# Relay Precoder Optimization in MIMO-Relay Networks With Imperfect CSI

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***Abstract*—In this paper, we consider robust joint designs of relay precoder and destination receive filters in a nonregenerative multiple-input multiple-output (MIMO) relay network. The network consists of multiple source-destination node pairs assisted by a MIMO-relay node. The channel state information (CSI) available at the relay node is assumed to be *imperfect*. We consider robust designs for two models of CSI error. The first model is a stochastic error (SE) model, where the probability distribution of the CSI error is Gaussian. This model is applicable when the imperfect CSI is mainly due to errors in channel estimation. For this model, we propose robust minimum sum mean square error (SMSE), MSE-balancing, and relay transmit power minimizing precoder designs. The next model for the CSI error is a norm-bounded error (NBE) model, where the CSI error can be specified by an uncertainty set. This model is applicable when the CSI error is dominated by quantization errors. In this case, we adopt a worst-case design approach. For this model, we propose a robust precoder design that minimizes total relay transmit power under constraints on MSEs at the destination nodes. We show that the proposed robust design problems can be reformulated as convex optimization problems that can be solved efficiently using interior-point methods. We demonstrate the robust performance of the proposed design through simulations.**

***Index Terms*—Imperfect CSI, MIMO relay, relay precoding, robust optimization.**

## I. Introduction

RELAY-ASSISTED wireless communication systems have been studied widely [1]–[3]. Improvement in link quality and reliability, and increase in coverage area are some of the benefits resulting from the use of relaying in wireless systems. Various relaying schemes have been proposed in the literature. Among them, regenerative and nonregenerative schemes have been studied widely [1], [2]. In regenerative relaying, the relay nodes decode the received signal, re-encode and then transmit it to the destination nodes. Whereas, in nonregenerative relaying, the relay nodes scale the received signal and transmit it to the destination nodes. The nonregenerative relaying is of practical interest as the signal processing involved is less complex and is easier to implement. A widely studied wireless relay-assisted system consists of a single source-destination pair and multiple relay nodes. Relay precoder designs for such a system have been reported in [4]–[6]. All the designs in [1]–[6] consider single-antenna relay nodes. Use of relay nodes with multiple-input multiple-output (MIMO) capability has the potential for further enhancing the spectral efficiency and the link reliability. Recently, studies on the application of MIMO techniques in relay networks have been reported in [7]–[9]. In [7], a relay precoder design that maximizes the capacity between the source and destination nodes in a nonregenerative relay system with single MIMO source-destination pair, and a MIMO-relay is considered. A MIMO point-to-multi-point system with a MIMO-relay is studied in [8]. In [9], a source and relay precoder design based on the minimization of mean-square error (MSE) for a three-node multi-carrier MIMO-relay network is reported.

Most of the works mentioned above assume the availability of perfect channel state information (CSI) at the relay node. In practice, the CSI available at the relay node is usually imperfect due to different factors such as estimation error, quantization, feedback delay, etc. Moreover, the performance of the precoders designed based on the assumption of perfect CSI degrades in the presence of errors in the CSI. Hence, it is of interest to develop relay precoder designs that are robust to errors in CSI. Robust precoder designs for the conventional broadcast channels have been widely studied [10]–[13]. Robust relay precoder designs for single-antenna nodes with partial/imperfect CSI have been studied in [14]–[16]. The robust precoder designs proposed in [14] are based on the second-order statistics of the CSI. Whereas, in [15], the robust designs consider only large-scale fading. A robust relay precoder for minimizing total relay transmit power under an SINR constraint at the destination node is considered in [16]. A study on robust MIMO-relay precoder design with SINR constraints for a multipoint-to-multipoint relay network has been reported in [17]. Relay precoder designs for a system with multiple source-destination pairs and multiple MIMO-relay nodes for perfect and imperfect knowledge of the second order statistics of channels are studied in [18].

In this paper, we propose *joint designs of robust relay precoder and destination filters* for a *MIMO-relay* system using nonregenerative relaying with *imperfect CSI at the MIMO-relay*. More specifically, we consider a system with multiple source-destination pairs assisted by a single MIMO-relay. The source and destination nodes are single antenna nodes, whereas the relay node has multiple receive and multiple transmit antennas. We consider two widely used models for the CSI error [19], and pro-

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pose robust designs suitable for these models. First, we consider a stochastic error (SE) model for the CSI error, which is applicable when the error is mainly due to inaccurate channel estimation. The error in this model is assumed to follow the Gaussian distribution. In this case, we adopt a statistically robust design. For this model, we consider robust precoder designs that are based on i) sum mean square error (SMSE) minimization with a constraint on total relay transmit power, ii) MSE-balancing with a constraint on total relay transmit power, and iii) minimization of total relay transmit power with constraints on the MSEs of the destination nodes. In case i), we show that the proposed robust precoder/receive filters design problem can be solved by iteratively solving a pair of subproblems. The first subproblem is formulated as a convex optimization problem that can be solved efficiently using interior-point methods [20]. The second subproblem can be solved analytically. The proposed iterative algorithm is not guaranteed to converge to the globally optimal solution. In case ii) and iii), we show that the robust design problems can be reformulated as convex optimization problems with globally optimal solutions. Further, we also consider the robust design of the minimum SMSE relay precoder and the receive filters with constraints on the power of individual relay transmit antenna. Next, we consider a norm-bounded error (NBE) model for the CSI error, which is applicable when the error is mainly due to quantization. In this case, we adopt a min-max (worst-case) approach to the robust design that is based on the minimization of the total relay transmit power with constraints on the MSEs at the destination nodes. We show that this design problem can be solved by solving an alternating sequence of minimization and worst-case analysis problems. The minimization problem is formulated as a convex optimization problem that can be solved efficiently using interior-point methods. The worst-case analysis problem can be solved analytically using an approximation for the MSEs at the destination nodes. Here again, the proposed iterative algorithm is not guaranteed to converge to the globally optimal solution.

The rest of the paper is organized as follows. The system model is presented in Section II. The proposed robust precoder/receive filters design for SE model is presented in Section III. The proposed robust precoder/receive filters design for NBE model is presented in Section IV. Section V presents the simulation results and comparisons. Conclusions are presented in Section VI.

## II. System Model

We consider a wireless relay system with $M$ source-destination node pairs, and a MIMO-relay node having $N$ receive and $N$ transmit antennas, as shown in Fig. 1. The source and destination nodes are each equipped with a single antenna. We assume that there is no direct link between the transmit and the destination nodes. We consider a nonregenerative relaying scheme with half-duplex relay mode. In this mode, during the first time slot, the $i$th source node transmits the symbol[1] $x_i \in \mathbb{C}$ with $\mathbb{E}\{|x_i|^2\} = 1$. Let $\alpha_{i,j} \in \mathbb{C}$ represent the channel gain from the $i$th source node to the $j$th receive antenna of the relay node. Define $\boldsymbol{\alpha_i} = [\alpha_{i,1}\alpha_{i,2}\cdots\alpha_{i,N}]^T$, $1 \le i \le M$, and $\mathbf{A} = [\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2\cdots\boldsymbol{\alpha}_M]$. The signal vector received at the relay node is given by

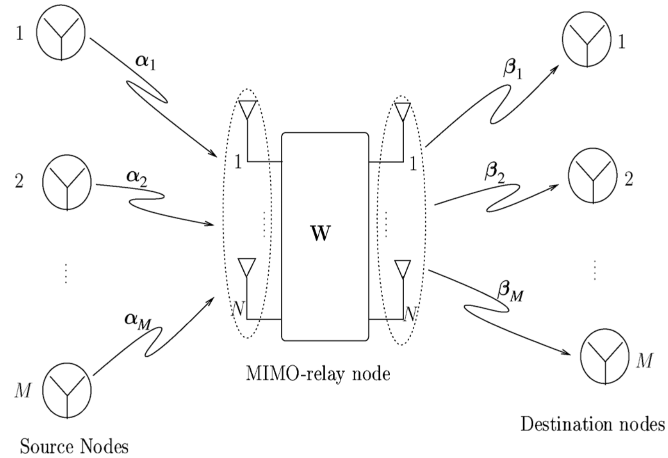


Fig. 1. MIMO-relay system model.

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\mu} \tag{1}$$

where $\mathbf{y} = [y_1\ y_2\cdots y_N]^T$, $\mathbf{x} = [x_1\ x_2\cdots x_M]^T$, $\boldsymbol{\mu} = [\mu_1\ \mu_2\cdots\mu_N]^T$, $\mu_k \in \mathbb{C}$ is the noise at the $k$th receive antenna of the relay node. The elements of $\boldsymbol{\mu}$ are independent and complex Gaussian random variables with zero mean and $\mathbb{E}\{|\mu_k|^2\} = \sigma_\mu^2, 1 \le k \le M$. During the second time slot, the relay node transmits the received signal vector after multiplying it by a precoding matrix $\mathbf{W} \in \mathbb{C}^{N\times N}$. Let $\beta_{i,j} \in \mathbb{C}$ represent the channel gain from the $j$th transmit antenna of the relay node to $i$th destination node. Define $\boldsymbol{\beta_i} = [\beta_{i,1}\ \beta_{i,2}\cdots\beta_{i,N}]$, $1 \le i \le M$, and $\mathbf{B} = [\boldsymbol{\beta}_1^T\ \boldsymbol{\beta}_2^T\cdots\boldsymbol{\beta}_M^T]^T$. The signals received by the destination nodes, $z_i$, $1 \le i \le M$, can be represented vectorially as

$$\mathbf{z} = \mathbf{B}\mathbf{W}\mathbf{y} + \boldsymbol{\nu} \tag{2}$$

where $\mathbf{z} = [z_1\ z_2\cdots z_M]^T$, $\boldsymbol{\nu} = [\nu_1\ \nu_2\cdots\nu_M]^T$, and $\nu_i \in \mathbb{C}$ is the noise at the $i$th destination node. The elements of $\boldsymbol{\nu}$ are independent and complex Gaussian with zero mean and $\mathbb{E}\{|\nu_k|^2\} = \sigma_\nu^2, 1 \le k \le M$. Let $\boldsymbol{\Gamma} = \operatorname{diag}(\gamma_1, \gamma_2, \cdots, \gamma_M)$, where $\gamma_i$ is the receive scaling factor at the $i$th destination node. The estimate of the transmitted signal vector can be expressed as

$$\begin{aligned}\widehat{\mathbf{x}} &= \boldsymbol{\Gamma}\mathbf{z}\\ &= \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{y} + \boldsymbol{\Gamma}\boldsymbol{\nu}\\ &= \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{A}\mathbf{x} + \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\boldsymbol{\mu} + \boldsymbol{\Gamma}\boldsymbol{\nu}\end{aligned} \tag{3}$$

where $\widehat{\mathbf{x}} = [\widehat{x}_1,\ \widehat{x}_2\cdots\widehat{x}_M]^T$, and $\widehat{x}_i$ is the signal estimate at the $i$th destination node. We consider CSI uncertainties that can be modeled as

$$\boldsymbol{\alpha}_i = \widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i, \quad 1 \le i \le M \tag{4}$$

$$\boldsymbol{\beta}_i = \widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i, \quad 1 \le i \le M \tag{5}$$

[1]Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[\cdot]^T$ denotes transpose operation, $[\cdot]^H$ denotes Hermitian operation, $[\cdot]^\star$ denotes complex conjugation, and $\mathbb{E}\{.\}$ denotes the expectation operator. $\otimes$ denotes the Kronecker product. $\Re(\cdot)$ and $\Im(\cdot)$ denote real part and imaginary part of the argument, respectively. $\operatorname{vec}(\cdot)$ stacks all the columns of the argument into a single column vector. $\operatorname{diag}(\cdot)$ generates a diagonal matrix with the argument on the diagonal. $\mathbf{I}_N$ denotes $N\times N$ identity matrix.

where $\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, 1 \le i \le M$ are the true CSI, $\widehat{\boldsymbol{\alpha}}_i, \widehat{\boldsymbol{\beta}}_i, 1 \le i \le M$, are the imperfect CSI available at the relay node, and $\boldsymbol{\phi}_i, \boldsymbol{\pi}_i, 1 \le i \le M$ represent the additive errors in the CSI. Equivalently,

$$\mathbf{A} = \widehat{\mathbf{A}} + \mathbf{E} \tag{6}$$

$$\mathbf{B} = \widehat{\mathbf{B}} + \mathbf{F} \tag{7}$$

$\widehat{\mathbf{A}} = [\widehat{\boldsymbol{\alpha}}_1\, \widehat{\boldsymbol{\alpha}}_2 \cdots \widehat{\boldsymbol{\alpha}}_M], \widehat{\mathbf{B}} = [\widehat{\boldsymbol{\beta}}_1\, \widehat{\boldsymbol{\beta}}_2 \cdots \widehat{\boldsymbol{\beta}}_M], \widehat{\mathbf{E}} = [\widehat{\boldsymbol{\phi}}_1\, \widehat{\boldsymbol{\phi}}_2 \cdots \widehat{\boldsymbol{\phi}}_M]$, and $\widehat{\mathbf{F}} = [\widehat{\boldsymbol{\pi}}_1\, \widehat{\boldsymbol{\pi}}_2 \cdots \widehat{\boldsymbol{\pi}}_M]$. In a stochastic error (SE) model, $\mathbf{E}$ and $\mathbf{F}$ are the channel estimation error matrices. Further, we assume that $\mathbf{E}$ and $\mathbf{F}$ are Gaussian distributed with zero mean and $\mathbb{E}\{\mathrm{vec}(\mathbf{E})\mathrm{vec}(\mathbf{E}^H)\} = \sigma_E^2\mathbf{I}$, and $\mathbb{E}\{\mathrm{vec}(\mathbf{F})\mathrm{vec}(\mathbf{F}^H)\} = \sigma_F^2\mathbf{I}$. Such a model is suitable when the CSI error is predominantly due to the channel estimation inaccuracies. An alternate error model is a norm-bounded error (NBE) model, where we assume that $\|\boldsymbol{\phi}_i\| \le \delta_{\alpha_i}$, and $\|\boldsymbol{\pi}_i\| \le \delta_{\beta_i}, 1 \le i \le M$. Equivalently, $\boldsymbol{\alpha}_i$ belongs to the uncertainty set $\mathcal{R}_{\alpha_i}$, and $\boldsymbol{\beta}_i$ belongs to the uncertainty set $\mathcal{R}_{\beta_i}$, where

$$\mathcal{R}_{\alpha_i} = \{\boldsymbol{\zeta} | \boldsymbol{\zeta} = \widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i, \|\boldsymbol{\phi}_i\| \le \delta_{\alpha_i}\} \tag{8}$$

and

$$\mathcal{R}_{\beta_i} = \left\{\boldsymbol{\zeta} | \boldsymbol{\zeta} = \widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i, \|\boldsymbol{\pi}_i\| \le \delta_{\beta_i}\right\}. \tag{9}$$

This model is suitable for systems where quantization of CSI is involved [12].

## III. Robust Precoder Design With Stochastic CSI Error

In this section, we describe the proposed robust design of the relay precoding matrix $\mathbf{W}$, and the receive filter $\boldsymbol{\Gamma}$ for the SE model of CSI error. We adopt a stochastic approach to the robust design by minimizing the expected values of those objective and constraint functions that depend on the CSI error. Such an approach ensures robust performance on the average, though it does not guarantee robust performance for each individual realization of the channel coefficients. We present relay precoder/receive filter designs based on minimum SMSE and MSE-balancing criteria.

### *A. Proposed Robust Minimum SMSE Design*

In this subsection, we describe the proposed robust design of the relay precoding matrix $\mathbf{W}$, and the receive filter $\boldsymbol{\Gamma}$ that minimize the SMSE under a constraint on the total relay transmit power. The CSI available at the relay node consists of $\widehat{\mathbf{A}}$ and $\widehat{\mathbf{B}}$, whereas the actual channel corresponds to $\mathbf{A} = \widehat{\mathbf{A}} + \mathbf{E}$, and $\mathbf{B} = \widehat{\mathbf{B}} + \mathbf{F}$. Further, the CSI error covariances are assumed to be known at the relay node. In this scenario, we adopt a stochastic approach to the robust design by minimizing the expected value of the SMSE[2] with respect to the CSI errors. Mathematically, the robust design can be expressed as

$$\begin{array}{ll} \min\limits_{\mathbf{W},\boldsymbol{\Gamma}} & \mathbb{E}\{\epsilon\} \\ \text{subject to} & P \le P_T \end{array} \tag{10}$$

[2]In the context of *robust designs* with stochastic CSI error, all the *MSE*-based designs actually use the average or expected value of MSE in the optimization, though we do not explicitly call them *average MSE*-based designs.

where $\epsilon$ is the SMSE, $P$ is the total relay transmit power, $P_T$ is the upper limit on the total relay transmit power, and the expectation is with respect to the CSI error $\mathbf{E}$ and $\mathbf{F}$. In the rest of this subsection, we reformulate the problem in (10) as a convex optimization problem. Based on (3), the SMSE can be expressed as

$$\begin{aligned} \epsilon = \sum_{i=1}^{M} \epsilon_i &= \mathbb{E}\left\{\|\widehat{\mathbf{x}} - \mathbf{x}\|^2\right\} \\ &= \mathrm{tr}(\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{A} - \mathbf{I})(\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{A} - \mathbf{I})^H \\ &\quad + \sigma_\mu^2 \mathrm{tr}(\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{W}^H\mathbf{B}^H\boldsymbol{\Gamma}^H) + \sigma_\nu^2\mathrm{tr}(\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H) \end{aligned} \tag{11}$$

where $\epsilon_i$ is the MSE at the $i$th destination node, and we have used the fact that $\mathbb{E}\{\boldsymbol{\mu}\boldsymbol{\mu}^H\} = \sigma_\mu^2\mathbf{I}$, and $\mathbb{E}\{\boldsymbol{\nu}\boldsymbol{\nu}^H\} = \sigma_\nu^2\mathbf{I}$. Substituting the CSI error model $\mathbf{A} = \widehat{\mathbf{A}} + \mathbf{E}, \mathbf{B} = \widehat{\mathbf{B}} + \mathbf{F}$ in (11), we can express the expected value of the SMSE as

$$\begin{aligned} \overline{\epsilon} &= \mathbb{E}\{\epsilon\} = \mathrm{tr}(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\widehat{\mathbf{A}}\widehat{\mathbf{A}}^H\mathbf{W}^H\widehat{\mathbf{B}}^H\boldsymbol{\Gamma}^H) - 2\Re\mathrm{tr}(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\widehat{\mathbf{A}}) \\ &\quad + \mathrm{tr}\left(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\mathbb{E}\left\{(\mathbf{E}\mathbf{E}^H)\right\}\mathbf{W}^H\widehat{\mathbf{B}}^H\boldsymbol{\Gamma}^H\right) \\ &\quad + \mathbb{E}\left\{\mathrm{tr}(\boldsymbol{\Gamma}\mathbf{F}\mathbf{W}\widehat{\mathbf{A}}\widehat{\mathbf{A}}^H\mathbf{W}^H\mathbf{F}^H\boldsymbol{\Gamma}^H)\right\} \\ &\quad \mathbb{E}\left\{\mathrm{tr}(\boldsymbol{\Gamma}\mathbf{F}\mathbf{W}\mathbf{E}\mathbf{E}^H\mathbf{W}^H\mathbf{F}^H\boldsymbol{\Gamma}^H)\right\} \\ &\quad + \sigma_\mu^2\mathrm{tr}(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\mathbf{W}^H\widehat{\mathbf{B}}^H\boldsymbol{\Gamma}^H) \\ &\quad + \mathbb{E}\left\{\mathrm{tr}(\boldsymbol{\Gamma}\mathbf{F}\mathbf{W}\mathbf{W}^H\mathbf{F}^H\boldsymbol{\Gamma}^H)\right\} + \sigma_\nu^2\mathrm{tr}(\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H) + M \end{aligned} \tag{12}$$

where the expectation is over $\mathbf{E}$ and $\mathbf{F}$. We use the following Lemma I to further simplify the terms in (12).

*Lemma I:* Let $\mathbf{X}$ be a $n \times n$ random matrix with $\mathbb{E}\{\mathrm{vec}(\mathbf{X})\mathrm{vec}(\mathbf{X})^H\} = \sigma^2\mathbf{I}_{n^2}$, and $\mathbf{U}$ and $\mathbf{V}$ be matrices of appropriate dimensions. Then $\mathbb{E}\{\mathrm{tr}(\mathbf{X}\mathbf{U}\mathbf{X}^H\mathbf{V}\}) = \sigma^2\mathrm{tr}(\mathbf{U})\mathrm{tr}(\mathbf{V})$.

*Proof:*

$$\begin{aligned} \mathbb{E}\left\{\mathrm{tr}(\mathbf{X}\mathbf{U}\mathbf{X}^H\mathbf{V})\right\} &= \mathbb{E}\left\{\mathrm{tr}(\mathbf{X}^H\mathbf{V}\mathbf{X}\mathbf{U})\right\} \\ &= \mathbb{E}\left\{\mathrm{vec}(\mathbf{X})^H\mathrm{vec}(\mathbf{V}\mathbf{X}\mathbf{U})\right\} \\ &= \mathrm{tr}\left((\mathbf{U}^T \otimes \mathbf{V})\mathbb{E}\left\{\mathrm{vec}(\mathbf{X})\mathrm{vec}(\mathbf{X})^H\right\}\right) \\ &= \sigma^2\mathrm{tr}(\mathbf{U})\mathrm{tr}(\mathbf{V}). \end{aligned}$$

The sequence of equalities given above follows from the following properties of $\mathrm{vec}(\cdot)$ and $\mathrm{tr}(\cdot)$ operators for any matrices $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ of appropriate dimensions: i) $\mathrm{tr}(\mathbf{A}\mathbf{B}) = \mathrm{vec}(\mathbf{A}^H)^H\mathrm{vec}(\mathbf{B})$, ii) $\mathrm{vec}(\mathbf{A}\mathbf{B}\mathbf{C}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, iii) $\mathrm{tr}(\mathbf{A} \otimes \mathbf{B}) = \mathrm{tr}(\mathbf{A})\mathrm{tr}(\mathbf{B})$, and iv) $\mathrm{tr}(\mathbf{A}^T) = \mathrm{tr}(\mathbf{A})$. ■

Application of Lemma I in (12), and a few algebraic manipulations lead to the following expression for the expected value of the SMSE:

$$\begin{aligned} \overline{\epsilon} &= \mathrm{tr}\left((\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\widehat{\mathbf{A}} - \mathbf{I})(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\widehat{\mathbf{A}} - \mathbf{I})^H\right) \\ &\quad + \sigma_E^2\mathrm{tr}(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\mathbf{W}^H\widehat{\mathbf{B}}^H\boldsymbol{\Gamma}^H) \\ &\quad + \sigma_F^2\mathrm{tr}(\mathbf{W}\widehat{\mathbf{A}}\widehat{\mathbf{A}}^H\mathbf{W}^H)\mathrm{tr}(\boldsymbol{\Gamma}^H\boldsymbol{\Gamma}) \\ &\quad + M\sigma_E^2\sigma_F^2\mathrm{tr}(\mathbf{W}\mathbf{W}^H)\mathrm{tr}(\boldsymbol{\Gamma}^H\boldsymbol{\Gamma}) \\ &\quad + \sigma_\mu^2\mathrm{tr}(\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\mathbf{W}^H\widehat{\mathbf{B}}^H\boldsymbol{\Gamma}^H) \\ &\quad + \sigma_F^2\sigma_\mu^2\mathrm{tr}(\mathbf{W}\mathbf{W}^H)\mathrm{tr}(\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H) + \sigma_\nu^2\mathrm{tr}(\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H). \end{aligned} \tag{13}$$

Note that when $\sigma_E^2 = 0$, and $\sigma_F^2 = 0$, (13) reduces to (11), which represents the SMSE with perfect CSI. The expected value of the total relay transmit power can be expressed as

$$\begin{aligned}P =& \mathbb{E}\left\{\mathrm{tr}(\mathbf{W}\mathbf{y}\mathbf{y}^H\mathbf{W}^H)\right\}\\ =& \mathbb{E}\left\{\mathrm{tr}\left(\mathbf{W}(\widehat{\mathbf{A}}+\mathbf{E})\mathbf{x}\mathbf{x}^H(\widehat{\mathbf{A}}+\mathbf{E})^H\mathbf{W}^H\right)\right\}\\ &+\sigma_\mu^2\mathrm{tr}(\mathbf{W}\mathbf{W}^H)\\ =& \mathrm{tr}(\mathbf{W}\widehat{\mathbf{A}}\widehat{\mathbf{A}}^H\mathbf{W}^H)+\left(M\sigma_E^2+\sigma_\mu^2\right)\mathrm{tr}(\mathbf{W}\mathbf{W}^H) \quad (14)\end{aligned}$$

where the expectation operation is with respect to $\mathbf{x}$, $\boldsymbol{\mu}$, and $\mathbf{E}$. We can observe from (13) and (14) that the constraint function in (10) is a convex function of the $\mathbf{W}$, whereas the objective function is not jointly convex in $\mathbf{W}$ and $\boldsymbol{\Gamma}$. However, the objective function is convex in $\mathbf{W}$ for a fixed value of $\boldsymbol{\Gamma}$ and vice versa. This implies that we can solve the optimization in (10) by solving two subproblems alternatively, each of which is a convex optimization problem. However, we note that the solution thus obtained may not be globally optimal as the problem in (10) is not a convex optimization problem. The first subproblem involves the computation of the relay precoder $\mathbf{W}$ for a fixed value of $\boldsymbol{\Gamma}$, and the second subproblem is the computation of $\boldsymbol{\Gamma}$ for a fixed value of $\mathbf{W}$.

*1) Robust Design of Relay Precoder Matrix* $\mathbf{W}$*:* The first subproblem involves the computation of the relay precoder $\mathbf{W}$ for a given value of the receive filter matrix $\boldsymbol{\Gamma}$. We can rewrite (13) as

$$\begin{aligned}\bar{\epsilon} =& \|\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\widehat{\mathbf{A}}-\mathbf{I}\|_F^2+\sigma_E^2\|\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\|_F^2+\sigma_F^2\|\mathbf{W}\widehat{\mathbf{A}}\|_F^2\|\boldsymbol{\gamma}\|^2\\ &+M\sigma_E^2\sigma_F^2\|\mathbf{W}\|_F^2\|\boldsymbol{\gamma}\|^2+\sigma_\mu^2\|\boldsymbol{\Gamma}\widehat{\mathbf{B}}\mathbf{W}\|_F^2\\ &+\sigma_F^2\sigma_\mu^2\|\mathbf{W}\|_F^2\|\boldsymbol{\gamma}\|^2+\sigma_\nu^2\|\boldsymbol{\gamma}\|^2. \quad (15)\end{aligned}$$

Further, based on (15), we can express $\bar{\epsilon}$ in terms of $\mathbf{w}$, where $\mathbf{w} = \mathrm{vec}(\mathbf{W})$, as

$$\begin{aligned}\bar{\epsilon} =& \|\mathbf{M}_1\mathbf{w}-\mathbf{r}\|^2+\left(\sigma_E^2+\sigma_\mu^2\right)\|\mathbf{M}_2\mathbf{w}\|^2\\ &+\sigma_F^2\|\boldsymbol{\gamma}\|^2\|\mathbf{M}_3\mathbf{w}\|^2+\sigma_F^2\left(\sigma_\mu^2+M\sigma_E^2\right)\|\boldsymbol{\gamma}\|^2\|\mathbf{w}\|^2\\ &+\sigma_\nu^2\|\boldsymbol{\gamma}\|^2 \quad (16)\end{aligned}$$

where $\mathbf{M}_1 = \widehat{\mathbf{A}}^T\otimes(\boldsymbol{\Gamma}\widehat{\mathbf{B}})$, $\mathbf{M}_2 = \mathbf{I}\otimes(\boldsymbol{\Gamma}\widehat{\mathbf{B}})$, and $\mathbf{M}_3 = \widehat{\mathbf{A}}^T\otimes\mathbf{I}$. Similarly, we can express the total relay transmit power as

$$P = \|\mathbf{M}_3\mathbf{w}\|^2+\left(M\sigma_E^2+\sigma_\mu^2\right)\|\mathbf{w}\|^2. \quad (17)$$

The subproblem of computing the robust relay precoder matrix $\mathbf{W}$ for a given value of $\boldsymbol{\Gamma}$ can now be reformulated as the following convex optimization program:

$$\begin{aligned}\min_{\mathbf{w},t_1,t_2,t_3,t_4}\quad & t_1^2+\left(\sigma_\mu^2+\sigma_E\right)^2t_2^2+\sigma_F^2\|\boldsymbol{\gamma}\|^2t_3^2\\ & +\sigma_F^2\left(\sigma_\mu^2+M\sigma_E^2\right)\|\boldsymbol{\gamma}\|^2t_4^2\\ \text{subject to}\quad & \|\mathbf{M}_1\mathbf{w}-\mathbf{r}\|\le t_1,\\ & \|\mathbf{M}_2\mathbf{w}\|\le t_2,\\ & \|\mathbf{M}_3\mathbf{w}\|\le t_3,\\ & \|\mathbf{w}\|\le t_4,\\ & t_3^2+\left(M\sigma_E^2+\sigma_\mu^2\right)t_4^2\le P_T \quad (18)\end{aligned}$$

where $t_1, t_2, t_3$, and $t_4$ are auxiliary variables. We have dropped the last term of (16) in the formulation in (18), as it does not depend on $\mathbf{W}$. The advantage of the convex formulation of the robust precoder design as in (18) is that it can be solved efficiently using interior-point methods [20], [21].

*2) Robust Design of Receive Filter* $\boldsymbol{\Gamma}$*:* The second subproblem in the proposed robust design involves the computation of the receive filter matrix $\boldsymbol{\Gamma}$ for a given $\mathbf{W}$. The computation of the receive filter matrix $\boldsymbol{\Gamma}$ that minimizes $\bar{\epsilon}$ is an unconstrained optimization problem unlike the other subproblem. Note that, the optimum value of $\gamma_i$, $1\le i\le M$, satisfies the following condition:

$$\frac{\partial\bar{\epsilon}}{\partial\gamma_i^*} = \frac{\partial\bar{\epsilon}_i}{\partial\gamma_i^*} = 0, \quad 1\le i\le M \quad (19)$$

where $\bar{\epsilon}_i = \mathbb{E}\{\epsilon_i\}$. The estimate of the transmitted signal $x_i$ at the $i$th destination node can be written as

$$\widehat{x}_i = \gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_ix_i+\gamma_i\sum_{k=1,k\ne i}^{M}\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_kx_k+\gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\mu}+\gamma_i\nu_i. \quad (20)$$

We can express the MSEs, $\epsilon_i$, $1\le i\le M$, as

$$\begin{aligned}\epsilon_i =& |x_i-\widehat{x}_i|^2\\ =& \left|\gamma_i(\widehat{\boldsymbol{\beta}}_i+\boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_i+\boldsymbol{\phi}_i)-1\right|^2\\ &+|\gamma_i|^2\sum_{k=1,k\ne i}^{M}\left|(\widehat{\boldsymbol{\beta}}_i+\boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k+\boldsymbol{\phi}_k)\right|^2\\ &+|\gamma_i|^2(\widehat{\boldsymbol{\beta}}_i+\boldsymbol{\pi}_i)\mathbf{W}\mathbf{R}_\mu\mathbf{W}^H(\widehat{\boldsymbol{\beta}}_i+\boldsymbol{\pi}_i)^H+|\gamma_i|^2\sigma_\nu^2. \quad (21)\end{aligned}$$

Based on (21), the expected value of the MSE at the $i$th user can be expressed as

$$\begin{aligned}\bar{\epsilon}_i =& |\gamma_i\widehat{\boldsymbol{\beta}}_i\mathbf{W}\widehat{\boldsymbol{\alpha}}_i-1|^2+|\gamma_i|^2\sum_{k=1}^{M}\sigma_F^2\|\mathbf{W}\widehat{\boldsymbol{\alpha}}_k\|^2\\ &+M|\gamma_i|^2\sigma_E^2\|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\|^2+M|\gamma_i|^2\sigma_E^2\sigma_F^2\|\mathbf{W}\|_F^2\\ &+|\gamma_i|^2\sum_{k=1,k\ne i}^{M}|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\widehat{\boldsymbol{\alpha}}_k|^2+|\gamma_i|^2\sigma_\mu^2\widehat{\boldsymbol{\beta}}_i\mathbf{W}\mathbf{W}^H\widehat{\boldsymbol{\beta}}_i^H\\ &+|\gamma_i|^2\sigma_F^2\sigma_\mu^2\mathrm{tr}(\mathbf{W}\mathbf{W}^H)+|\gamma_i|^2\sigma_\nu^2. \quad (22)\end{aligned}$$

From the optimality condition in (19) we have

$$\begin{aligned}&\frac{\partial\mathbb{E}\{\bar{\epsilon}\}}{\partial\gamma_i^*} = \frac{\partial\mathbb{E}\{\bar{\epsilon}\}_i}{\partial\gamma_i^*} = 0\\ &\Longrightarrow\gamma_i = \frac{(a_i^*-1)}{b_i}, \quad 1\le i\le M \quad (23)\end{aligned}$$

where $a_i = \widehat{\boldsymbol{\beta}}_i\mathbf{W}\widehat{\boldsymbol{\alpha}}_i$, and

$$\begin{aligned}b_i =& \sum_{k=1}^{M}\sigma_F^2\|\mathbf{W}\widehat{\boldsymbol{\alpha}}_k\|^2+M\sigma_E^2\|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\|^2+M\sigma_E^2\sigma_F^2\|\mathbf{W}\|_F^2+\sigma_\nu^2\\ &+\sum_{k=1,k\ne i}^{M}|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\widehat{\boldsymbol{\alpha}}_k|^2+\sigma_\mu^2\widehat{\boldsymbol{\beta}}_i\mathbf{W}\mathbf{W}^H\widehat{\boldsymbol{\beta}}_i^H+\sigma_F^2\sigma_\mu^2\|\mathbf{W}\|_F^2. \quad (24)\end{aligned}$$

TABLE I
ALGORITHM FOR COMPUTATION OF PRECODING MATRIX $\mathbf{W}$ AND RECEIVE FILTER $\mathbf{\Gamma}$

Select $N_{max}$ (maximum number of iterations), $T_{th}$(convergence threshold), Initialize $\mathbf{X}^0 = [\mathbf{W}^0 \ \ \mathbf{\Gamma}^0]$

1) $n = 0$
2) **while** $n \leq N_{max}$
3) Compute $\mathbf{W}^{n+1}$ using $\mathbf{\Gamma}^n$ in (18)
4) Compute $\mathbf{\Gamma}^{n+1}$ using $\mathbf{W}^{n+1}$ in (23)
5) $\mathbf{x}^{n+1}$=$[\mathbf{w}^{n+1} \ \ \boldsymbol{\gamma}^{n+1}]$
6) **if** $\left\|\mathbf{x}^{n+1} - \mathbf{x}^n\right\| \leq T_{th}$ **then**
7) **break**
8) **endif**
9) $n \leftarrow n+1$
10) **endwhile**

*3) Iterative Algorithm for Solving (10):* The complete algorithm to compute the robust relay precoder matrix $\mathbf{W}$, and the matrix of receive filters for the destination nodes $\mathbf{\Gamma}$, which essentially solves (10), is given in Table I. At the $(n+1)$th iteration, the value of $\mathbf{W}$ is the solution to the following problem:

$$\mathbf{W}^{n+1} = \underset{\mathbf{W}:P\leq P_T}{\arg\min}\ \overline{\epsilon}(\mathbf{W}, \mathbf{\Gamma}^{n+1}) \tag{25}$$

which can be solved using (18). Having computed $\mathbf{W}^{n+1}, \mathbf{\Gamma}^{n+1}$ is obtained as the solution to the following problem:

$$\mathbf{\Gamma}^{n+1} = \underset{\mathbf{\Gamma}}{\arg\min}\ \overline{\epsilon}(\mathbf{W}^{n+1}, \mathbf{\Gamma}). \tag{26}$$

The iterative optimization over $\{\mathbf{W}\}$ and $\{\mathbf{\Gamma}\}$ can be repeated until optimization variables converge. From (25) and (26), we have

$$\overline{\epsilon}(\mathbf{W}^{n+1}, \mathbf{\Gamma}^{n+1}) \leq \overline{\epsilon}(\mathbf{W}^{n+1}, \mathbf{\Gamma}^{n}) \leq \overline{\epsilon}(\mathbf{W}^{n}, \mathbf{\Gamma}^{n}). \tag{27}$$

Coupled with the fact that the SMSE is lower bounded, (27) implies that the proposed algorithm is guaranteed to converge to a limit as $n \rightarrow \infty$. However, as noted earlier, convergence to the globally optimal solution is not guaranteed. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached.

*4) Robust Design With Per-Antenna Power Constraints:* As each antenna at the MIMO-relay node usually has its own amplifier, it is important to consider precoder design with constraints on power transmitted from each antenna. For incorporating per-antenna power constraint in the proposed robust minimum SMSE design, only the precoder matrix design (18) has to be modified by including the constraints on power transmitted from each antenna as given below:

$$\begin{aligned}
\min_{\mathbf{w},t_1,t_2,t_3,t_4} \quad & t_1^2 + \left(\sigma_\mu^2 + \sigma_E\right)^2 t_2^2 + \sigma_F^2\|\boldsymbol{\gamma}\|^2 t_3^2 \\
& + \sigma_F^2\left(\sigma_\mu^2 + M\sigma_E^2\right)\|\boldsymbol{\gamma}\|^2 t_4^2 \\
\text{subject to} \quad & \|\mathbf{M}_1\mathbf{w} - \mathbf{r}\| \leq t_1, \\
& \|\mathbf{M}_2\mathbf{w}\| \leq t_2, \\
& \|\mathbf{M}_3\mathbf{w}\| \leq t_3, \\
& \|\mathbf{w}\| \leq t_4, \\
& \left(M\sigma_E^2 + \sigma_\mu^2\right)\|(\mathbf{I}\otimes\boldsymbol{\phi}_k)\mathbf{w}\|^2 \\
& + \left\|(\widehat{\mathbf{A}}^T\otimes\boldsymbol{\phi}_k)\mathbf{w}\right\|^2 \leq P_T^k, \ \ 1\leq k\leq M
\end{aligned} \tag{28}$$

where $\boldsymbol{\phi}_k = [\mathbf{0}_{1\times k-1} \ 1 \ \mathbf{0}_{1\times N-k}]$, and $P_T^k$ is the upper limit on the power for the $k$th relay transmit antenna. The receive filter $\mathbf{\Gamma}$ can be computed using (23). Hence, the algorithm in Table I can be used to obtain the robust design with per-antenna power constraints with $\mathbf{W}$ computed using (28) and $\mathbf{\Gamma}$ computed using (23). As in the case with the total power constraint, the resulting solution is not guaranteed to be globally optimal. Further, the robust design with per-antenna power constraints is computationally more complex compared to that with total power constraint due to the additional SOC constraints in (28).

## B. Robust MSE-Balancing Relay Precoder Design

When the CSI available at the relay node is imperfect, the problem of robust design of relay precoder based on MSE-balancing with a constraint on the total relay transmit power can be expressed as

$$\begin{aligned}
\min_{\mathbf{W},\mathbf{\Gamma}} \quad & \max_i \quad \overline{\epsilon}_i \\
\text{subject to} \quad & P \leq P_T.
\end{aligned} \tag{29}$$

The problem given above is equivalent to

$$\begin{aligned}
\min_{\mathbf{W},\mathbf{\Gamma},t} \quad & t \\
\text{subject to} \quad & \overline{\epsilon}_i \leq t, \quad 1\leq i\leq M, \\
& P \leq P_T
\end{aligned} \tag{30}$$

where $t$ is a slack variable. The expression for $\overline{\epsilon}_i$ given in (22) can be rewritten as

$$\begin{aligned}
\overline{\epsilon}_i = & \|\gamma_i\boldsymbol{\beta}_i\mathbf{W}\mathbf{A} - \mathbf{e}_i\|^2 + |\gamma_i|^2\left(M\sigma_E^2 + \sigma_\mu^2\right)\|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\|^2 \\
& + |\gamma_i|^2\sigma_F^2\left(M\sigma_E^2 + \sigma_\mu^2\right)\|\mathbf{W}\|_F^2 \\
& + |\gamma_i|^2\sigma_F^2\|\mathbf{W}\mathbf{A}\|_F^2 + |\gamma_i|^2\sigma_\nu^2
\end{aligned} \tag{31}$$

where $\mathbf{e}_i$ is a column vector whose $i$th component is 1, and the rest are 0s. The problem in (30) can be reformulated as the following optimization problem:

$$\begin{aligned}
\min_{\mathbf{W},\mathbf{\Gamma},r_1,r_2,r_3,r_4,t} \quad & t \\
\text{subject to} \quad & \left(r_1^i\right)^2 + \left(r_2^i\right)^2 + \sigma_F^2(r_3)^2 + (r_4)^2 \leq \psi_i^2 t, \\
& \|\boldsymbol{\beta}_i\mathbf{W}\mathbf{A} - \psi_i\mathbf{e}_i\| \leq r_1^i \\
& \sqrt{(M\sigma_E^2 + \sigma_\nu^2)}\|\widehat{\boldsymbol{\beta}}_i\mathbf{W}\| \leq r_2^i, 1\leq i\leq M \\
& \|\mathbf{M}_3\mathbf{w}\| \leq r_3 \\
& \|\mathbf{w}\| \leq r_4 \\
& \left\| r_3 + \sqrt{(M\sigma_E^2 + \sigma_\nu^2)}r_4 \right\| \leq \sqrt{P_T}
\end{aligned} \tag{32}$$

where $\psi_i = 1/\gamma_i$, and $r_1^i, r_2^i, r_3, r_4, \ 1\leq i\leq M$ are slack variables. The problem in (32) is a quasi-convex optimization problem, which can be solved through a sequence of bisection search and solution of a convex feasibility problem [20]. Suppose $t^*$ is the optimal solution of the problem in (32). For a fixed

TABLE II
ITERATIVE ALGORITHM FOR THE ROBUST MSE-BALANCING RELAY PRECODER DESIGN

Set the interval $[b_l \quad b_u]$ that contains the optimum value of the objective. Set desired tolerance $\kappa$

1) $n = 0$
2) **while** $b_u - b_l \leq \kappa$
3) $r \leftarrow (b_l + b_u)/2$
4) Solve (32) for $t = r$
5) **If** feasible, $b_u \leftarrow r$, **else** $b_l \leftarrow r$.
6) **endwhile**

value of $t$, the problem in (32) is a convex feasibility problem, i.e., to find the set of optimization variables that satisfy all the constraints. If the problem is feasible for a fixed value of $t$, we can see that $t^* \leq t$, otherwise $t^* \geq t$. Based on this fact, we can devise an iterative algorithm to solve the problem in (32). The iterative algorithm given in Table II involves a bisection search in $t$, and a solution of a convex feasibility problem. As the objective $t$ represents MSE, $b_l$ can be initialized as zero, and $b_u$ can be initialized as a sufficiently large value. The size of the interval for search is reduced by half at each step, and so the iteration is guaranteed to converge in $\lceil \log_2((b_u - b_l)/\kappa) \rceil$ steps, where $\kappa$ is the convergence threshold, and $\lceil x \rceil$ represents the lowest integer greater than or equal to $x$. Further, as the feasibility problem is convex, this algorithm will converge to a globally optimal solution.

*1) Robust Relay Power Minimizing Precoder Design:* The design of a robust relay precoder that minimizes the total relay transmit power with constraints on MSEs at the destination nodes is closely related to that of the MSE-balancing precoder considered earlier. Such a precoder is of interest when there is a requirement to maintain a specific quality-of-service (QoS) at the destination nodes. This design problem can be stated as

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma}} \quad & P \\ \text{subject to} \quad & \bar{\epsilon}_i \leq \eta_i, \quad 1 \leq i \leq M \end{aligned} \tag{33}$$

where $\eta_i$ is the maximum allowed MSE (MSE target) at the $i$th destination. Based on the earlier developments, this problem can be reformulated as the following convex optimization problem:

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma},r_1,r_2,r_3,r_4,t} \quad & t^2 \\ \text{subject to} \quad & \left\| r_3 + \sqrt{(M\sigma_E^2 + \sigma_\nu^2)} r_4 \right\| \leq t \\ & \left(r_1^i\right)^2 + \left(r_2^i\right)^2 + \sigma_F^2 (r_3)^2 + (r_4)^2 \leq \psi_i^2 \eta_i \\ & \|\boldsymbol{\beta}_i \mathbf{W}\mathbf{A} - \psi_i \mathbf{e}_i\| \leq r_1^i \\ & \sqrt{(M\sigma_E^2 + \sigma_\nu^2)} \|\widehat{\boldsymbol{\beta}}_i \mathbf{W}\| \leq r_2^i, \ 1 \leq i \leq M \\ & \|\mathbf{M}_3 \mathbf{w}\| \leq r_3 \\ & \|\mathbf{w}\| \leq r_4. \end{aligned} \tag{34}$$

Being a convex optimization problem, it has a unique optimal solution and can be efficiently solved using interior-point methods. However, the precoder designed using (34) for given CSI satisfies the MSE constraints only on the average, not for each individual realization of the CSI error. The problem of robust design that satisfies MSE constraints for each realization of the CSI error in a specified uncertainty set is addressed in the next section.

## IV. ROBUST PRECODER DESIGN WITH NORM-BOUNDED CSI ERROR

In this section, we propose a robust design of the relay precoding matrix $\mathbf{W}$ and the receive filter $\mathbf{\Gamma}$ in the presence of CSI error that follows the NBE model. The design seeks to minimize the total relay transmit power while meeting MSE constraints at all the destination nodes. When the CSI available at the relay is imperfect with errors of bounded norm, we can make the relay precoder and the receive filter robust by ensuring that the precoder and the receive filter satisfy the MSE constraints for all possible CSI errors satisfying the norm bound. Mathematically, this problem can be represented as

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma}} \quad & P \\ \text{subject to} \quad & \epsilon_i \leq \eta_i, \\ & \boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \boldsymbol{\beta}_i \in \mathcal{R}_{\beta_i} \quad 1 \leq i \leq M \end{aligned} \tag{35}$$

where $P$ is the total relay transmit power, and $\epsilon_i$ is the actual MSE at the $i$th destination node. The problem above is equivalent to

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma},\tau} \quad & \tau \\ \text{subject to} \quad & \max_{\boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \forall i} P \leq \tau \\ & \max_{\boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \boldsymbol{\beta}_i \in \mathcal{R}_{\beta_i}, \forall i} \epsilon_i \leq \eta_i \end{aligned} \tag{36}$$

where $\tau$ is an auxiliary optimization variable. Though an exact solution to this problem is difficult, we propose a tractable solution based on the cutting-set method [22]. The cutting-set method is an effective technique to solve worst-case convex optimization problems with parameter uncertainty. The uncertain parameters are assumed to belong to some given uncertainty sets. In this method, the worst-case optimization alternates between an optimization step and a worst-case analysis step. The optimization step involves the computation of the optimal solution for fixed values of the parameters, and the worst-case analysis step involves maximization of the constraint functions over the uncertainty sets. The cutting-set method leads to the robust optimal solution if the worst-case analysis step results in exact solution. The proposed solution involves solving an alternating sequence of two subproblems, viz., i) precoder/receive filter design with fixed channel vectors, and ii) computation of worst-case channel vectors for a fixed precoder and receive filter. We note that the precoder/receive filter design with perfect CSI is a special case of the problem described above, which involves only solving once the first subproblem using the perfectly known channel vectors. The following subsections describe the solutions of the first and second subproblems and the iterative algorithm to solve the overall robust design problem.

### A. Precoder/Receive Filter Design for Given Channel Vectors

The first subproblem in the proposed robust design is the computation of the relay precoder $\mathbf{W}$ and the receive filter $\mathbf{\Gamma}$

for a given set of channel vectors $\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, 1 \le i \le M$. This computation involves the minimization of the total relay transmit power under MSE constraints at the destination nodes. Mathematically, this problem can be written as

$$\begin{aligned} \min_{\mathbf{W},\boldsymbol{\Gamma},\tau} \quad & \tau \\ \text{subject to} \quad & P \le \tau \\ & \epsilon_i \le \eta_i, \quad 1 \le i \le M. \end{aligned} \tag{37}$$

The total relay transmit power can be expressed as

$$\begin{aligned} P &= \mathbb{E}\left\{\|\mathbf{W}\mathbf{y}\|^2\right) \\ &= \mathbb{E}\left\{\widetilde{\mathbf{w}}^H(\mathbf{I}_{N^2}\otimes \mathbf{y}^T)^H(\mathbf{I}_{N^2}\otimes \mathbf{y}^T)\widetilde{\mathbf{w}}\right\} \\ &= \widetilde{\mathbf{w}}^H(\mathbf{I}_{N^2}\otimes \left(\mathbb{E}\left\{(\mathbf{y}^T)^H\mathbf{y}^T\right\}\right)\widetilde{\mathbf{w}} \\ &= \widetilde{\mathbf{w}}^H\left(\mathbf{I}_{N^2}\otimes(\mathbf{A}\mathbf{A}^H+\mathbf{R}_\mu)^T\right)\widetilde{\mathbf{w}} \end{aligned} \tag{38}$$

where $\widetilde{\mathbf{w}} = \mathrm{vec}(\mathbf{W}^T)$, and $\mathbf{R}_\mu = \mathbb{E}\{\boldsymbol{\mu}\boldsymbol{\mu}^H\}$. The estimate of the transmitted signal $x_i$ at the $i$th destination node can be written as

$$\begin{aligned} \widehat{x}_i &= \gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_i x_i + \gamma_i\sum_{k=1,k\neq i}^{M}\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_k x_k + \gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\mu} + \gamma_i\nu_i \\ &= \gamma_i\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_i^T\right)\widetilde{\mathbf{w}}x_i + \sum_{k=1,k\neq i}^{M}\gamma_i\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_k^T\right)\widetilde{\mathbf{w}}x_k \\ &\quad + \sum_{k=1}^{N}\gamma_i\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\mathbf{i}_k^T\right)\widetilde{\mathbf{w}}\mu_k + \gamma_i\nu_i. \end{aligned} \tag{39}$$

From the expression given above, the MSE at the $i$th destination node can be written as

$$\begin{aligned} \epsilon_i &= \mathbb{E}\left\{|\widehat{x}_i - x|^2\right\} \\ &= \left|\gamma_i\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_i^T\right)\widetilde{\mathbf{w}} - 1\right|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1,k\neq i}^{M}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_k^T\right)\widetilde{\mathbf{w}}\right|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1}^{N}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\mathbf{i}_k^T\right)\widetilde{\mathbf{w}}\right|^2\sigma_{\mu_k}^2 + |\gamma_i|^2\sigma_{\nu_i}^2. \end{aligned} \tag{40}$$

The MSE constraints at the destination nodes can be written as

$$\begin{aligned} &\left|\gamma_i\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_i^T\right)\widetilde{\mathbf{w}} - 1\right|^2 + |\gamma_i|^2\sum_{k=1,k\neq i}^{M}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_k^T\right)\widetilde{\mathbf{w}}\right|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1}^{N}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\mathbf{i}_k^T\right)\widetilde{\mathbf{w}}\right|^2\sigma_{\mu_k}^2 + |\gamma_i|^2\sigma_{\nu_i}^2 \le \eta_i, \quad \forall i \\ &\Rightarrow \left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_i^T\right)\widetilde{\mathbf{w}} - \psi_i\right|^2 + \sum_{k=1,k\neq i}^{M}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\boldsymbol{\alpha}_k^T\right)\widetilde{\mathbf{w}}\right|^2 \\ &\quad + \sum_{k=1}^{N}\left|\boldsymbol{\beta}_i\left(\mathbf{I}_N\otimes\mathbf{i}_k^T\right)\widetilde{\mathbf{w}}\right|^2\sigma_{\mu_k}^2 + \sigma_{\nu_i}^2 \le \eta_i|\psi_i|^2\ \forall i, \end{aligned} \tag{41}$$

where $\psi_i = 1/\gamma_i$. The precoder/receive filter design problem as obtained by substituting (41) in (37) is not a convex optimization problem. We can transform this problem into a convex optimization problem as follows. Let

$$\mathbf{p}^i = \left[p_1^i \cdots p_{i-1}^i\left(p_i^i - \psi_i\right)p_{i+1}^i \cdots p_M^i\right] \tag{42}$$

where $p_k^i = \boldsymbol{\beta}_i(\mathbf{I}_N\otimes\boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}$. Let $\mathbf{q}^i = [q_1^i q_2^i \cdots q_M^i]$, where $q_k^i = \boldsymbol{\beta}_i(\mathbf{I}_N\otimes\mathbf{i}_k^T)\widetilde{\mathbf{w}}\sigma_{\mu_i}$. The constraints in (41) can be reformulated as

$$\left\|\begin{bmatrix}\mathbf{p}^i & \mathbf{q}^i & \sigma_{\nu_i}\end{bmatrix}\right\|^2 \le \eta_i|\psi_i|^2, \quad \forall i. \tag{43}$$

The precoder design that minimizes the total relay transmit power under MSE constraints at the destination nodes can be expressed as

$$\begin{aligned} \min_{\widetilde{\mathbf{w}},\{\psi_i\},\tau} \quad & \tau \\ \text{subject to} \quad & \widetilde{\mathbf{w}}^H\left(\mathbf{I}_{N^2}\otimes(\mathbf{A}\mathbf{A}^H+\mathbf{R}_\mu)^T\right)\widetilde{\mathbf{w}} \le \tau \\ & \left\|\begin{bmatrix}\mathbf{p}^i & \mathbf{q}^i & \sigma_{\nu_i}\end{bmatrix}\right\| \le \sqrt{\eta_i}\psi_i, \ \ 1 \le i \le M \end{aligned} \tag{44}$$

where we have assumed $\psi_i$, $1 \le i \le M$, are nonnegative real numbers. As the first constraint in the problem in (44) is a convex quadratic constraint and the rest are SOC constraints, the problem given above is a convex optimization program. The assumption that $\{\psi_i\}$ are nonnegative real numbers is required in order to express (43) as SOC constraints in the convex formulation in (44). If there is only a single constraint of the form (43), i.e., $M = 1$, then (37) and (44) are equivalent as the phase factor of $\psi$ can be absorbed into $\widetilde{\mathbf{w}}$ without affecting the objective and other constraints. However, when there are multiple constraints of the form (43), i.e., $M > 1$, then the solution of (44) can provide only an approximate solution of (37) as the phase factors of all the constraints cannot be simultaneously absorbed by $\widetilde{\mathbf{w}}$.

### B. Computation of Worst-Case Channels

The second subproblem in the proposed robust design involves the computation of the worst-case channels, i.e., those channel vectors that belong to the uncertainty region and maximize the total relay transmit power and the MSEs at the destination nodes.

First, we consider the computation of the worst case channels that maximize the MSEs for a given precoder and receive filter. If the worst-case analysis problem can be solved exactly, then the exact robust optimal solution to the problem above is possible. But, in the present problem, it turns out that an exact solution to the worst-case analysis, i.e., the computation of $\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i$, $1 \le i \le M$ that maximize $P$ and $\epsilon_i, 1 \le i \le M$, is not possible due to the nonconvexity of this problem. Hence, we propose an approximate solution to the worst-case analysis problem. We can express the MSEs, $\epsilon_i$, $1 \le i \le M$, as

$$\begin{aligned} \epsilon_i &= \left|\gamma_i(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i) - 1\right|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1,k\neq i}^{M}\left|(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)\right|^2 \\ &\quad + |\gamma_i|^2(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}\mathbf{R}_\mu\mathbf{W}^H(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)^H + |\gamma_i|^2\sigma_\nu^2. \end{aligned} \tag{45}$$

Let $\overline{\boldsymbol{\phi}}_k^i$, $1 \leq k \leq M$, and $\overline{\boldsymbol{\pi}}^i$ be the optimal solution of the following problem:

$$\begin{aligned} \max_{\{\boldsymbol{\phi}_k\}_{k=1}^M, \boldsymbol{\pi}_i} \quad & \epsilon_i \\ \text{subject to} \quad & \|\boldsymbol{\phi}_k\|^2 \leq \delta_{\alpha_k}, \quad 1 \leq k \leq M, \\ & \|\boldsymbol{\pi}_i\|^2 \leq \delta_{\beta_i}. \end{aligned} \tag{46}$$

Then $\overline{\boldsymbol{\alpha}}_k^i = \widehat{\boldsymbol{\alpha}}_i + \overline{\boldsymbol{\phi}}_k^i, 1 \leq k \leq M$, and $\overline{\boldsymbol{\beta}}^i = \widehat{\boldsymbol{\beta}}_i + \overline{\boldsymbol{\pi}}^i$ correspond to the worst-case channels that give rise to the worst-case MSE at the $i$th destination node, given the imperfect CSI $(\{\widehat{\boldsymbol{\alpha}}_k\}_{k=1}^M, \widehat{\boldsymbol{\beta}}_i)$ at the relay, and the CSI error norm bounds $\{\delta_{\alpha_k}\}_{k=1}^M$ and $\delta_{\beta_i}$. Note that the MSE at the $i$th destination node is a function of the source-to-relay channel vectors of all the source nodes, i.e., $\boldsymbol{\alpha}_k$, $1 \leq k \leq M$. Referring to (45), we can see that solving (46) exactly is quite difficult. To significantly simplify the problem, we approximate $\epsilon_i$ in (46) by neglecting those terms in (45) that involve second and higher orders of $\{\boldsymbol{\phi}_k\}_{k=1}^M$ and $\boldsymbol{\pi}_i$. We can approximate the MSE at the $i$th source node as

$$\begin{aligned} \epsilon_i \approx \widehat{\epsilon}_i &+ 2\Re\left\{(\theta_{ii}^\star - 1)\,\boldsymbol{\beta}_i \mathbf{W}\boldsymbol{\phi}_i\right\} + 2\Re\left\{(\theta_{ii}^\star - 1)\,\boldsymbol{\pi}_i \mathbf{W}\boldsymbol{\alpha}_i\right\} \\ &+ 2\sum_{k=1,k\neq i}^M \Re\left\{\theta_{ik}^\star \boldsymbol{\beta}_i \mathbf{W}\boldsymbol{\phi}_k\right\} \\ &+ 2\Re\left\{\boldsymbol{\pi}_i\left(\sum_{k=1,k\neq i}^M \theta_{ik}^\star \mathbf{W}\boldsymbol{\alpha}_k + \mathbf{W}\mathbf{R}_\mu \mathbf{W}\boldsymbol{\beta}^H\right)\right\} \end{aligned} \tag{47}$$

where $\theta_{ij} = \boldsymbol{\beta_i}\mathbf{W}\boldsymbol{\alpha_j}$, and $\widehat{\epsilon}_i$ is obtained by setting $\boldsymbol{\phi}_i = \boldsymbol{\pi}_i = \mathbf{0}$, $1 \leq i \leq M$, in (45). Considering the terms involving $\boldsymbol{\phi}_k$ and $\boldsymbol{\pi}_k$ in (47), and applying Cauchy–Schwartz inequality, we can determine the worst-case CSI error vectors as follows:

$$\overline{\boldsymbol{\phi}}_i^i = \frac{\delta_{\alpha_i}}{\left\|(\theta_{ii} - 1)\mathbf{W}^H \boldsymbol{\beta}_i^H\right\|}(\theta_{ii} - 1)\mathbf{W}^H \boldsymbol{\beta}_i^H, \quad \forall i \tag{48}$$

$$\overline{\boldsymbol{\phi}}_k^i = \frac{\delta_{\alpha_k}}{\left\|\theta_{ik}\mathbf{W}^H \boldsymbol{\beta}_i^H\right\|}\theta_{ik}\mathbf{W}^H \boldsymbol{\beta}_i^H, \quad \forall i, \forall k \neq i \tag{49}$$

$$\overline{\boldsymbol{\pi}}^i = \frac{\delta_{\beta_i}}{\|\mathbf{f}_i\|}\mathbf{f}_i, \quad \forall i \tag{50}$$

where

$$\mathbf{f}_i = (\theta_{ii}^\star - 1)\,\mathbf{W}\boldsymbol{\alpha}_i + \left(\sum_{k=1,k\neq i}^M \theta_{ik}^\star \mathbf{W}\boldsymbol{\alpha}_k + \mathbf{W}\mathbf{R}_\mu \mathbf{W}\boldsymbol{\beta}^H\right). \tag{51}$$

Next, we consider the computation of the worst case channels that maximize the total relay transmit power. The total relay transmit power can be expressed as

$$P = \sum_{k=1}^M (\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H \mathbf{W}^H \mathbf{W}(\widehat{\boldsymbol{\alpha}} + \boldsymbol{\phi}_k)_k + \operatorname{trace}(\mathbf{W}\mathbf{R}_\mu \mathbf{W}^H). \tag{52}$$

As the last term in (52) does not depend on the CSI error, the worst-case channel vectors maximizing the total relay transmit power is obtained by solving

$$\max_{\left\{\boldsymbol{\phi}_i : \|\boldsymbol{\phi}_i\| \leq \delta_{\alpha_i}\right\}_{i=1}^M} \sum_{k=1}^M (\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H \mathbf{W}^H \mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k). \tag{53}$$

The problem in (53) is equivalent to the following individual problems for $1 \leq k \leq M$:

$$\max_{\boldsymbol{\phi}_k : \|\boldsymbol{\phi}_k\| \leq \delta_{\alpha_k}} (\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H \mathbf{W}^H \mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k). \tag{54}$$

The constraint in (54) is always active. So, the optimality conditions [20] can be written as

$$\nabla \mathcal{L}_k = 0 \tag{55a}$$

$$\boldsymbol{\phi}_k^H \boldsymbol{\phi}_k - \delta_{\alpha_k}^2 = 0 \tag{55b}$$

$$\rho > 0 \tag{55c}$$

where $\nabla$ is the gradient operator, $\rho$ is the Lagrange multiplier, and $\mathcal{L}_k$ is the Lagrangian associated with (54),

$$\mathcal{L}_k = (\widehat{\boldsymbol{\alpha}}_k + \mathbf{a}_k)^H \mathbf{W}^H \mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k) + \rho\left(\boldsymbol{\phi}_k^H \boldsymbol{\phi}_k - \delta_{\alpha_k}^2\right). \tag{56}$$

From (55a) and (56), we get

$$\boldsymbol{\phi}_k = -(\mathbf{W}^H \mathbf{W} + \rho \mathbf{I})^{-1} \mathbf{W}^H \mathbf{W}\boldsymbol{\alpha}_k. \tag{57}$$

Let $\mathbf{W}^H \mathbf{W} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H$ be the singular value decomposition of $\mathbf{W}^H \mathbf{W}$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the singular values $\lambda_i$ $1 \leq i \leq N$. Then, based on (57) and (55b), we can show that

$$\begin{aligned} \boldsymbol{\phi}_k^H \boldsymbol{\phi}_k &= \mathbf{g}_k^H \boldsymbol{\Lambda}(\boldsymbol{\Lambda} + \rho \mathbf{I})^{-2} \boldsymbol{\Lambda}\mathbf{g}_k = \delta_{\alpha_k}^2 \\ \Rightarrow \sum_{i=1}^N \frac{|g_{ki}|^2 \lambda_i^2}{(\lambda_i + \rho)^2} &- \delta_{\alpha_k}^2 = 0, \end{aligned} \tag{58}$$

where $\mathbf{g}_k = \mathbf{V}^H \boldsymbol{\alpha}_k$, and $g_{ki}$ is the $i$th element of $\mathbf{g}_k$. The Lagrange multiplier $\rho$ can be determined by solving (58).

We note that since the worst-case design in this subsection is based on the approximate expression for MSE in (47), even if the channel is in the assumed uncertainty region, some of the resulting solutions might have slightly violated the MSE constraints. However, such violations are negligible as the effect of second and higher order terms of CSI error is quite insignificant in the case of small CSI error.

### *C. Iterative Algorithm for the Robust Design*

The proposed robust precoder design involves iterating over a sequence of minimization and worst-case analysis steps described in the previous two subsections till a stopping criterion is met. We start with the set $\mathcal{S}$ of channel vectors, which initially contains only the imperfect CSI $\widehat{\boldsymbol{\alpha}}_i, \widehat{\boldsymbol{\beta}}_i, 1 \leq i \leq M$ available at the relay node. The first step involves the solution of the optimization problem in (44) for all elements of the set $\mathcal{S}$. This step computes $\mathbf{W}$, $\boldsymbol{\Gamma}$, and $\tau$ for a given $\mathcal{S}$. The second step is the worst-case analysis as described in the previous subsection. If the resulting worst-case channels violate the constraints in (44) for the values of $\mathbf{W}$, $\boldsymbol{\Gamma}$, and $\tau$ computed in the previous step, these channel vectors are added to $\mathcal{S}$. So, during the worst-case analysis step in each iteration, the set $\mathcal{S}$ of the worst-case channels may be expanded depending on the constraint violations. During the minimization step in each iteration, the precoder and the receive filter are optimized to meet MSE constraints for increasing number of worst-case channels resulting in increased robustness. These two steps are iterated till maximum constraint violation $\max_i(\tilde{\epsilon}_i - \eta_i)$, where $\tilde{\epsilon}_i$ is the worst-case MSE at the $i$th destination node, becomes less than a certain threshold. When the worst-case analysis problem has an exact solution, these iterations lead to the robust optimal solution [22]. For the problem considered here, the worst-case analysis is approximate, and the iteration is not guaranteed to lead to the robust optimal solution. However, our simulations show that the proposed design is robust to the CSI errors.

## V. Results and Discussions

In this section, we illustrate the performance of the proposed robust designs of the MIMO-relay precoder and receive filter, evaluated through simulations. We compare the performance of the proposed robust designs with that of the nonrobust designs. The channel fading is modeled as Rayleigh, with the channel vectors $\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k$, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each node is assumed to be zero-mean complex Gaussian random variable.

First, we consider the performance of the robust MIMO-relay precoder designs presented in Section III for the stochastic CSI error model. For this model, results for the perfect CSI case is obtained assuming $\mathbf{A} = \widehat{\mathbf{A}}$, $\mathbf{B} = \widehat{\mathbf{B}}$, and using $\sigma_E = \sigma_F = 0$ in the algorithm in Table I. Results for the nonrobust design is obtained assuming $\mathbf{A} = \widehat{\mathbf{A}} + \mathbf{E}$, $\mathbf{B} = \widehat{\mathbf{B}} + \mathbf{F}$, but setting $\sigma_E = \sigma_F = 0$ in the algorithm in Table I and Table II. In other words, in the case of nonrobust design, even though the actual channels are different from the CSI estimates available at the relay node, the precoder is designed neglecting this difference. For a comparison of the performance of the proposed robust design with that of the nonrobust design, first we consider the average SMSE in the presence of CSI errors. For this purpose, we consider a system with $M = 3$ transmit nodes communicating with $M = 3$ destination nodes, and a MIMO-relay with $N = 3$ Tx/Rx antennas. The average SMSE versus the SNR is compared for different values of $\sigma_E$, and $\sigma_F$, and the results are shown in Fig. 2. The SNR is defined as $\text{SNR} = P/(N\sigma_\mu^2)$. The results show that the proposed robust design outperforms the nonrobust design. It is found from the results that the difference between the performance of the robust and nonrobust designs increases with the SNR. This can be observed in (16), where the CSI error variances are amplified by the squared norm of the precoding matrix, which is proportional to the relay transmit power. Further, we compare the performance of nonrobust design and the proposed robust design in terms of average SMSE versus CSI error variance. For this comparison, we consider two system configurations, one with $N = M = 3$, SNR = 30 dB, and the other with $N = M = 4$, SNR = 20 dB. The results are shown in Fig. 3. In both the configurations, the robust design is found to perform better than the nonrobust design. At higher CSI error variances, the performance gain achieved by the robust design in terms of the average SMSE increases significantly. For example, the difference in the average SMSE between nonrobust and robust design for $\sigma_E = \sigma_F = 0.25$, is more than six times the difference for $\sigma_E = \sigma_F = 0.15$. The convergence behavior of the proposed design is shown in Fig. 4. We consider a setup with $M = 2$ source-destination pairs and a MIMO-relay with $N = 2$ Tx/Rx antennas. We have shown the convergence results for different values CSI error variances and different initialization methods of the algorithm. We have shown the results for initialization of the precoder matrix with a random matrix and a unit matrix. Results show that initialization with a random matrix results in faster convergence compared to initialization with a unit matrix. We can observe that the algorithm converges in less than 10 iterations when initialized with a random matrix, whereas it converges in around 13 iterations when initialized with a unit matrix. The performance of the robust MSE-balancing precoder design is shown in Fig. 5. For this study, we consider a system with $M = N = 3$. The maximum MSE among the destination nodes versus the SNR is obtained

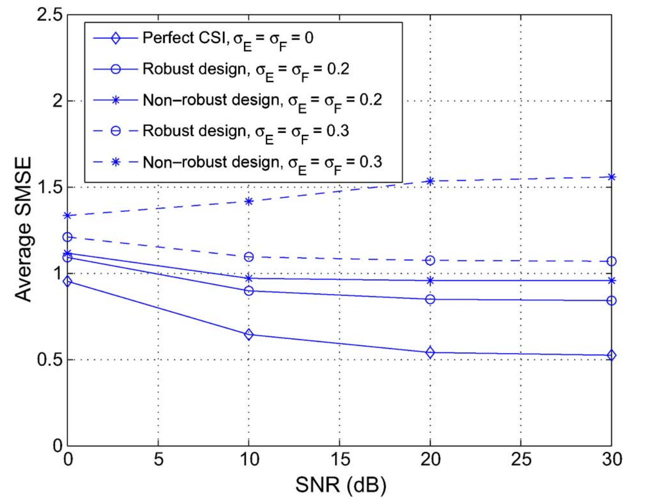


Fig. 2. Average SMSE versus relay SNR. $N = M = 4$. $\sigma_E = \sigma_F = 0$, 0.2, 0.3.

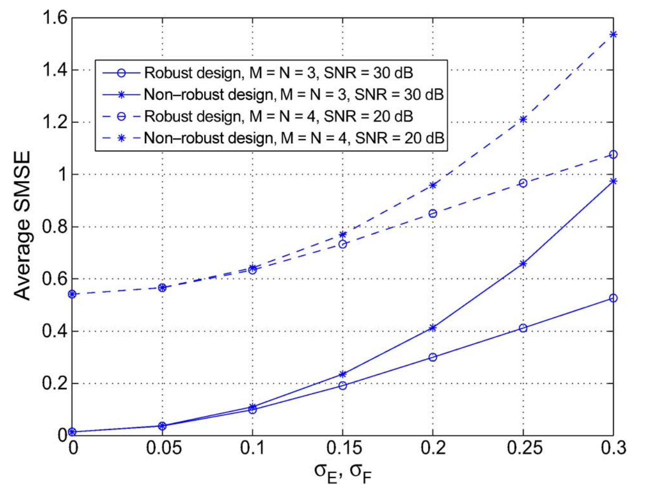


Fig. 3. Average SMSE versus CSI error variance. $N = M = 3, 4$, Relay SNR = 20 dB, 30 dB.

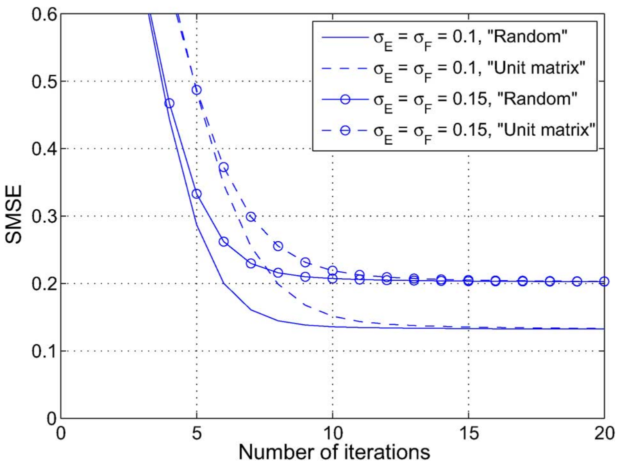


Fig. 4. Convergence behavior of the proposed robust design. $M = 2, N = 2$, Relay SNR $=$ 20 dB. "Random" implies initialization of the precoder matrix with a random matrix, and "Unit matrix" implies initialization with the unit matrix.

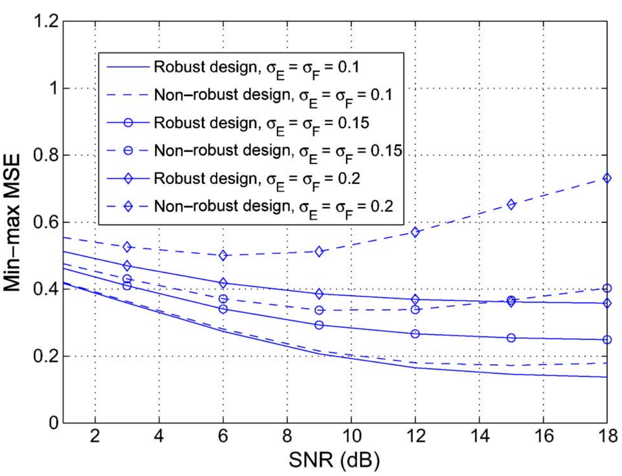


Fig. 5. Min-max MSE at the destination nodes versus relay SNR. $M = 3$, $N = 3$.

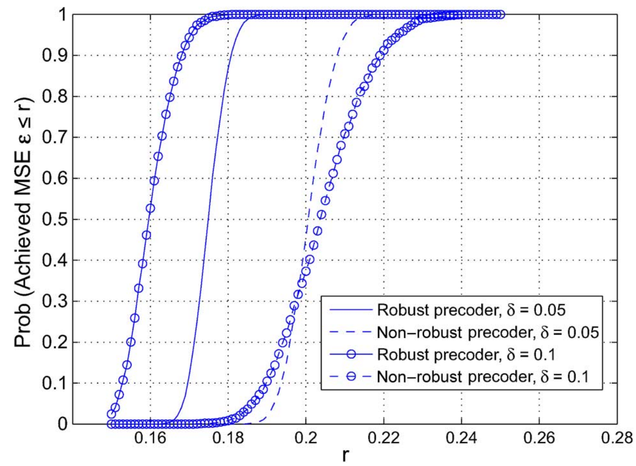


Fig. 6. Cumulative distribution of achieved MSE $\epsilon_i = \epsilon, 1 \leq i \leq M$. Target MSE $\eta = 0.2, \delta = 0.05, 0.1, M = N = 4, \sigma_\mu^2 = \sigma_\nu^2 = 0.1$.

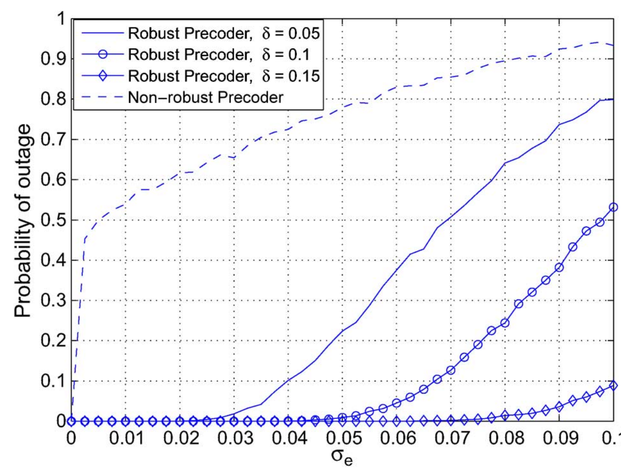


Fig. 7. Probability of outage $Pr\{\epsilon_i > \eta_i\}$ versus channel estimation error variance. Target MSE $\eta = 0.1, \delta = 0.05, 0.1, 0.15, M = N = 4, \sigma_\mu^2 = \sigma_\nu^2 = 0.1$.

for different values of CSI error variance. As CSI error variances increases, the performance improvement achieved by the robust design compared to the nonrobust design is found to increase.

Next, we present the performance of the robust designs proposed in Section IV for the norm-bound model of CSI error. We compare the performance of the proposed robust design with that of the nonrobust design. The nonrobust design of the precoder and receive filter are obtained by solving (44) using $\widehat{mmb}\alpha_k, \widehat{\boldsymbol{\beta}}_k, 1 \leq k \leq M$. In all the simulations for the NBE model, we have assumed $\eta_i = \eta, \delta_{\alpha_i} = \delta_{\beta_i} = \delta, 1 \leq i \leq M$. For a comparison of the performance of the proposed robust design with that of the nonrobust design, first we consider the cumulative distribution of achieved MSEs in the presence of CSI errors. For this purpose, we consider a system with $M = 4$ transmit nodes communicating with $M = 4$ destination nodes, and a MIMO-relay with $N = 4$ Tx/Rx antennas. The target MSE is set as 0.2 for all destination nodes. To estimate the cumulative distribution, we use $\boldsymbol{\phi}_k, \boldsymbol{\pi}_k, 1 \leq k \leq M$, satisfying the norm constraints. The results are shown in Fig. 6. The results show that the nonrobust design fails to meet the MSE target with higher probabilities for larger values of the CSI error bounds. The robust design results in MSE less than the target MSE even in the presence of CSI errors. We also evaluate the performance of the proposed design in the presence of CSI errors that are Gaussian distributed. For this study, we consider a system with $M = N = 4$, and the target MSE $\eta = 0.1$ for all destination nodes. The components of the CSI error vectors $\boldsymbol{\phi}_k$, and $\boldsymbol{\pi}_k$ are generated as independent and identically distributed complex Gaussian random variables with zero mean and variance $\sigma_e^2$. We compare the performance of the nonrobust design and robust design in terms of the probability of outage defined as $Pr\{\epsilon_i > \eta_i\}$ versus CSI estimation error variance. Probability of outage for the nonrobust design and the robust design with $\delta = 0.05, 0.1, 0.15$ are shown in Fig. 7. The probability of outage of the nonrobust design significantly increases with increase in the CSI error variance, whereas the robust design result in zero or very low outage depending on design value of $\delta$. For example, when $\sigma_e = 0.06$, the probability of outage is negligibly small for the robust precoder designed with $\delta = 0.15$, and it is 0.045 for $\delta = 0.1$, whereas it is 0.83 for the nonrobust

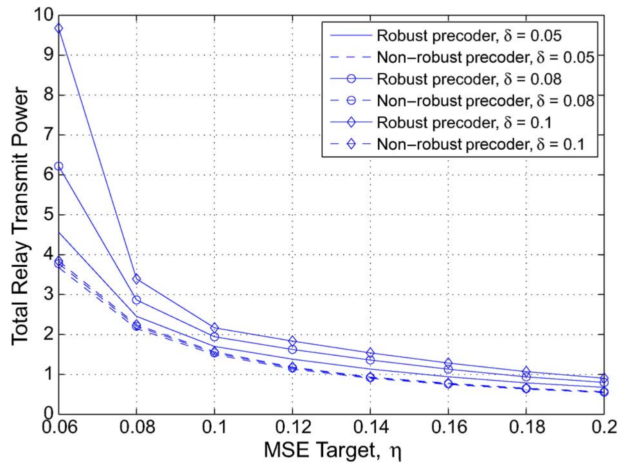


Fig. 8. Total relay transmit power $P$ versus maximum allowed MSEs at the destination relays. $\delta = 0.05, 0.1$, $M = N = 4$, $\sigma_\mu^2 = \sigma_\nu^2 = 0.1$.

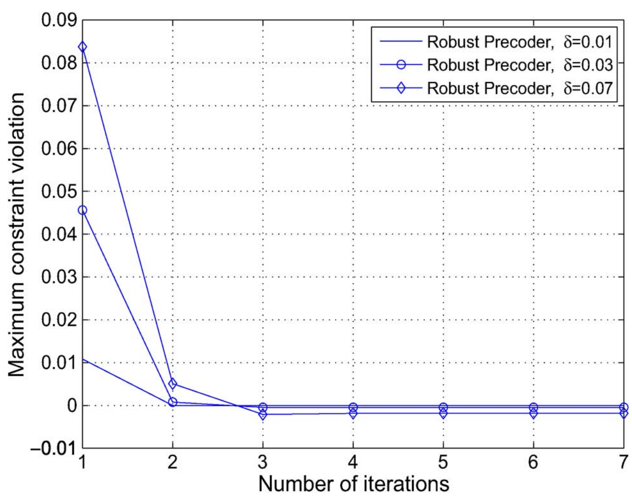


Fig. 9. Convergence of the proposed robust design. $M = N = 6$, $\sigma_\mu^2 = \sigma_\nu^2 = 0.1$, $\eta = 0.3$, $1 \leq i \leq M$, $\delta = 0.01, 0.03, 0.07$.

design. Further, we study the performance of the proposed design in terms of total relay transmit power versus MSE target for different values of CSI error bounds. For this purpose, we consider a set-up with system parameters set as $M = N = 4$, $\sigma_\mu^2 = \sigma_\nu^2 = 0.1$. The total relay transmit power resulting from the robust and the nonrobust designs in the presence of CSI errors is estimated through simulations. The results are shown in Fig. 8. The results show that the total relay transmit power required to achieve a given MSE target increases with increase in the CSI error norm bound. Comparing with the results in Fig. 6, we observe that this increase in transmit power is the price to pay for ensuring that the MSE constraints are satisfied in the presence of CSI errors. Finally, the convergence behavior of the proposed design is shown in Fig. 9. We consider a set-up with $M = 6$ source-destination pairs and a MIMO-relay with $N = 6$ Tx/Rx antennas. The target MSE is 0.3 at all the destination nodes. We have shown the convergence results for CSI error norms $\delta = 0.01$, 0.03, and 0.07. From the results, we can observe that the algorithm converges in less than four iterations of the minimization and worst case analysis steps.

## VI. Conclusion

We presented MIMO-relay precoder/receive filter designs that are robust to CSI errors following SE and NBE models. For the SE model of CSI errors, we presented robust designs based on minimum SMSE with a constraint on relay transmit power, MSE balancing with a constraint on relay transmit power, and minimization of total relay transmit power with MSE constraints at the destination nodes. For the NBE model of CSI errors, we presented a robust design based on relay transmit power minimization with MSE constraints at the destination nodes. We showed that these robust design problems can be formulated as convex optimization problems that can be solved efficiently. We presented simulation results that illustrate the improved performance of the proposed robust designs compared to the nonrobust designs in the presence of CSI imperfections at the MIMO-relay.

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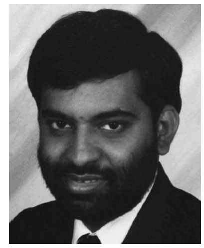

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