On Generalized Spatial Modulation

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***Abstract*— Generalized spatial modulation (GSM) is a relatively new modulation scheme for multi-antenna wireless communications. It is quite attractive because of its ability to work with less number of transmit RF chains compared to traditional spatial multiplexing (V-BLAST system). In this paper, we show that, by using an optimum combination of number of transmit antennas (N_t) and number of transmit RF chains (N_{rf}), GSM can achieve better throughput and/or bit error rate (BER) than spatial multiplexing. First, we quantify the percentage savings in the number of transmit RF chains as well as the percentage increase in the rate achieved in GSM compared to spatial multiplexing; 18.75% savings in number of RF chains and 9.375% increase in rate are possible with 16 transmit antennas and 4-QAM modulation. A bottleneck, however, is the complexity of maximum-likelihood (ML) detection of GSM signals, particularly in large MIMO systems where the number of antennas is large. We address this detection complexity issue next. Specifically, we propose a Gibbs sampling based algorithm suited to detect GSM signals. The proposed algorithm yields impressive BER performance and complexity results. For the same spectral efficiency and number of transmit RF chains, GSM with the proposed detection algorithm achieves better performance than spatial multiplexing with ML detection.**

Keywords – *Generalized spatial modulation, RF chain savings, detection, Gibbs sampling.*

I. INTRODUCTION

Multi-antenna wireless communication systems have become popular due to their advantages of high throughput and enhanced reliability. Practical multi-antenna systems are faced with the problem of maintaining multiple radio frequency (RF) chains at the transmitter and receiver, and the associated cost, hardware complexity and inter-antenna synchronization issues [1]. Multiple RF chains are, in general, expensive. Spatial modulation (SM) is a relatively new modulation technique which can resolve these issues by using only one transmit RF chain and choosing one antenna element in a multiple transmit antenna array and sending the information symbol through the chosen antenna [2],[3]. The SM schemes proposed in [3]-[5] use the index of active antenna in the array as well as the symbol transmitted by that antenna to convey the information bits. That is, $\log_2 N_t$ bits and $\log_2 M$ bits are simultaneously carried in the antenna index and the M-ary modulation symbol, respectively, where N_t is the total number of transmit antenna elements. Therefore, the spectral efficiency of SM is given by is $(\log_2 N_t + \log_2 M)$ bpcu. The detector at the receiver in SM has to identify the transmit antenna index through which the symbol was transmitted jointly with the symbol. SM enjoys the advantages of complete removal of inter-channel interference and reduced hardware requirement.

A drawback in SM is its low spectral efficiency. A recently proposed extension (or generalization) of SM, termed *generalized spatial modulation (GSM)*, allows multiple transmit antennas to be active simultaneously [6]-[8]. By choosing a combination of total number of transmit antenna elements and number of transmit RF chains, GSM can achieve higher spectral efficiencies than SM. We present the GSM system model in Section II. We present the maximum achievable rate and transmit RF chain saving in GSM compared to spatial multiplexing, by selecting optimum combination of total number of transmit antenna elements (N_t) and number of transmit RF chains (N_{rf}). We present the optimum (N_t, N_{rf}) combination, and quantify the percentage savings in the number of transmit RF chains as well as the percentage increase in the rate achieved in GSM compared to spatial multiplexing – about 18.75% savings in number of RF chains and 9.375% increase in rate are possible with 4-QAM. The achievable rate and RF chain saving results are presented in Section III.

A challenge that arises in GSM with high rate is in the detection of the transmitted signal. For large number of transmit antenna elements, maximum-likelihood (ML) detection becomes computationally infeasible. Hence, there is a need for low complexity detection schemes with good performance for GSM with large number of transmit antenna elements. Low-complexity detection strategies used in the MIMO detection literature can not be directly used in GSM signal detection due to the special nature of GSM transmit vector having a fixed number of non-zero elements. Detection problem in sparse multiuser scenario is similar in nature to this problem, as, in both cases, the receiver has to detect a signal vector that has some zero entries [9]. But the difference lies in the fact that in sparse multiuser case the number of non-zero elements is variable and unknown to the receiver. Hence the detector tries to find a solution vector as sparse as possible, and thus the scaled zeroth norm of the test vector is added to the cost function to be minimized. But in GSM all possible transmitted vectors have same level of sparsity which makes the detection strategies used in sparse multiuser detection not directly applicable in GSM. To address this issue, we propose a low complexity detection scheme based on Gibbs sampling approach suited for GSM signal detection. The proposed Gibbs sampling based GSM detection algorithm, its BER performance and complexity results are presented in Section IV. For the same spectral efficiency and number of transmit RF chains, GSM with the proposed detection algorithm performs better than spatial multiplexing with ML detection.

Notations: Bold lowercase and uppercase letters denote column vectors and matrices, respectively. For a vector $\mathbf{r}$, r_j denotes its jth coordinate. The entry in ith row and jth column is of a matrix $\mathbf{R}$ is denoted by $r_{i,j}$. $\|\mathbf{r}\|_p$ denotes the p-norm of vector $\mathbf{r}$. $\binom{\cdot}{\cdot}$ denotes the binomial coefficient. $\lfloor x \rfloor$

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denotes the largest integer less than equal to x. $|\mathbb{A}|$ denotes the cardinality of set $\mathbb{A}$. $[\mathbf{x}]_{\mathbb{A}}$ denotes the element-wise quantization of $\mathbf{x}$ to its nearest point in $\mathbb{A}$. $(.)^H$ and $(.)^T$ denote Hermitian and transpose operations, respectively.

II. System Model

Consider a multi-antenna system with N_t transmit antennas and N_r receive antennas. The transmitter has N_{rf} RF chains and a $N_{rf} \times N_t$ switch that connects the RF chains to the transmitting antennas. Let us denote the $N_t \times 1$-sized transmitted vector as $\mathbf{x}$. N_{rf} active transmit antennas are chosen among the N_t antennas, and information symbols are loaded on these chosen antennas and the other $N_t - N_{rf}$ antennas remain silent. Therefore, $\|\mathbf{x}\|_0 = N_{rf}$. Let $k \triangleq \lfloor \log_2 \binom{N_t}{N_{rf}} \rfloor$ and $K \triangleq 2^k$. Let $\mathbb{A}$ denote the modulation alphabet from which information symbols are chosen, and $M \triangleq |\mathbb{A}|$ and $m \triangleq \log_2 M$. Let us denote the ath element in alphabet $\mathbb{A}$ as $\mathbb{A}^a$. Average symbol energy for this alphabet is denoted by E_s. We consider standard alphabets like PAM, QAM, PSK, which do not include zero, in order to avoid the confusion between an antenna transmitting an information symbol zero and it being inactive (i.e., remaining silent). We denote $\mathbb{A}_0 = \mathbb{A} \cup 0$. In this GSM system, we can transmit $k + mN_{rf}$ information bits per channel use (bpcu). Let $R \triangleq k + mN_{rf}$. Out of the total R information bits, k bits are used as control bits in the switch to choose a particular combination of N_{rf} active antennas out of N_t available ones. The remaining mN_{rf} bits are divided into N_{rf} groups, where each group of m bits is converted into a symbol from alphabet $\mathbb{A}$. Note that $K < \binom{N_t}{N_{rf}}$, and so, in general, not all antenna activity patterns are valid. Let the set of valid antenna activity patterns with cardinality K be denoted by $\mathbb{S}$. For a transmit vector $\mathbf{x}$, let the corresponding antenna activity pattern vector be denoted by $\mathbf{t}^{\mathbf{x}}$, where $t_j^{\mathbf{x}} = 1, \;$ iff $\; x_j \neq 0 \;, \forall j = 1, 2, \cdots, N_t$. $\mathbb{S}$ and mapping between elements of $\mathbb{S}$ and k-length control bit sequences are known at both transmitter and receiver.

As an example, consider $N_t = 4, N_{rf} = 2$, and 4-QAM alphabet. Here, $k = 2$ and $R = 6$. We take $\mathbb{S} = \{[1,1,0,0]^T, [1,0,1,0]^T, [0,1,0,1]^T, [0,0,1,1]^T\}$[1]. For an information bit sequence $[0,1,0,0,1,1]$, the first 2 bits are used to choose the activity pattern, and the two 4-QAM symbols are generated by combining the third & fourth bits and fifth & sixth bits, respectively. Using the usual Gray mapping the transmitted vector becomes $[1 + \sqrt{-1}, 0, -1 - \sqrt{-1}, 0]^T$.

The $N_r \times N_t$ channel matrix is denoted by $\mathbf{H}$. We assume rich scattering environment. Hence, the entries of $\mathbf{H}$ are modeled as circularly symmetric complex Gaussian with zero mean and unit variance. $h_{i,j}$ denotes the channel gain from jth transmit antenna to ith receive antenna. Let us denote the $N_r \times 1$-sized received vector as $\mathbf{y}$, which is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

[1] Here, we could have used any 4 out of $\binom{4}{2} = 6$ possible activity patterns.

where $\mathbf{n}$ is the $N_r \times 1$-sized additive white Gaussian noise vector at the receiver. We assume $n_i \sim \mathcal{CN}(0, \sigma^2)$, $\forall i = 1, 2, \cdots, N_r$. The average received SNR is $10 \log_{10} \frac{N_{rf} E_s}{\sigma^2}$. We assume that the receiver has perfect knowledge of $\mathbf{H}$ and σ^2. The ML detection output can be written as

$$\mathbf{x}_{ML} = \arg \min_{\hat{\mathbf{x}} \in \mathbb{U}} \|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2, \tag{2}$$

where $\mathbb{U} = \{\mathbf{x} | \mathbf{x} \in {\mathbb{A}_0}^{N_t \times 1}, \|\mathbf{x}\|_0 = N_{rf}, \mathbf{t}^{\mathbf{x}} \in \mathbb{S}\}$ is the set of all possible transmitted vectors. Note that, $|\mathbb{U}| = 2^R$. For small N_t and N_{rf}, the set $\mathbb{U}$ may be completely tabulated and ML detection is possible. But for large N_t and N_{rf}, this brute force computation of $\mathbf{x}_{ML}$ in (2) becomes computationally infeasible.

III. Achievable rates in GSM

The rate R (bpcu) achieved in GSM with N_t transmit antennas, N_{rf} transmit RF chains, and M-QAM is given by

$$R = \left\lfloor \log_2 \left(\frac{N_t!}{N_{rf}!(N_t - N_{rf})!} \right) \right\rfloor + mN_{rf}, \tag{3}$$

where the first and second terms on the RHS correspond to number of control bits and number of modulation bits, respectively. Let us examine how R varies as a function of its variables. Figure 1 shows the variation of R as a function of N_{rf} for different values of $N_t = 4, 8, 12, 16, 22, 32$ and 4-QAM. The figure shows that for each N_t, there is an optimum N_{rf} that maximizes the achievable rate R. Note that the last x-axis value for each plot corresponds to $N_{rf} = N_t$. Also note that the traditional spatial multiplexing MIMO system (i.e., V-BLAST system) operates with $N_{rf} = N_t$, i.e., with equal number of transmit antennas and transmit RF chains. It is interesting to see that the maximum R does not occur at $N_{rf} = N_t$, but at some $N_{rf} < N_t$ for $N_t \geq 8$. This brings the following interesting observations to the fore: i) by choosing the optimum (N_t, N_{rf}) combination (i.e., using less RF chains than transmit antennas, $N_{rf} < N_t$), GSM can achieve a higher rate than that of traditional spatial multiplexing MIMO (i.e., V-BLAST system with $N_{rf} = N_t$), and ii) one can operate GSM at the same rate as that of spatial multiplexing MIMO but with even lesser number of RF chains. This reason for this behavior can be explained as follows. The first term in (3) increases from $N_{rf} = 0$ to $N_{rf} = \lfloor \frac{N_t}{2} \rfloor$ and decreases then onwards, whereas the second term increases linearly with N_{rf}. Hence, intuitively the optimum value of N_{rf} for which R attains maximum should be somewhere between $\lfloor \frac{N_t}{2} \rfloor$ and N_t, and it should achieve $R = mN_t$ at a lesser value than this optimum. Note that, the first term in (3) is independent of m and the second term is directly proportional to it. Hence, as we increase m, the second term will dominate and the maximum rate will shift towards $N_{rf} = N_t$, thus reducing the advantages of GSM. Let the value of N_{rf} for which GSM achieves maximum rate be defined as N_{rf}^{opt} and the value of N_{rf} for which GSM achieves $R = mN_t$ be defined as N_{rf}^{mid}. In Table I, we show the percentage saving in

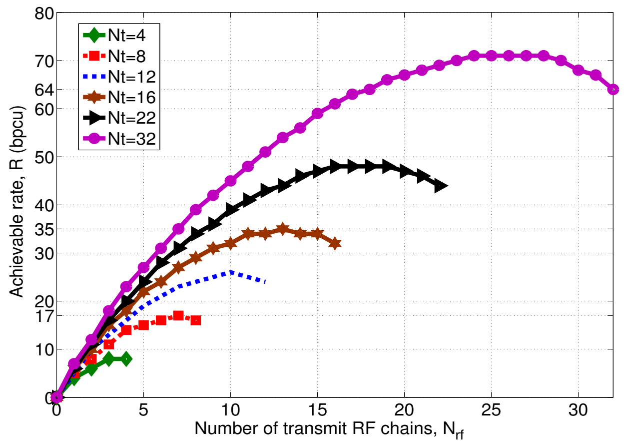

Fig. 1. Achievable rate R using GSM as a function of N_{rf} for different values of N_t and 4-QAM.

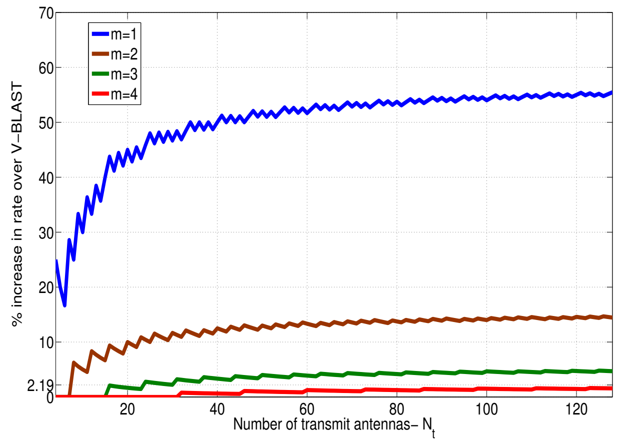

Fig. 2. Variation of percentage increase in rate compared to V-BLAST system as a function of N_t for different values of $m = 1, 2, 3, 4$.

m	% saving in # RF chains for $R = mN_t$		% increase in rate for $R = R_{opt}$		% saving in # RF chains for $R = R_{opt}$	
	N_t=16	N_t=32	N_t=16	N_t=32	N_t=16	N_t=32
1	68.75	71.88	43.75	46.88	31.25	40.63
2	37.5	43.75	9.385	10.94	18.75	25
3	18.75	21.88	2.08	3.13	6.25	12.5
4	6.25	9.38	0	.78	6.25	3.13

TABLE I
% SAVING IN RF CHAINS AND % INCREASE IN RATE FOR $m = 1, 2, 3, 4$, $N_t = 16$ AND 32.

RF chains and percentage increase in rate for $m = 1, 2, 3, 4$, $N_t = 16$ and 32.

In Figs. 2 and 3, we show the variation in rate gains and RF savings with increasing number of transmit antennas. In Fig. 2, we see that the percentage increase in rate over V-BLAST system shows an increasing a trend with increasing N_t. The zigzag nature of the curves is due to the flooring operation in the first term in (3). Similarly, in Fig. 3, we observe the variation in percentage saving in RF chains to achieve $R = R_{opt}$ and $R = mN_t$ with increasing N_t. From Table-I and Figs. 2 and 3, we note that the gains from GSM become less prominent as m increases, which is expected. With $m = 1$, i.e., BPSK modulation, one does not effectively use the complex plane, resulting in good potential for increased throughput in GSM.

In the next section, we present a low complexity detection algorithm for GSM. The BER results we present in the next section using this algorithm is for $m = 2$ (4-QAM), where GSM has good gains in rate and RF saving. The algorithm, however, is valid for any m.

IV. Proposed Gibbs Sampling Based Algorithm for GSM Detection

The GSM detection algorithm presented in this section is based on Gibbs sampling based approach, where a Markov chain is formed with all possible transmitted vectors as states. As the total number of non-zero entries in the solution vector has

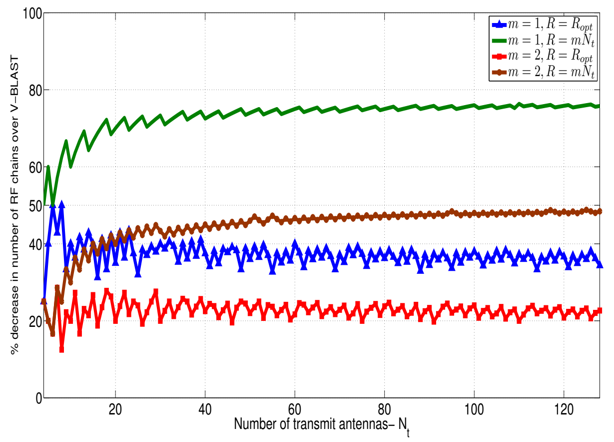

Fig. 3. Variation of percentage saving in RF chains compared to V-BLAST system as a function of N_t to achieve $R = R_{opt}$ and $R = mN_t$, for $m = 1, 2$.

to equal to N_{rf}, one can not sample each coordinate individually as in the case of conventional MIMO detection. To solve this problem, we propose the following approach: *sample two coordinates at a time jointly, keeping other* $(N_t - 2)$ *coordinates fixed which contain* $(N_{rf} - 1)$ *non-zero entries.* For any vector $\mathbf{x}^{(t)} \in \mathbb{A}_0^{N_t}, \|\mathbf{x}^{(t)}\|_0 = N_{rf}$, where the t in the superscript of $\mathbf{x}^{(t)}$ refers to the iteration index in the algorithm. Let $i_1, i_2, \cdots, i_{N_{rf}}$ denote the locations of non-zero entries and $j_1, j_2, \cdots, j_{(N_t - N_{rf})}$ denote the locations of zero entries in $\mathbf{x}^{(t)}$. We will sample $x_{i_l}^{(t)}$ and $x_{j_k}^{(t)}$ jointly, keeping other coordinates fixed, where $l = 1, 2, \cdots, N_{rf}$ and $k = 1, 2, \cdots, (N_t - N_{rf})$. As any possible transmitted vector can have only N_{rf} non-zero entries, the next possible state $\mathbf{x}^{(t+1)}$ can only be any one of the following $2|\mathbb{A}|$ candidate vectors denoted by $\{\mathbf{z}^w, w = 1, 2, \cdots, 2|\mathbb{A}|\}$, which can be partitioned into two sets. In the first set corresponding to $w = 1, 2, \cdots, |\mathbb{A}|$, we enlist the vectors which has the same activity pattern as $\mathbf{x}^{(t)}$. Hence, $z_{i_l}^w = \mathbb{A}^w, z_{j_k}^w = 0, z_q = x_q^{(t)}, q = 1, 2, \cdots, N_t, q \neq i_j, j_k, \forall w = 1, 2, \cdots, |\mathbb{A}|$. For $w = |\mathbb{A}| + 1, |\mathbb{A}| + 2, \cdots, 2|\mathbb{A}|$, we enlist the vectors whose activity pattern differs from that of $\mathbf{x}^{(t)}$ in locations j_k and i_l. Hence, $z_{j_k}^w = \mathbb{A}^{(w-|\mathbb{A}|)}, z_{i_l}^w = 0, z_q = x_q^{(t)}, q = 1, 2, \cdots, N_t, q \neq$

$i_j, j_k, \forall w = |\mathbb{A}|+1, |\mathbb{A}|+2, \cdots, 2|\mathbb{A}|$.

To simplify the sampling process, we calculate the best vectors from the two sets corresponding to not swapping and swapping the zero and non-zero locations, and choose among these two vectors. Let $\mathbf{x}^{NS}$ denote the best vector from the first set corresponding to no swap. We set $\mathbf{x}^{NS} = \mathbf{x}^{(t)} + \lambda \mathbf{e}_{i_l}$ and minimize $\|\mathbf{y} - \mathbf{H}\mathbf{x}^{NS}\|^2$ over λ. For this, we have

$$\begin{aligned} \|\mathbf{y} - \mathbf{H}\mathbf{x}^{NS}\|^2 &= \|\mathbf{y} - \mathbf{H}(\mathbf{x}^{(t)} + \lambda \mathbf{e}_{i_l})\|^2 \\ = \mathbf{y}^H\mathbf{y} - 2\Re\left(\mathbf{y}^{MF}\mathbf{x}^{(t)}\right) &+ \mathbf{x}^{(t)^H}\mathbf{R}\mathbf{x}^{(t)} \\ - 2\Re\left(\lambda \mathbf{y}^{MF}\mathbf{e}_{i_l}\right) + 2\Re\left(\lambda \mathbf{x}^{(t)^H}\mathbf{R}\mathbf{e}_{i_l}\right) &+ |\lambda|^2 R_{i_l,i_l}, \end{aligned} \quad (4)$$

where $\mathbf{y}^{MF} = \mathbf{y}^H\mathbf{H}$ and $\mathbf{R} = \mathbf{H}^H\mathbf{H}$. Differentiating (4) w.r.t λ and equating it to zero, we get

$$\lambda_{opt} = \frac{\left(y_{i_l}^{MF} - \mathbf{x}^{(t)^H}\mathbf{r}_{i_l}\right)^H}{R_{i_l,i_l}}, \quad (5)$$

where $\mathbf{r}_{i_l}$ is the i_lth column vector of $\mathbf{R}$. We obtain $\mathbf{x}^{NS} = [\mathbf{x}^{(t)} + \lambda_{opt}\mathbf{e}_{i_l}]_{\mathbb{A}}$. Similarly we obtain $\mathbf{x}^S$, the best vector from the second set corresponding to swap. The next state $\mathbf{x}^{(t+1)}$ is chosen between $\mathbf{x}^S$ and $\mathbf{x}^{NS}$ with probability p^S and p^{NS}, respectively, where $p^S = \alpha \widetilde{p}^S + \frac{1-\alpha}{2}$, $p^{NS} = 1 - p^S$, and

$$\widetilde{p}^S = \frac{\exp(-\frac{\|\mathbf{y}-\mathbf{H}\mathbf{x}^S\|^2 - \|\mathbf{y}-\mathbf{H}\mathbf{x}^{NS}\|^2}{\sigma^2})}{1 + \exp(-\frac{\|\mathbf{y}-\mathbf{H}\mathbf{x}^S\|^2 - \|\mathbf{y}-\mathbf{H}\mathbf{x}^{NS}\|^2}{\sigma^2})}. \quad (6)$$

Here, α gives the probability of mixing between Gibbs sampling and sampling from uniform distribution. We use $\alpha = 1 - \frac{1}{N_t}$, which is found to give very good performance. After sampling, the best vector obtained so far is updated. The above sampling process is repeated for all l and k. The algorithm is stopped after it meets the stopping criterion or reaches the maximum number of allowable iterations, and outputs the best vector in terms of ML cost obtained so far. Let us denote the best vector as $\mathbf{z}$. The stopping criterion works as follows: we compute a metric $\Theta_s(\mathbf{z}) = \left\lceil \max\left(c_{min}, c_1 \exp(\phi(\mathbf{z}))\right)\right\rceil$, where $\phi(\mathbf{z}) = \frac{\|\mathbf{y}-\mathbf{H}\hat{\mathbf{x}}\|^2 - N_r\sigma^2}{\sqrt{N_r}\sigma^2}$ is the normalized ML cost of $\mathbf{z}$. If $\mathbf{z}$ has not changed for $\phi(\mathbf{z})$ iterations, the iterations are stopped. This concludes one restart and $\mathbf{z}$ is declared as the output of this restart. Now, we check whether $\mathbf{t}^{\mathbf{x}}$ belongs to $\mathbb{S}$ or not to check its validity. Multiple such runs starting from different initial vectors are done till the best valid output obtained so far is satisfactory in terms of ML cost. Let us denote the best vector among restarts as $\mathbf{s}$ and the number of restarts that has given $\mathbf{s}$ as output as r_s. We calculate another metric $\Theta_r(\mathbf{s}) = \lfloor \max(0, c_2\phi(\mathbf{s}))\rfloor + 1$ and compare r_s with this. If r_s is equal to $\Theta_r(\mathbf{s})$ or maximum number of restarts have been reached, we terminate the algorithm. The pseudo-code of the proposed algorithm is given in **Algorithm 1**.

Algorithm 1 Proposed Gibbs sampling based algorithm for GSM detection

1: **input:** $\mathbf{y}$, $\mathbf{H}$, N_t, N_{rf}; MAX-ITR: max. # iterations; MAX-RST: max. # restarts;
2: $r = 0$, $\kappa = 10^{10}$, $\alpha = 1 - \frac{1}{N_t}$;
3: Compute $\mathbf{y}^{MF} = \mathbf{y}^H\mathbf{H}$ and $\mathbf{R} = \mathbf{H}^H\mathbf{H}$;
4: $\phi(.)$: ML cost fn; $\Theta_s(.)$: *stopping criterion* fn; $\Theta_r(.)$: *restart criterion* fn;
5: **while** $r <$ MAX-RST **do**
6: $\quad$ $\mathbf{x}^{(0)}$: initial vector $\in \mathbb{A}_0{}^{N_t \times 1}$; $\|\mathbf{x}^{(0)}\|_0 = N_{rf}$;
7: $\quad$ $\beta = \phi(\mathbf{x}^{(0)})$; $\mathbf{z} = \mathbf{x}^{(0)}$; $t = 0$;
8: $\quad$ **while** $t <$ MAX-ITR **do**
9: $\quad\quad$ **for** $l = 1$ to N_{rf} **do**
10: $\quad\quad\quad$ **for** $k = 1$ to $N_t - N_{rf}$ **do**
11: $\quad\quad\quad\quad$ find i_l and j_k indices;
12: $\quad\quad\quad\quad$ Compute λ_{opt} from (5); compute $\mathbf{x}^{NS} = [\mathbf{x}^{(t)} + \lambda_{opt}\mathbf{e}_{i_l}]_{\mathbb{A}}$;
13: $\quad\quad\quad\quad$ set $\mathbf{x}^{temp} = \mathbf{x}^{(t)}$; $\mathbf{x}_{i_l}^{temp} = 0$;
14: $\quad\quad\quad\quad$ $\mathbf{x}^S = [\mathbf{x}^{(t)} + \frac{\left(y_{j_k}^{MF} - \mathbf{x}^{temp^H}\mathbf{r}_{j_k}\right)^H}{R_{j_k,j_k}}\mathbf{e}_{j_k}]_{\mathbb{A}}$
15: $\quad\quad\quad\quad$ Compute $\widetilde{p}^S$ from (6);
16: $\quad\quad\quad\quad$ Compute $p^S = \alpha\widetilde{p}^S + \frac{1-\alpha}{2}$, $p^{NS} = 1 - p^S$;
17: $\quad\quad\quad\quad$ Choose $\mathbf{x}^{(t+1)}$ between $\mathbf{x}^S$ and $\mathbf{x}^{NS}$ with probability p^S & p^{NS};
18: $\quad\quad\quad\quad$ $\gamma = \phi(\mathbf{x}^{(t+1)})$;
19: $\quad\quad\quad\quad$ **if** $(\gamma \leq \beta)$ **then**
20: $\quad\quad\quad\quad\quad$ $\mathbf{z} = \mathbf{x}^{(t+1)}$; $\beta = \gamma$; calculate $\Theta_s(\mathbf{z})$;
21: $\quad\quad\quad\quad$ **end if**
22: $\quad\quad\quad\quad$ $t = t + 1$;
23: $\quad\quad\quad\quad$ $\beta_v^{(t)} = \beta$;
24: $\quad\quad\quad$ **end for**
25: $\quad\quad$ **end for**
26: $\quad\quad$ **if** $\Theta_s(\mathbf{z}) < t$ **then**
27: $\quad\quad\quad$ **if** $\beta_v^{(t)} == \beta_v^{(t-\Theta_s(\mathbf{z}))}$ **then**
28: $\quad\quad\quad\quad$ goto step 32
29: $\quad\quad\quad$ **end if**
30: $\quad\quad$ **end if**
31: $\quad$ **end while**
32: $\quad$ $r = r + 1$;
33: $\quad$ **if** $\mathbf{t}^{\mathbf{z}} \in \mathbb{S}$ **then**
34: $\quad\quad$ **if** $\beta < \kappa$ **then**
35: $\quad\quad\quad$ $\kappa = \beta$; $r_s = 1$; $\mathbf{s} = \mathbf{z}$; Compute $\Theta_r(\mathbf{s})$;
36: $\quad\quad$ **end if**
37: $\quad\quad$ **if** $\beta == \kappa$ **then**
38: $\quad\quad\quad$ $r_s = r_s + 1$;
39: $\quad\quad$ **end if**
40: $\quad\quad$ **if** $r_s == \Theta_r(\mathbf{s})$ **then**
41: $\quad\quad\quad$ goto step 45
42: $\quad\quad$ **end if**
43: $\quad$ **end if**
44: **end while**
45: **output:** $\mathbf{s}$. $\quad$ $\mathbf{s}$: output solution vector

A. Complexity

The complexity of the proposed Gibbs sampling based detector can be separated into three parts: i) computation of starting vectors, ii) computation of $\mathbf{y}^{MF}$ and $\mathbf{R}$, and iii) computations involved in sampling and updating process. In our simulations, we use MMSE output as the starting vector for the first restart, and random vectors for the subsequent restarts.The MMSE output needs the computation of $\left(\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I}_{N_t}\right)^{-1}\mathbf{H}^H\mathbf{y}$, whose complexity is $\mathcal{O}(N_t^3)$. Note that this operation includes the computations of $\mathbf{y}^{MF}$ and $\mathbf{R}$. For the sampling and updating process, in each iteration, i.e., for each choice of l and k, the algorithm needs to compute $\mathbf{x}^{(t)^H}\mathbf{r}_{i_l}$ and $\mathbf{x}^{(t)^H}\mathbf{r}_{j_k}$ which requires $\mathcal{O}(N_{rf})$ computations. The rest of the computations are $\mathcal{O}(1)$. The number of iterations before the algorithm terminates is found to be $\mathcal{O}(N_{rf}(N_t - N_{rf}))$ by computer simulations. Thus, the total number of computations involved in $iii)$ is $\mathcal{O}(N_{rf}^2(N_t - N_{rf}))$. Hence, the total complexity of the proposed algorithm for GSM detection is $\mathcal{O}(N_t^3) + \mathcal{O}(N_{rf}^2(N_t - N_{rf}))$. Figure 4 shows the complexity comparison between the proposed algorithm and brute force ML detection.

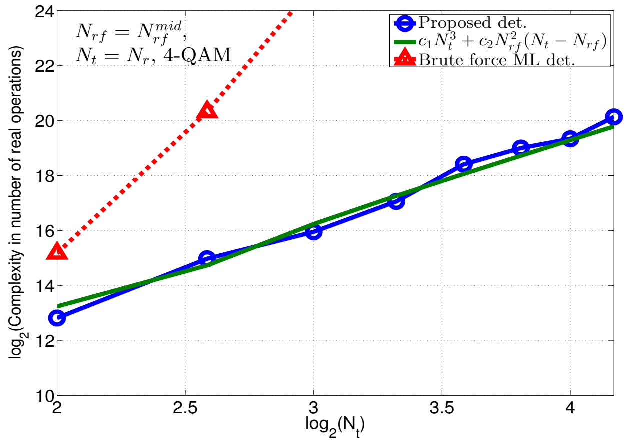

Fig. 4. Complexities of the proposed algorithm and brute force ML detection for N_t, $N_{rf} = N_{rf}^{mid}$, $N_r = N_t$ and $M = 4$ at a BER of 0.01.

B. Results and Discussions

In this section, we present simulation results of the uncoded BER performance achieved by the proposed Gibbs sampling based algorithm for detecting GSM signals. We compare this performance with ML performance (where ever ML complexity permits its simulation). We also compare the performance of GSM with the performance of traditional spatial multiplexing MIMO system. For notation purpose, a GSM system with N_t transmit antennas and N_{rf} transmit RF chains is referred to as "(N_t, N_{rf})-GSM" system. Also, we use the term "(N_t, N_{rf})-V-BLAST" system to refer the MIMO system with spatial multiplexing (where $N_t = N_{rf}$). The following parameters are used in the simulations of the proposed detection algorithm: $c_{min} = 10N_{rf}(N_t - N_{rf})$, $c_1 = 10mN_{rf}(N_t - N_{rf})$, MAX-ITR = $8N_tN_{rf}(N_t - N_{rf})\sqrt{M}$, MAX-RST= 20, $c_2 = 0.5(m+1)$.

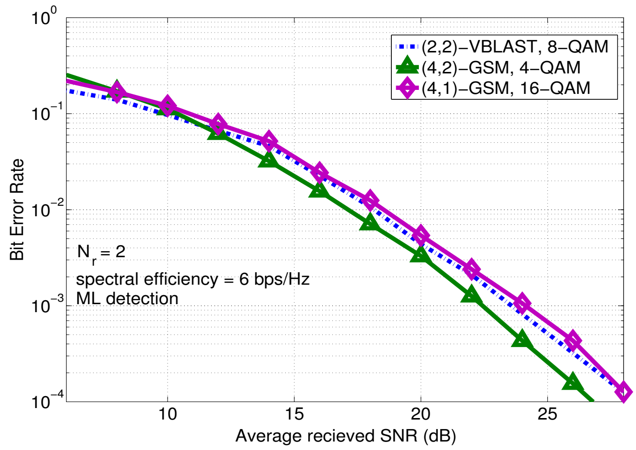

Fig. 5. BER comparison between $(4, 2)$-GSM, $(4, 1)$-GSM, and $(2, 2)$-V-BLAST systems with 6 bps/Hz, $N_r = 2$, and ML detection.

In Fig. 5, we show the BER comparison between $i)$ $(4, 2)$-GSM with 4-QAM, $ii)$ $(4, 1)$-GSM with 16-QAM, and $iii)$ $(2, 2)$-V-BLAST with 8-QAM, using $N_r = 2$. Note that in all the three systems, the modulation alphabets are chosen such that the spectral efficiency is the same 6 bps/Hz. Since the systems are small, brute force ML detection is used. It can be seen that (4,2)-GSM system performs better than (2,2)-V-BLAST system. That is, for the same spectral efficiency (6 bps/Hz) and $N_{rf} = 2$, GSM achieves better performance than V-BLAST (about 1 dB better performance at 0.01 BER). The additional resources used in GSM are not the transmit RF chains (which are expensive), but only the transmit antenna elements (which are not expensive). It can also be seen that even (4,1)-GSM performs almost the same as (2,2)-V-BLAST. This shows that GSM can save RF transmit chains without losing much performance compared to V-BLAST.

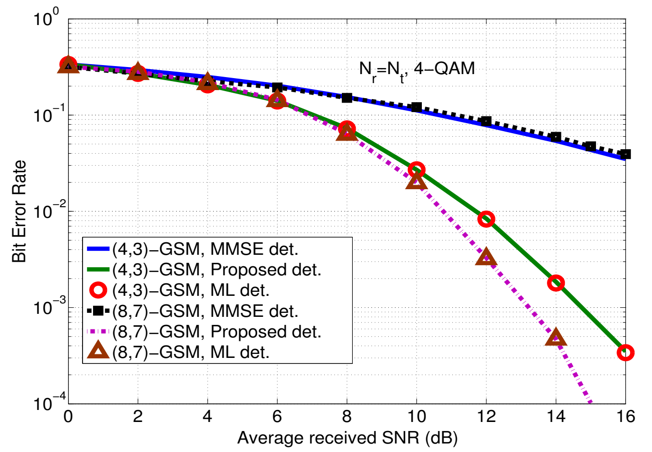

Fig. 6. BER comparison between different detectors for GSM – MMSE detector, proposed detector and ML detector – in (4,3)-GSM and (8,7)-GSM with $N_r = N_t$ and 4-QAM.

Figure 6 shows the BER performance of different detection schemes for GSM. (4,3)-GSM and (8,7)-GSM with $N_r = N_t$ and 4-QAM are considered. Note that both the N_{rf} choices correspond to N_{rf}^{opt}. Three detectors, namely, MMSE detector, proposed detector and ML detector, are considered. ML detection is done by enumerating all possible transmitted vectors and comparing their ML costs. It can be seen that MMSE detector yields very poor performance, but the proposed detector yields a performance which almost matches the ML detector performance. The proposed detector achieves this almost ML performance in just cubic complexity in N_t, whereas ML detection has exponential complexity in N_t.

In Fig. 7, we compare the performance of three systems achieving 24 bps/Hz: $i)$ (8,8)-V-BLAST with 8-QAM and ML detection using sphere Decoder (SD), $ii)$ (12,8)-GSM with 4-QAM and proposed detector, and $iii)$ (12,12)-V-BLAST with 4-QAM and ML detection using generalized sphere decoder (GSD) in [10]. All systems use $N_r = 8$. It can be seen that (12,8)-GSM performs close to (12,12)-V-BLAST performance, using much lesser transmit RF chains. It also shows that the (12,8)-GSM with proposed detector outperforms (8,8)-V-BLAST with SD employing same RF resources.

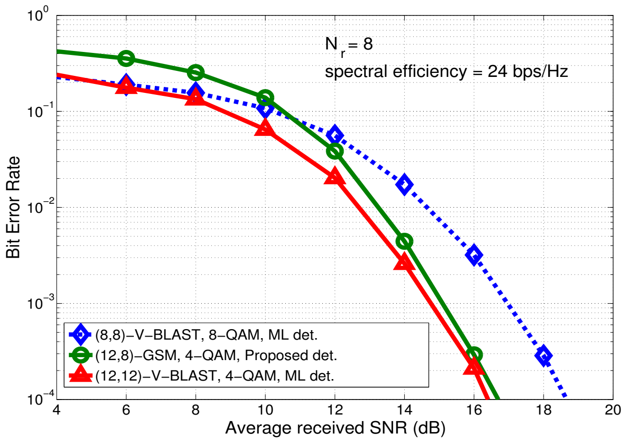

Fig. 7. BER comparison among three systems achieving 24 bps/Hz: i) (8,8)-V-BLAST with 8-QAM, ii) (12,8)-GSM with 4-QAM, iii) (12,12)-V-BLAST with 4-QAM. $N_r = 8$.

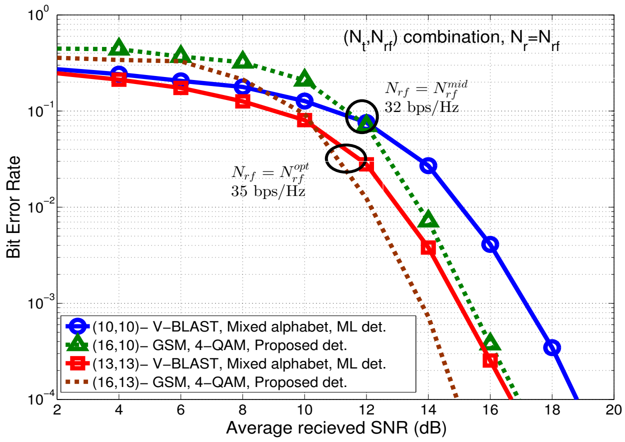

Fig. 8. BER comparison between GSM and V-BLAST schemes using same RF resources for $N_{rf} = N_{rf}^{mid}$, and $N_{rf} = N_{rf}^{opt}$, $N_r = N_{rf}$ to achieve 32 bps/Hz and 35 bps/Hz, respectively.

Figure 8 shows the BER comparison between GSM and V-BLAST schemes using same RF resources for $N_{rf} = N_{rf}^{mid}$, and $N_{rf} = N_{rf}^{opt}$, $N_r = N_{rf}$ to achieve 32 bps/Hz and 35 bps/Hz, respectively. GSM scheme uses $N_t = 16$ and 4-QAM whereas V-BLAST schemes use a mixture of 4-QAM, 8-QAM and 16-QAM to match the rate of GSM scheme. For $N_{rf} = N_{rf}^{mid} = 10$, target rate of 32, V-BLAST scheme uses 4 antennas transmitting 4-QAM symbols and 6 antennas transmitting 16-QAM symbols. For $N_{rf} = N_{rf}^{opt} = 13$, target rate of 32, V-BLAST scheme uses 4 antennas transmitting 4-QAM and 9 antennas transmitting 8-QAM. GSM schemes use the proposed detector whereas V-BLAST schemes use SD for detection. It can be seen that for both choices of N_{rf}, GSM outperforms the corresponding V-BLAST using same RF resources in medium to high SNR regime.

V. Conclusion

We made two new contributions in this paper regarding generalized spatial modulation, an emerging and promising modulation technique for multi-antenna communication. We first showed that, with optimum choice of (N_t, N_{rf}) combination, GSM can achieve increased rates and saving in number of transmit RF chains compared to spatial multiplexing (V-BLAST system). The gains are high for BPSK and 4-QAM; 9.375% gain in rate and 18.75% savings in RF chains are possible with $N_t = 16$ and 4-QAM. These gains diminish as the modulation order is increased. Then, motivated by the complexity involved in ML detection of GSM signals in large MIMO systems with large number of antennas, we proposed a Gibbs sampling based low complexity algorithm suited for GSM signal detection. The proposed algorithm was shown to yield impressive BER performance and complexity results. For the same spectral efficiency, and number of transmit RF chains, GSM with the proposed detector was shown to achieve better performance than spatial multiplexing with ML detection.

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