Random-Restart Reactive Tabu Search Algorithm for Detection in Large-MIMO Systems

Tanumay Datta, N. Srinidhi, A. Chockalingam, and B. Sundar Rajan

***Abstract*—We present a low-complexity algorithm based on reactive tabu search (RTS) for near maximum likelihood (ML) detection in large-MIMO systems. The conventional RTS algorithm achieves near-ML performance for 4-QAM in large-MIMO systems. But its performance for higher-order QAM is far from ML performance. Here, we propose a *random-restart RTS (R3TS)* algorithm which achieves significantly better bit error rate (BER) performance compared to that of the conventional RTS algorithm in higher-order QAM. The key idea is to run multiple tabu searches, each search starting with a random initial vector and choosing the best among the resulting solution vectors. A criterion to limit the number of searches is also proposed. Computer simulations show that the R3TS algorithm achieves almost the ML performance in 16×16 V-BLAST MIMO system with 16-QAM and 64-QAM at significantly less complexities than the sphere decoder. Also, in a 32×32 V-BLAST MIMO system, the R3TS performs close to ML lower bound within 1.6 dB for 16-QAM (128 bps/Hz), and within 2.4 dB for 64-QAM (192 bps/Hz) at 10^{-3} BER.**

***Index Terms*—Large-MIMO systems, maximum likelihood detection, reactive tabu search, random-restart, low-complexity detection.**

I. Introduction

CAPACITY of multiple-input multiple-output (MIMO) wireless channels is known to increase linearly with the minimum of the number of transmit and receive antennas [1]. Very high spectral efficiencies can be achieved if large number of antennas are employed. A key challenge in practically realizing large-MIMO systems with tens of antennas is the detection complexity at the receiver. Recently, large-MIMO systems have attracted increased research attention. This is because certain algorithms from machine learning/artificial intelligence have been shown recently to achieve near-optimal performance in large-MIMO systems with tens of antennas at low complexities [2]-[8]. In [2],[3], a low-complexity local neighborhood search algorithm, termed as likelihood ascent search (LAS) algorithm, for large-MIMO detection has been proposed for BPSK [2] and M-QAM [3]. In [4],[5], another local neighborhood search algorithm based on reactive tabu search (RTS) [9] has been proposed for large-MIMO systems with M-QAM, which achieved improved bit error rate (BER) performance. In [6], a Gibbs sampling based large-MIMO detection algorithm was presented for BPSK modulation. In [7], a factor graph based belief propagation (BP) algorithm that used a Gaussian approximation of the interference was reported for large-MIMO systems with BPSK modulation. In [8], the LAS algorithm in [2],[3] was extended using multiple searches to achieve better performance.

Manuscript received August 30, 2010. The associate editor coordinating the review of this letter and approving it for publication was H. Liu.

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Digital Object Identifier 10.1109/LCOMM.2010.101210.101587

All the above algorithms exhibit large-system behavior, where the BER performance improved and got increasingly closer to ML performance for increasing number of antennas. However, very close to ML performance has been reported only for BPSK and 4-QAM using these algorithms. For e.g., the RTS algorithm in [4],[5] achieved very close to ML performance at 10^{-3} BER with tens of antennas for 4-QAM (within 0.5 dB of unfaded SISO AWGN performance, which is a lower bound on ML performance). However, its performance was far from ML for higher-order QAM like 16-QAM and 64-QAM (e.g., about 8 dB away from ML lower bound at 7×10^{-4} BER for 16-QAM in 32×32 V-BLAST MIMO). The 16-QAM performance of the multiple search LAS algorithm in [8] is also quite far from the ML performance (about 12.5 dB away from ML lower bound at 7×10^{-4} BER for 16-QAM in 32×32 V-BLAST MIMO).

In this letter, we report a detection algorithm, termed as *random-restart RTS (R3TS)* algorithm, which achieves close to ML performance in large-MIMO systems with higher-order QAM. The key idea in R3TS is to run conventional RTS multiple times, each time with a random initial vector and choose the best among the resulting solution vectors. Computer simulations show that the R3TS algorithm achieves performance close to ML lower bound for 16×16, 32×32 and 64×64 V-BLAST MIMO systems with 16-QAM and 64-QAM at low complexities.

Consider a V-BLAST MIMO system with n_t transmit and n_r receive antennas, $n_r \geq n_t$. The transmitted symbols take values from a modulation alphabet $\mathbb{A}$. Let $\mathbf{x} \in \mathbb{A}^{n_t}$ denote the transmitted vector. Let $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix, whose entries are assumed to be i.i.d. Gaussian with zero mean and unit variance. The received vector $\mathbf{y}$ is given by

$$\mathbf{y} \;=\; \mathbf{Hx} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the noise vector whose entries are modeled as i.i.d. $\mathbb{C}\mathcal{N}(0, \sigma^2)$. We assume perfect channel estimation and synchronization at the receiver.

II. Conventional RTS Algorithm

In this section, we present a brief summary of the conventional RTS detection algorithm in [4],[5]. For a detailed description of the RTS algorithm, please refer [4],[5].

The conventional RTS algorithm starts with an initial solution vector, defines a neighborhood around it (i.e., defines a set of neighboring vectors based on a neighborhood criteria), and moves to the best vector among the neighboring vectors (even if the best neighboring vector is worse, in terms of ML cost $\|\mathbf{y} - \mathbf{Hx}\|^2$, than the current solution vector); this allows the algorithm to escape from local minima. This process is continued for a certain number of iterations, after which the algorithm is terminated and the best among the solution vectors in all the iterations is declared as the final solution vector. In defining the neighborhood of the solution vector in a given iteration, the algorithm attempts to avoid cycling by making the moves to solution vectors of the past few iterations as 'tabu' (i.e., prohibits these moves), which ensures efficient search of the solution space. The number of these past iterations is parametrized as the 'tabu period,' which is dynamically changed depending on the number of repetitions of the solution vectors that are observed in the search path (e.g., increase the tabu period if more repetitions are observed).

III. Proposed R3TS Algorithm

The following three parameters are defined to limit the number of RTS searches in the proposed R3TS algorithm: MAX, Θ, p. The R3TS algorithm is described below.

- ***Step 1:*** Choose a random initial vector. Run RTS algorithm using this initial vector and obtain the corresponding solution vector.
- ***Step 2:*** Check if MAX number of RTS searches have been done. If yes, go to Step 5; else go to Step 3.
- ***Step 3:*** If the ML cost of the solution vector from Step 1 is less than Θ, then output the solution vector from Step 1 as the final solution vector and stop; else go to Step 4.
- ***Step 4:*** Let K denote the number of searches done so far. Let L denote the number of distinct solution vectors from Step 1 so far. If $L/K \leq p$, go to Step 5; else go to Step 1.
- ***Step 5:*** Output the best (in terms of ML cost) among the solution vectors obtained so far and stop.

The choice of the value of Θ is made as follows. If the solution vector is same as the transmitted vector, then the ML cost is $\|\mathbf{n}\|^2$, which has a non-central chi-square distribution with mean $n_r\sigma^2$ and variance $n_r\sigma^4$. The Θ value is taken empirically to be $n_r\sigma^2 + 2\sqrt{n_r\sigma^4}$, i.e., the Θ value is taken to be the mean plus twice the standard deviation of the ML cost variable corresponding to error-free detection. The threshold comparison in Step 3 reduces the number of searches and hence the complexity. Also, the motivation to do Step 4 is to reduce complexity in realizations where $\|\mathbf{n}\|^2$ happens to be greater than Θ. We have used $p = 0.2$ and MAX $= 50$ in the simulations, which are found to result in good performance.

IV. Simulation Results

We evaluated the BER performance of the R3TS algorithm through simulations. In all the RTS and R3TS simulations, we employed the real-valued system model corresponding to (1), i.e., the system model $\mathbf{y}_r = \mathbf{H}_r\mathbf{x}_r + \mathbf{n}_r$, where

$$\mathbf{H}_r = \begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix}, \quad \mathbf{y}_r = \begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix},$$

$$\mathbf{x}_r = \begin{bmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{bmatrix}, \quad \mathbf{n}_r = \begin{bmatrix} \Re(\mathbf{n}) \\ \Im(\mathbf{n}) \end{bmatrix}. \tag{2}$$

Following RTS parameters are used in simulations: $max\_rep = 75$, $max\_iter = 300$, $\beta = 0.1$, $P_0 = 2$ for 4-QAM, $max\_rep = 250$, $max\_iter = 1000$, $\beta = 0.01$, $P_0 = 2$ for 16-QAM, and $max\_rep = 1000$, $max\_iter = 3000$, $\beta = 0.01$, $P_0 = 2$ for 64-QAM. MMSE initial vector is used in the conventional RTS.

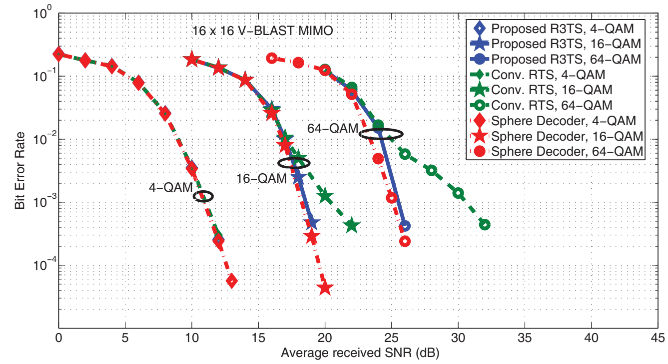

Fig. 1. BER performance of the proposed R3TS algorithm in comparison with those of conventional RTS algorithm and sphere decoder for 16×16 V-BLAST MIMO with 4-, 16- and 64 QAM.

In Figs. 1, 2, 3, we present the simulated BER performance of the proposed R3TS algorithm with 4-, 16- and 64-QAM for 16×16, 32×32 and 64×64 V-BLAST MIMO systems, respectively. In Fig. 1, the R3TS performance is compared with the performances of conventional RTS as well as sphere decoder (SD). We show the comparison of R3TS with SD only for 16×16 MIMO, and not for 32×32 and 64×64 MIMO, because of the prohibitively high complexity of SD in such large dimensions. So, for 32×32 and 64×64 MIMO, in Figs. 2 and 3, we compare the R3TS performance with unfaded SISO AWGN performance, which is a lower bound on the ML performance.

From Fig. 1, we see that the proposed R3TS algorithm performs almost the same as SD in 16×16 MIMO for all the modulations considered (4-, 16-, 64-QAM). The R3TS algorithm achieves this excellent performance (i.e., almost SD performance) at a much lesser complexity than that of SD; in our simulations we found that the SD complexity in average number of real operations for 16×16 MIMO with 16-QAM and 64-QAM to be about one to two orders more than that of R3TS at 10^{-2} BER. Also, the performance improvement achieved by R3TS compared to conventional RTS is quite significant for 16- and 64-QAM (e.g., for 64-QAM in 16×16 MIMO, R3TS outperforms conventional RTS performance by about 5 dB at 10^{-3} BER). From Figs. 2 and 3, we see that R3TS performs very well in 32×32 and 64×64 MIMO as well.

In Table I, we present a performance (in terms of SNR required to achieve 10^{-2} BER) and complexity (in terms of number of real operations at 10^{-2} BER) comparison between R3TS, conventional RTS, and a low-complexity variant of SD, namely fixed complexity sphere decoder (FSD) in [10] for 16×16, 32×32, 64×64 MIMO with 16- and 64-QAM. It is seen that, at 10^{-2} BER, R3TS outperforms conventional RTS by about 5.3 dB in 32×32 MIMO with 64-QAM, and by about 6.6 dB in 64×64 MIMO with 64-QAM, at additional complexities incurred due to multiple restarts. In [10], FSD was shown to achieve almost the SD performance for 4×4 MIMO with 4-, 16- and 64-QAM, and for 8×8 MIMO with 4- and 16-QAM, at lower complexities compared to SD. The FSD algorithm can result in sub-optimum performance because of

TABLE I

COMPLEXITY AND PERFORMANCE COMPARISON OF THE R3TS ALGORITHM WITH OTHER ALGORITHMS FOR 16×16, 32×32, 64×64 MIMO WITH 16-QAM AND 64-QAM. $\star$: THESE POINTS ARE NOT SIMULATED DUE TO THE PROHIBITIVELY HIGH COMPLEXITY OF FSD IN SUCH LARGE n_t AND M.

Modulation	Algorithm	Complexity in average number of real operations $\times 10^6$ and SNR required to achieve 10^{-2} BER					
		16×16 MIMO		32×32 MIMO		64×64 MIMO	
		Complexity	SNR	Complexity	SNR	Complexity	SNR
16-QAM	RTS	3.780112	17.1 dB	6.014432	17.9 dB	12.539648	19 dB
	R3TS	3.968	17 dB	7.40464	17 dB	37.750656	16.6 dB
	FSD in [10]	4.836432	17.6 dB	4599.531168	17.8 dB	$\star$	$\star$
64-QAM	RTS	23.7264	25 dB	27.635104	29.4 dB	32.863872	32 dB
	R3TS	25.429504	24.2 dB	77.08784	24.1 dB	467.373248	25.4 dB
	FSD in [10]	305.7204	24.3 dB	$\star$	$\star$	$\star$	$\star$

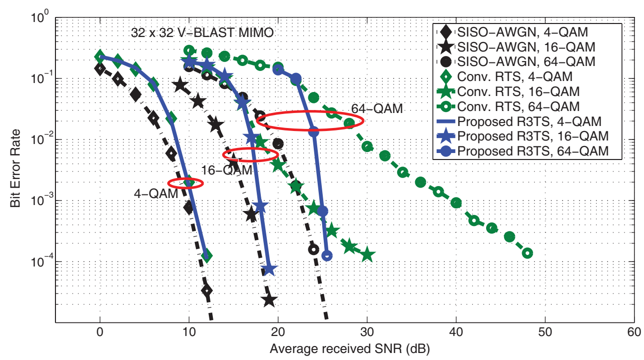

Fig. 2. BER performance of R3TS in comparison with those of conventional RTS and ML lower bound (i.e., unfaded SISO AWGN performance) for 32×32 V-BLAST MIMO with 4-, 16- and 64 QAM.

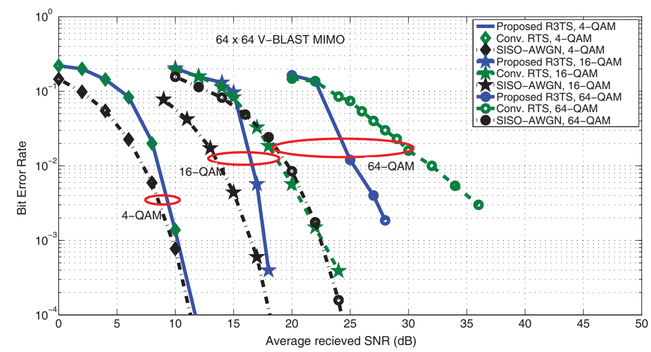

Fig. 3. BER performance of R3TS in comparison with those of conventional RTS and ML lower bound for 64×64 V-BLAST MIMO with 4-, 16- and 64 QAM.

restricting its search in order to keep the complexity fixed irrespective of the SNR. In Table I, we see that the FSD does not scale well in complexity for large n_t and M (e.g., for $n_t = 32, 64$ and $M = 16, 64$), whereas R3TS scales well in complexity as well as achieves very good performance for these large n_t and M. Finally, in Fig. 4, we see that, while the performance of detectors including semi-definite relaxation (SDR) in [11] and Gaussian tree approximation (GTA) in [12] are far from the SD performance, R3TS achieves almost SD performance.

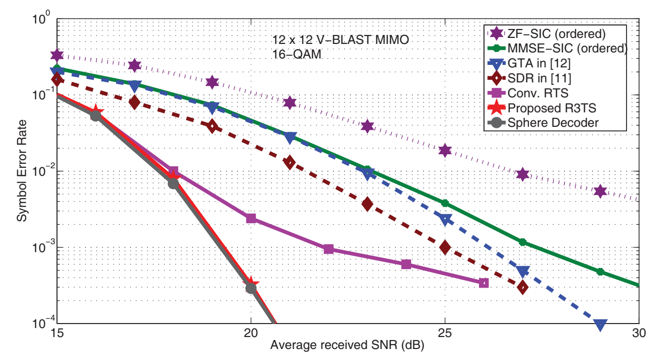

Fig. 4. SER performance comparison of R3TS algorithm with other detectors for 12×12 V-BLAST MIMO with 16-QAM.

V. CONCLUSIONS

We proposed a low-complexity random-restart reactive tabu search (R3TS) algorithm which achieved close to ML performance for large-MIMO systems with higher-order QAM. The achieved performance of the proposed R3TS algorithm is quite attractive in large-MIMO systems like 16×16, 32×32, 64×64 MIMO, with higher order QAM including 16- and 64-QAM.

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