcal{S}1$ results in $\widetilde{\mathbf{b}}^{n+1}$ and $\widetilde{\mathbf{g}}^{n+1}$ such that $f_k(\widetilde{\mathbf{b}}^{n+1}, \widetilde{\mathbf{g}}_k^{n+1}, \widetilde{\mathbf{c}}_k^n) \le \eta_k,\ 1 \le k \le M$, where

$$f_k = \max_{\widetilde{\mathbf{h}}_k:\widetilde{\mathbf{h}}_k \in \mathcal{R}_k} \epsilon_k. \tag{38}$$

Also the transmit power $P_{Tr}^{n+1} = \|\widetilde{\mathbf{b}}^{n+1}\|^2 \le \|\widetilde{\mathbf{b}}^n\|^2$. Solving the problem $\mathcal{S}2$ in the $n+1$th iteration, we obtain $\widetilde{\mathbf{c}}^{n+1}$ such that

$$f_k(\widetilde{\mathbf{b}}^{n+1}, \widetilde{\mathbf{g}}_k^{n+1}, \widetilde{\mathbf{c}}_k^{n+1}) \le f_k(\widetilde{\mathbf{b}}^{n+1}, \widetilde{\mathbf{g}}_k^{n+1}, \widetilde{\mathbf{c}}_k^{n}). \tag{39}$$

Since the transmit power P_{Tr} is lower-bounded and monotonically decreasing, we conclude that the sequence $\{P_{Tr}^n\}$ converges to a limit as the iteration proceeds.

B. Robust MSE balancing

In this subsection, we consider the problem of MSE balancing under a constraint on the total BS transmit power in the presence of CSIT imperfections. When the CSIT is known perfectly, the problem of MSE balancing can be written as

$$\min_{\mathbf{B},\mathbf{G},\mathbf{C}} \quad \max_k \epsilon_k \tag{40}$$

$$\text{Subject to} \quad \text{tr}(\mathbf{B}\mathbf{B}^H) \le P_T. \tag{41}$$

This problem is related to the SINR balancing problem due to the inverse relationship that exists between the MSE and SINR. The MSE balancing problem in the context of MISO downlink with perfect CSIT has been addressed in [19], [20]. Here, we consider the MSE balancing problem in a multiuser MIMO downlink with THP in the presence of CSIT errors.

When the CSIT is imperfect, this problem can be written as the following SOCP with infinite constraints:

$$\min_{\widetilde{\mathbf{b}},\widetilde{\mathbf{g}},\widetilde{\mathbf{c}}} \quad r^2 \tag{42}$$

$$\text{Subject to} \quad \left\|\begin{bmatrix}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k \\ \sigma_n\widetilde{\mathbf{c}}_k\end{bmatrix}\right\| \le r,$$

$$\forall\widetilde{\mathbf{h}}_k \in \mathcal{R}_k,\ 1 \le k \le M,$$

$$\|\widetilde{\mathbf{b}}\| < \sqrt{P_T}.$$

To solve this problem, we propose an iterative algorithm which involves the solution of problems $\mathcal{Q}1$ and $\mathcal{Q}2$ in each iteration. Applying Lemma 1, we can see that the problem $\mathcal{Q}1$, which involves optimization over $\widetilde{\mathbf{b}}$ and $\widetilde{\mathbf{g}}$, for fixed $\widetilde{\mathbf{c}}$ is equivalent to the following convex optimization problem:

$$\min_{\mathbf{c},t,\mu} \quad t^2 \tag{43}$$

$$\text{Subject to} \quad \begin{bmatrix} t-\mu_k & \mathbf{0} & (\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k)^T \\ \mathbf{0} & \mu_k\mathbf{I} & (\widetilde{\mathbf{D}}_k\delta_k)^T \\ (\widetilde{\mathbf{D}}_k\widetilde{\mathbf{h}}_k-\widetilde{\widetilde{\mathbf{g}}}_k) & (\widetilde{\mathbf{D}}_k\delta_k) & t\mathbf{I} \end{bmatrix} \succeq \mathbf{0}$$

$$1 \le k \le M,$$

$$\|\widetilde{\mathbf{b}}\| < \sqrt{P_T}.$$

Problem $\mathcal{Q}2$ is same as the problem $\mathcal{S}2$ and can be reformulated as in (23). By similar arguments as in the MSE-constrained problem, we can see that this iterative algorithm converges to a limit.

IV. Simulation Results

In this section, we illustrate the performance of the proposed robust THP transceiver algorithms, evaluated through simulations. We compare the performance of the proposed robust THP designs with the robust linear transceiver designs in [16]. The channel fading is modeled as Rayleigh, with the channel matrices $\mathbf{H}_k$, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each antenna of each user terminal is assumed to be zero-mean complex Gaussian random variable.

In the first experiment, we compare the performance of the proposed THP design with MSE constraints and the linear design in [16], in terms of the total BS transmit power $P_T = \|\mathbf{B}\|_F^2$ for different MSE constraints at the user terminals. The simulation results are shown in Fig. 2. The performances of the linear design, and the proposed robust THP design are evaluated for $N_t = 4$, $M = 2$ and $N_t = 6$, $M = 2$. Also, the effect of different CSIT uncertainty size δ on the transmit power are studied. From the simulation results, we can observe the superior performance of the proposed THP design compared to the linear design in [16]. By comparing the results for $N_t = 4$, $M = 2$ and $N_t = 6$, $M = 2$, we find that the performance gap between the robust linear and the robust THP transceivers reduces when the number of transmit antennas is increased while keeping the number of receive antennas fixed. In the second experiment, we evaluate the performance in terms of transmit power required to meet the MSE constraints for different values of CSIT uncertainty size δ. We compare the performance for different values of maximum allowed MSE $\eta = \eta_1 = \eta_2 = 0.1, 0.2, 0.3$. The results, which are shown in Fig. 3, illustrate the superior performance of the proposed design. The proposed robust THP design meets the desired MSE costraint with less transmit power compared to the robust linear design. In the third experiment, we compare the performance of the proposed robustt MSE balancing design with THP and the corresponding linear design in [16]. We compare the performance in terms of min-max MSE for different transmit power constraints. The results for this experiment are shown in Fig. 4. The performance of the robust designs are evaluated for different values of δ. The results show that, for the same transmit power constraint, the proposed design achieves lower min-max MSE compared to the robust linear design.

V. Conclusions

We presented two robust THP transceiver designs for multiuser MIMO downlink with imperfect CSIT. The first design is based on the minimization of the total BS transmit power under MSE constraints. The second design is based on

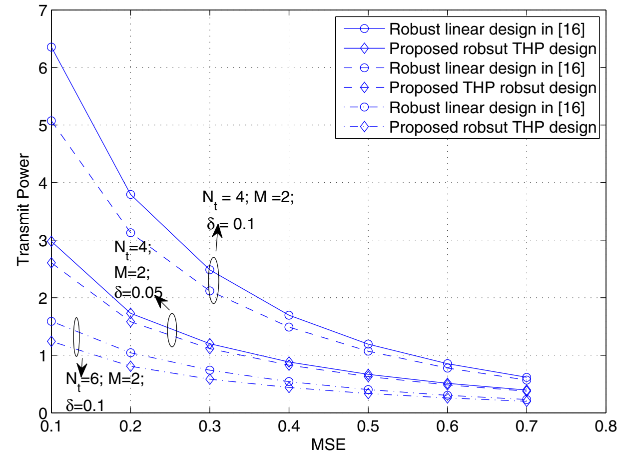

Fig. 2. Total transmit power $P_{Tr} = \|\mathbf{B}\|_F^2$ versus maximum allowed MSE. $N_t = 4, 6$, $M = 2$, $N_{r_1} = N_{r_2} = 2$, $L_1 = L_2 = 2$, $\delta = 0.05, 0.1$.

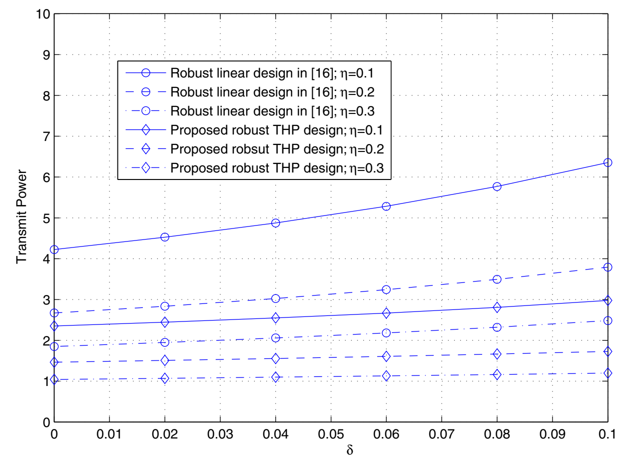

Fig. 3. Transmit power $P_T = \|\mathbf{B}\|_F^2$ versus CSIT uncertainty size δ. $N_t = 4$, $M = 2$, $N_{r_1} = N_{r_2} = 2$, $L_1 = L_2 = 2$, $\eta = \eta_1 = \eta_2 = 0.1, 0.2, 0.3$.

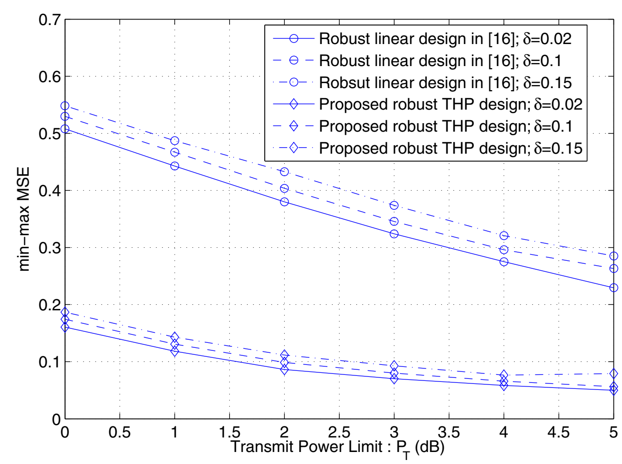

Fig. 4. Minimax MSE versus transmit power limit. $N_t = 4$, $M = 2$, $N_{r_1} = N_{r_2} = 2$, $L_1 = L_2 = 2$, $\delta = 0.02, 0.1, 0.15$.

balancing MSE among the users under a constraint on the total BS transmit power. We proposed iterative algorithms which involve solutions of convex optimization problems. Through simulation results, we illustrated the superior performance of the proposed robust designs compared to linear robust designs in the presence of CSIT imperfections.

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